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*Supplement of*

## **Concept for an electrostatic focusing device for continuous ambient pressure aerosol concentration**

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### Derivation of Dimensionless term, $\kappa$

We begin with a total force balance for a particle in a charged electric field:

$$m_p \frac{d\mathbf{v}}{dt} = \frac{3\pi\mu D_p}{C_c} (\mathbf{u} - \mathbf{v}) + q\mathbf{E}$$

$$\frac{\rho\pi D_p^3}{6} \frac{d\mathbf{v}}{dt} = \frac{3\pi\mu D_p}{C_c} (\mathbf{u} - \mathbf{v}) + q\mathbf{E}$$

The equation is rendered dimensionless. We scale  $\mathbf{u}$  and  $\mathbf{v}$  to the initial gas velocity entering the electrostatic focusing region,  $u_0$ . Time is scaled to a dimensionless value  $\tau$ , set here to be the time for gas to travel straight across the length of the gap between the inlet orifice and collection probe of the virtual impactor geometry,  $L$ . The electric field,  $\mathbf{E}$ , is scaled to some dimensionless value  $E_0$ , that will depend on the electric field being implemented over the impactor geometry.

$$\frac{\rho\pi D_p^3 u_0}{6} \frac{d\hat{\mathbf{v}}}{d\hat{t}} = \frac{3\pi\mu D_p}{C_c} u_0 (\hat{\mathbf{u}} - \hat{\mathbf{v}}) + qE_0 \hat{\mathbf{E}}$$

Rearrangement allows us to isolate a Reynolds number and neglect viscous forces, assuming high gas velocities.

$$\frac{\pi D_p^3}{6L} \frac{d\hat{\mathbf{v}}}{d\hat{t}} = \frac{3\pi\mu D_p}{\rho C_c u_0} (\hat{\mathbf{u}} - \hat{\mathbf{v}}) + \frac{qE_0}{\rho u_0^2} \hat{\mathbf{E}}$$

$$\frac{D_p C_c}{18L} \frac{d\hat{\mathbf{v}}}{d\hat{t}} = \frac{\mu}{\rho D_p u_0} (\hat{\mathbf{u}} - \hat{\mathbf{v}}) + \frac{C_c q E_0}{3\pi \rho D_p^2 u_0^2} \hat{\mathbf{E}}$$

$$\frac{D_p C_c}{18L} \frac{d\hat{\mathbf{v}}}{d\hat{t}} \approx \frac{C_c q E_0}{3\pi \rho D_p^2 u_0^2} \hat{\mathbf{E}}$$

We can now derive a term for  $\kappa$ , balancing out the necessary electrical force-to-inertia balance as a function of particle size and electric field strength.

$$\frac{d\hat{\mathbf{v}}}{d\hat{t}} \approx \kappa \hat{\mathbf{E}}$$

$$\kappa = \frac{Z \cdot E_0}{St \cdot u_0} = \frac{6LqE_0}{\rho\pi D_p^3 u_0^2}$$

### ***Comparison of Voltage fields***

As mentioned in the main text, it is expected that there is a significant degree of electrical distortion within the focusing region of our electrostatic focuser. Here, we provide a discussion of the possible effects of such distortion.

We expect that the collection probe will be the primary source of distortion, due to its proximity to the charged outlet lens. Assuming a common characteristic electric field of the applied voltage divided by the distance between its two constituent elements (between the outlet lens and the inlet lens or collection probe for the cylindrical lens field and distortion field, respectively), we estimate that their relative values of deflective force on aerosol particles to be roughly equal in magnitude. We took the sum of the potential fields, and estimate the gradient of this composite field to be the effective electric field within the charged region.

The effect of distortion due to the collection probe was estimated by assuming its fringe field with the outlet lens to behave similarly to a finite parallel plate capacitor, of infinite width and finite length. Charge density and electrical potential field functions have been estimated by Parker (2002) and Pillai (1970). For purposes of these calculations, we assumed the collection probe to exist at its external diameter only ( $\hat{r} = 0.317$ ) with negligible thickness.

The composite generated electric field generated by adding the fields of Parker (2002) and Bertram (1942) was then used in the theoretical model used to predict enrichment. Using the flow parameters of our experimental system, negligible enrichment was predicted for particles between 30-200nm in size.