Supplement of

# Characterization of the planar differential mobility analyzer (DMA P5): resolving power, transmission efficiency and its application to atmospheric relevant cluster measurements 

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## S1: Theoretical derivation of the operation principle of planar DMA



Figure S1 operation principle of planar DMA
$V_{D M A}$ : Voltage between the two electrodes;
$h$ : Distance between the two electrodes;
The electric field between the two electrodes: $E=\frac{V_{D M A}}{h}$
During the scanning period of planar DMA, the electric field is applied on the z-direction, a laminar particle-free sheath flow is circulating thorough the capacitor along the x-direction at the flow rate of $Q_{s h}$, and the aerosol flow is fed into the capacitor thorough input slit located at the
top electrode at the flow rate of $Q_{a}$. The direction of aerosol flow is parallel to the electric field and perpendicular to the sheath flow.
The particle velocity in x -direction is given as: $u_{x}(z)=\frac{d_{x}}{d_{t}}$
The equation can be transformed as $d_{x}=u_{x}(\mathrm{z}) \cdot d_{t}$
The particle velocity in z -direction is given as:

$$
\begin{equation*}
u_{z}=\frac{d z}{d t}=\frac{Q_{a}}{s_{s l i t}}+E \cdot Z_{p}=\frac{Q_{a}}{s_{s l i t}}+\frac{V_{D M A} \cdot Z_{p}}{h} \tag{S4}
\end{equation*}
$$

where $Z_{p}$ represent the electric mobility of the particle and $S_{s l i t}$ represent the cross-section area of inlet slit.
Since $\frac{Q_{a}}{S_{s l i t}}$ is much smaller than $\frac{V_{D M A} \cdot Z_{p}}{h}$, equation (S4) can be written as: $u_{z}=\frac{d z}{d t}=\frac{V_{D M A} \cdot Z_{p}}{h}$
Equation (S5) can be transformed as $d_{t}=\frac{h}{V_{D M A} \cdot Z_{p}} \cdot d_{z}$
Combined equation (S3) and (S6), we can get the relation that

$$
\begin{equation*}
d_{x}=\frac{u_{x}(z) \cdot h}{V_{D M A} \cdot Z_{p}} \cdot d_{z} \tag{S7}
\end{equation*}
$$

Integrating equation (S7), we can get the equation that $\int_{0}^{L} d_{x}=\frac{h}{V_{D M A} \cdot Z_{p}} \int_{0}^{h} u_{x}(z) d_{z}$
where L represent the distance between the inlet slit and the monodispersed particle exit.
Assuming that $\bar{u}_{x}(z)=\frac{Q_{s h}}{w \cdot h}$, where w represents the width of the capacitor and $\mathrm{w} \cdot \mathrm{h}$ represent the cross-section area of the capacitor, the integral equation can be transformed as $\int_{0}^{L} d_{x}=L=\frac{Q_{s h}}{w \cdot h} \cdot \frac{h}{V_{D M A} \cdot Z_{p}} \int_{0}^{h} d_{z}=\frac{Q_{s h \cdot h}}{w \cdot V_{D M A} \cdot Z_{p}}$

Equation (S9) can be written as $Z_{p}=\frac{Q_{s h \cdot h}}{w \cdot U \cdot L}$, and combined with the assumption that $Q_{s h}=\bar{u}_{x}(z) \cdot w \cdot h$, we can get the expression of

$$
\begin{equation*}
Z_{p}=\frac{\bar{u}_{x}(z) \cdot h^{2}}{V_{D M A} \cdot L} \tag{S10}
\end{equation*}
$$

In equation (S10) $\bar{u}_{x}(z)$ represent the average speed of sheath flow along $z$-direction; L and h represent the horizontal distance of inlet the exit and between the two electrodes, respectively; $V_{D M A}$ represent the voltage applied between the two electrodes.

Account for the planar DMA P5, the sheath flow speed is uniform along z-direction $\left(\bar{u}_{x}(z)=u_{x}\right)$, the physical dimension of L and h are 40 mm and 10 mm , respectively. The relation of the electric mobility $\left(\mathrm{Z}_{\mathrm{p}}\right)$ and the voltage applied by planar DMA P5 ( $\mathrm{V}_{\text {DMA }}$ ) can be expressed as:

$$
\begin{equation*}
Z_{p}=\frac{u_{x} \cdot h^{2}}{V_{D M A} \cdot L} \tag{S11}
\end{equation*}
$$

