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*Supplement of*

## **Computational efficiency for the surface renewal method**

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## Supplementary Materials

### S.1 Despiking method using convolution (*despike.m*)

```
1 function [data_ds, ns, index] = despike(data, nw, sig, buffer, varargin)
2
3 % DESPIKE filters out spikes from a data vector using a Gaussian convolution.
4 %   INPUTS: data: nx1 vector
5 %           nw: sample size of "sliding window" used for convolution
6 %           sig: # of standard deviations considered significant (& removed)
7 %           buff: number of adjacent samples to remove
8 %           varargin{1}: 'interp' option interpolates over nans (cubic)
9 %           varargin{2}: timestamps for data
10 %           varargin{3}: (optional) passes interpolation method ('cubic', etc.)
11 %                       w/o 3rd varargin, reverts to default 'linear'
12 %   OUTPUTS:           data_ds: despiked data
13 %           ns: number of (removed) spikes
14 %           index: logical vector with TRUE = spike
15 %   REQUIRES: setnan.m (function that sets flagged values to NaN for index with buffer)
16 %
17 % Jason Kelley NEWAg Lab OSU
18 % Written 29 FEB 2016
19 % Last modified 27JUL2016 (Jewell)
20
21 % check for pre-existing non-number points (errors) and interpolate during despiking
22 nn = isnan(data);
23 xs = 1:length(data);
24 if nnz(isnan(data))>0
25     data = interp1(xs(~nn),data(~nn),xs,'nearest');
26 end
27
28 w = gausswin(nw,1);           % Matlab function generates Gaussian filter
29 sw = sum(w);                 % total area under window function
30 w = w./sw;                  % normalize window
31
32 filter = conv(data,w,'same'); % filtered data using _convolution_
33
34 ii = true(length(data),1);   % eliminate edge bias: index to original data
35     hw = ceil(length(w)/2);   % data to ignore is 50% of filter window size
36     ii(1:hw) = false;
37     ii(end-hw:end) = false;
38
39 mstd = mw_std(data,nw).*sig; % significance level in terms of standard deviation
40 fluc = zeros(length(data),1); % normalize fluctuations by absolute value
41 % fluc(ii) = (data(ii)-filter(ii))./data(ii); % alternate def for significance
```

```

42 fluc(ii) = data(ii)-filter(ii); % spikes are fluctuations exceeding signif. threshold
43 index = abs(fluc)>mstd;          % index the spikes
44 ns = nnz(index);                % count the spikes
45
46 data_ds = setnan(data,index,buffer); % set spikes and adjacent values to NaN
47 if nnz(nn)>0                    % reset pre-existing NaNs in data vector
48     data_ds(nn) = NaN;
49     data(nn) = NaN;
50 end
51 fprintf(' %i spikes removed ; ',ns)
52 fprintf('%3.3f%% of data NaN'\ed\n',(sum(isnan(data_ds))/length(data))*100)
53
54 % optional plotting for visual inspection (not included here for brevity)
55
56 % optional interpolation between nan'd points using timestamp for ordinates
57 if nargin > 4 && strcmp(varargin{1},'interp')
58     ind = isnan(data_ds);
59     switch nargin
60         case 5
61             time = 1:length(data);
62         case 6;
63             time = varargin{2};
64     end % switch
65     if nargin == 7
66         data_ds(ind) = interp1(time(~ind),data_ds(~ind),time(ind),varargin{3});
67     else
68         data_ds(ind) = interp1(time(~ind),data_ds(~ind),time(ind));
69     end
70 end % end interp option
71
72 end % end main despiking function
73
74 % sub function for moving window standard deviation
75 function mstd = mw_std(signal,w)
76     % adapted from http://matlabtricks.com/post-20/
77     % "calculate-standard-deviation-case-of-sliding-window"
78
79     N = length(signal);
80     n = conv(ones(N,1),ones(w,1),'same'); % counts no. elements in each window
81     s = conv(signal, ones(1, w), 'same'); % s vector
82     q = signal .^ 2;
83     q = conv(q, ones(1, w), 'same');      % q vector
84     mstd = (q - s.^2./n)./(n-1);          % variance of moving window
85     mstd = mstd.^0.5;                    % standard deviation
86 end % moving window mw_std sub-function
87
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```

## S.2 Structure function calculation using convolution (*strfnc.m*)