## S2: Mobility diameter calculation

Calculation of diameter from mobility (Tammet, 1995; Wiedensohler et al., 2012)

$$
\begin{gather*}
Z_{p}=\frac{n e C_{c}\left(D_{p}\right)}{3 \pi \mu D_{p}} \\
C_{c}=1+\frac{2 \lambda}{D_{p}}\left(1.165+0.483 \exp \left(-0.997 \frac{D_{p} 2 \lambda}{2 \lambda}\right)\right)  \tag{S13}\\
\lambda=\lambda_{0}\left(\frac{T}{T_{0}}\right)^{2}\left(\frac{P_{0}}{P}\right)\left(\frac{T_{0}+110.4 K}{T+110.4 K}\right)
\end{gather*}
$$

$$
\begin{equation*}
\mu=\mu_{0}\left(\frac{T}{T_{0}}\right)^{3 / 2}\left(\frac{T_{0}+110.4 K}{T+110.4 K}\right) \tag{S15}
\end{equation*}
$$

$n$ is Number of elementary charges on the particle; e is Elementary charge $=1.60 \times 10^{-19} \mathrm{C} ; \mathrm{C}_{\mathrm{c}}$ is Cunningham slip correction; $\mathrm{D}_{\mathrm{p}}$ is Mobility diameter; $\mu$ is Dynamic gas viscosity; $\lambda$ is Mean free path of gas; T is Temperature, and is set as 298.15 K ; P is Pressure, assuming P equals to $1 \mathrm{~atm} ; \mathrm{T}_{0}$ is Reference temperature $(296.15 \mathrm{~K}) ; \mathrm{P}_{0}$ is the Reference pressure $=1 \mathrm{~atm}=101325 \mathrm{~Pa} ; \lambda_{0}$ is Mean free path at 296.15 K and $1 \mathrm{~atm}=67.3$ $\times 10^{-9} \mathrm{~m} ; \mu_{0}$ is the gas viscosity at 296.15 K and latm, which is equals to $1.83245 \times 10^{-5} \mathrm{~kg} \mathrm{~m}^{-1} \mathrm{~s}^{-1}$

## S3: Theoretical calculation of the resolving power of planar DMA

Assuming that variances are additive, the resolving power for a planar DMA is given by the following formula:

$$
\begin{equation*}
R^{-2}=\left(\frac{\Delta Z_{F W H M}}{Z}\right)^{2}=\left(\frac{W}{L_{\text {slit }}} \frac{Q_{\text {in }}+Q_{o u t}}{Q_{c}+Q_{m}}\right)^{2}+\left(\frac{2 \sigma \sqrt{2 \ln 2}}{L} \frac{\sqrt{L^{2}+h^{2}}}{h}\right)^{2} \tag{S16}
\end{equation*}
$$

Where $W$ is the width of the DMA channel at the outlet slit ( 14.9 mm ), $L_{s l i t}$ is the monodisperse sampling slit length ( 6.5 mm ), $Q_{\text {out }}$ is the monodisperse sampling flow rate, $Q_{i n}$ is the polydisperse aerosol flow rate, $Q_{c}$ is the flow rate at the DMA sheath gas inlet, $Q_{m}$ is the flow rate at the DMA sheath gas outlet, $2 \sigma(2 \ln 2)^{1 / 2}$ is the width at half maximum of a Gaussian distribution, $L$ is the separation channel length, $h$ is the normal distance between electrodes and $\left(\left(L^{2}+h^{2}\right)^{1 / 2}\right) / h$ is the projection of the Gaussian distribution width into the outlet electrode. In this planar DMA, the polydisperse sample is electrically pushed through the inlet slit; indeed, a small counterflow ( $0.5-1 \mathrm{~L} / \mathrm{min}$ in this work) is exhausted through the inlet slit in order to prevent droplets from entering the DMA. Since $Q_{i n}$ and $Q_{o u t}$ are much lower than $Q_{c}$ and $Q_{m}$, the following simplification
may be assumed with little error: $Q_{c} \sim Q_{m} \rightarrow Q_{c}=Q_{m}=Q$, where $Q$ is the sheath gas flow rate through the separation channel. Hereafter only $Q$ will be considered.

The variance of a Gaussian distribution $\sigma^{2}=2 D t$, is controlled by the ion time of residence in the DMA $t$ and the diffusion coefficient of the ions in the sheath gas $D$. The Einstein relation ( $D=Z k T / N_{e}$ ), relates $D$ with the electrical mobility $Z$, the Boltzmann's constant $k$, the gas absolute temperature $T$ and the net charge on the particle $N e$. The time of residence $t$ in the planar DMA can be expressed as follows:

$$
\begin{equation*}
t=\frac{L}{U}=\left(\frac{h}{Z E L}\right) L=\frac{h^{2}}{Z V_{D M A}} \tag{S17}
\end{equation*}
$$

Where $E$ is the electric field between the DMA electrodes and $V_{D M A}$ is the voltage between the electrodes. So $\sigma^{2}$ can be expressed as:

$$
\begin{equation*}
\sigma^{2}=2 D t=2 \frac{Z k T}{N_{e}} t=2 \frac{Z k T}{N_{e}} \frac{h^{2}}{Z V_{D M A}}=\frac{2 k T h^{2}}{V_{D M A} N_{e}} \tag{S18}
\end{equation*}
$$

And $Q$ can be expressed as a function of the Reynolds number:

$$
\begin{equation*}
R e=\frac{U h}{v}=\frac{Q}{W v} \tag{S19}
\end{equation*}
$$

Where $U$ is the sheath gas velocity in the DMA channel and $v$ is the kinematic viscosity of the gas. Then R can be rewritten as:

$$
\begin{equation*}
R^{-1}=\sqrt{\left[\left(\frac{Q_{\text {in }}+Q_{\text {out }}}{L_{\text {slit }} 2 \operatorname{Rev} v}\right)^{2}+\frac{16 \ln 2 k T}{V_{D M A} N_{e}}\left(1+\left[\frac{h}{L}\right]^{2}\right)\right]} \tag{S20}
\end{equation*}
$$

Convective diffusion problems at large Reynolds numbers are well known to be governed by the Peclet number $P e$ defined as:

$$
\begin{equation*}
P e=R e \frac{v}{D}=\frac{U h}{D}=\frac{Z V_{D M A} L}{h D}=\frac{V_{D M A} L N e}{h k T} \tag{S21}
\end{equation*}
$$

Therefore, R can be expressed as a function of $P e$ number:

$$
\begin{equation*}
R^{-1}=\sqrt{\left[\left(\frac{Q_{\text {in }}+Q_{\text {out }}}{L_{\text {slit }} 2 \text { Rev }}\right)^{2}+\frac{16 \ln 2}{P e}\left(\frac{L}{h}+\frac{h}{L}\right) \sqrt{\frac{16 \ln 2}{P e}\left(\frac{L}{h}+\frac{h}{L}\right)}\right]} \tag{S22}
\end{equation*}
$$

## S4: Supplementary figures and tables



Figure S2 The relation of blower control voltage with sheath flow rate and corresponding DMA P5 sizing range


Figure $\mathbf{S 3}$ The positive ion mobility spectrum of $\mathbf{T H A B}$ under suction mode ( $\mathrm{V}_{\mathrm{blower}}=\mathbf{5 V}, \mathrm{Q}_{\mathrm{in}}=\mathbf{5 L} / \mathrm{min}, Q_{\text {out }}=\mathbf{1 . 5 L} / \mathrm{min}$ ) with different solution concentrations


Figure S4 THA ${ }^{+}$Signal intensity normalized by monodispersed flow rate


Figure S5 Positive ion mobility spectrum of electrospraied THAB solution obtained from HalfMini + Lynx E12


Figure S6 Schematic diagram of tandem DMA system


Figure S7 The distribution of transmission efficiency of the DMA P5 when classifying THA ${ }^{+}$_under different sheath flow rate, with $Q_{\text {out }}=2.5 \mathrm{~L} / \mathrm{min}$


Figure S8 The distribution of transmission efficiency of the DMA P5 when classifying THA ${ }^{+}$_under different sheath flow rate, with $Q_{\text {out }}=3.0 \mathrm{~L} / \mathrm{min}$


Figure S9 Ion mobility distribution of the main identified clusters. Table S1 Inverse mobilities $1 / \mathbf{Z}\left(\mathrm{V} \mathrm{s}^{\prime} / \mathrm{cm}^{2}\right)$ for four tetra-alkyl ammonium positive ions

Table S1 Inverse mobilities $1 / \mathbf{Z}\left(\mathrm{V} \mathrm{s}^{\prime} / \mathrm{cm}^{2}\right)$ for four tetra-alkyl ammonium positive ions

| Peak ${ }^{+}$ | TMAI |  | TBAI |  | THAB |  | TDAB |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | this work | Ude et al. $2005$ | this work | Ude et al. $2005$ | this work | Ude et al. 2005 | this work | Ude et al. $2005$ |
| $\mathbf{A}^{+}$ | 0.458 | 0.459 | 0.723 | 0.718 | 1.03 | 1.03 | 1.269 | 1.285 |
| $\mathbf{A}^{+}(\mathbf{A B})$ | 0.667 | 0.677 | 1.164 | 1.153 | 1.533 | 1.529 | 1.811 | 1.846 |
| $\mathbf{A}^{+}(\mathbf{A B})_{2}$ | - |  | 1.475 | 1.450 | 1.898 | 1.893 | - |  |

Table S2 Inverse mobilities $\mathbf{1 / Z}\left(\mathbf{V ~ s} / \mathbf{c m}^{2}\right)$ for four tetra-alkyl ammonium negative ions

| Peak $^{-}$ | TMAI | TBAI | THAB | TDAB |
| :---: | :---: | :---: | :---: | :---: |
| B $^{-}$ | 0.423 | 0.422 | 0.436 | 0.436 |

## References

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