```

1 function [S, max_i, fluxdir] = strfnc( trace, freq, maxlag )
2 %STRFNC Structure function calculation (following Van Atta, 1977)
3 % Last modified 09mar16
4 % INPUTS 'trace' data to analyze (Nx1 vector array)
5 % 'freq' sampling frequency (Hz)
6 % 'maxlag' maximum lag time, (seconds)
7 % OUTPUTS 'S' structure functions  $S(r)^n$  and lag r (in seconds) for rows
8 % only calculates 2nd 3rd 5th order to save memory,
9 % column order corresponds to SFs, also calculates  $-S^3(r)/r$ 
10 % format: [r  $S^2(r)$   $S^3(r)$   $-S^3(r)/r$   $S^5(r)$ ]
11 % 'max_i' relative location (iteration) at which  $S^3(r)/r$  is maximum
12 % 'fluxdir' sign of  $S^3(r)/r$ , used to determine vertical flux direction
13
14 m = length(trace);
15 lags = 1:maxlag*freq; % vector of lags from 1 to maxlag
16 rn = length(lags);
17 S = zeros(rn,5); % initialize array S to store str funcs
18
19 % method by nested iterative loops -----
20 for j = lags
21     r = lags(j); early = trace(1:end-r); later = trace(r+1:end);
22     diffs = later-early;
23     for i = [2 3 5]
24         S(j,i) = sum((diffs).^i)/(m-r);
25     end %structure functions at lag j
26 end % lags j
27 S(:,6) = lags./freq;
28
29 % method using convolution -----
30 filt = [ones(1,rn); -eye(rn)]; % singleton comparators at 1:rn lags e.g. [1 0 0 -1]
31 cT = conv2(trace,filt); % conv filter with trace to get all lags
32 cT = cT(rn+1:end-rn,:); % trim edges. 'same' does not work as with conv1.m
33 cTp(:,:,1) = power(cT,ones(m-rn,rn).*2); % for second order SF
34 cTp(:,:,2) = cTp(:,:,1).*cT; % third order SF
35 cTp(:,:,3) = cTp(:,:,1).*cTp(:,:,2); % fifth order SF
36 w = (m-rn-(1:rn))-1; % unbiased weighting vector 1/(N-1)
37 S(:,2) = sum(cTp(:,:,1),1)./w; % column order corresponds to SF order
38 S(:,3) = sum(cTp(:,:,2),1)./w; % i.e. 3rd order SF is S(:,3)
39 S(:,5) = sum(cTp(:,:,3),1)./w;
40 S(:,1) = lags./freq; % sample N -> dt
41 S(:,4) = -S(:,3)./S(:,1); % ratio used to detect lag max'ing S3r
42 % identify time lag at which  $S_3(r)/r$  is maximized, flux direction by +/-  $S^3(r)/r$ 
43 fluxdir = sign(nanmean(S(:,4),1));
44 [~,max_i] = max(fluxdir.*(S(:,4)));
45
46 end %function
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```

### S.3 Cardanos Method for roots of depressed cubic polynomial (*cardanos.m*)

```
1 function [REALrts, ALLrts] = cardanos(p,q)
2 % CARDANOS(p,q) root finding algorithm for depressed cubic polynomial with real
3   valued p and q. This has reduced functionality of CardanRoots.m for limited
4   cases required for the surface renewal method. Vectors p and q (from structure
5   functions) used to determine ramp Amplitudes. polynomial should be of form  $A^3 +$ 
6    $p*A + q = 0$ , p & q real valued
7 % RETURNS REALrts: only positive real valued solutions, complex and negative
8   solutions replaced with NaN
9   ALLrts: includes positive and negatively valued and complex solutions
10  % References
11 %refs:<ahref="matlab:web('https://en.wikipedia.org/wiki/Cubic_function#Cardano.27s_me
12   thod','-browser')">Wiki</a>
13 %<ahref="matlab:web('https://www.mathworks.com/matlabcentral/newsreader/view_thre
14   ad/165013?requestedDomain=www.mathworks.com','-browser')">Source Code</a>
15
16   D = q.^2 + (4/27)*p.^3; % the discriminant
17   Dneg = D<0;
18   Dpos = ~Dneg;
19       n = size(D,1);
20       rts = zeros(n, 3); % initialization
21       a = -q(Dneg);      b = sqrt(-D(Dneg));
22       r2 = a.^2-D(Dneg);
23       rho = (4^(1/3))*exp(log(r2)/6);
24       theta = atan2(b,a)/3;
25       a = rho.*cos(theta);  b = rho.*sin(theta);
26       S1 = a;
27       x = (-0.5)*a;        y = (sqrt(3)/2)*b;
28       S2 = x-y;           S3 = x+y;
29
30   rts(Dneg,1:3) = [S1 S2 S3];
31       E = sqrt(D(Dpos));
32       u3 = (-q(Dpos)+E)/2;
33       v3 = (-q(Dpos)-E)/2;
34       u = sign(u3).*exp(log(abs(u3))/3); % Cubic roots of u3 and v3
35       v = sign(v3).*exp(log(abs(v3))/3);
36       S1 = u+v;
37       j = complex(-0.5,sqrt(3)/2); % Complex solutions
38       j2 = complex(-0.5,-sqrt(3)/2);
39       S2 = j*u+j2*v;      S3 = conj(S2);
40
41   rts(Dpos,1:3) = [S1 S2 S3];
42   ALLrts = rts;
43   rts(imag(rts)~=0) = NaN;
44   REALrts = rts;
45   end
46 Published with MATLAB® R2016a
47
```