



*Supplement of*

## **Multiple-scattering effects on single-wavelength lidar sounding of multi-layered clouds**

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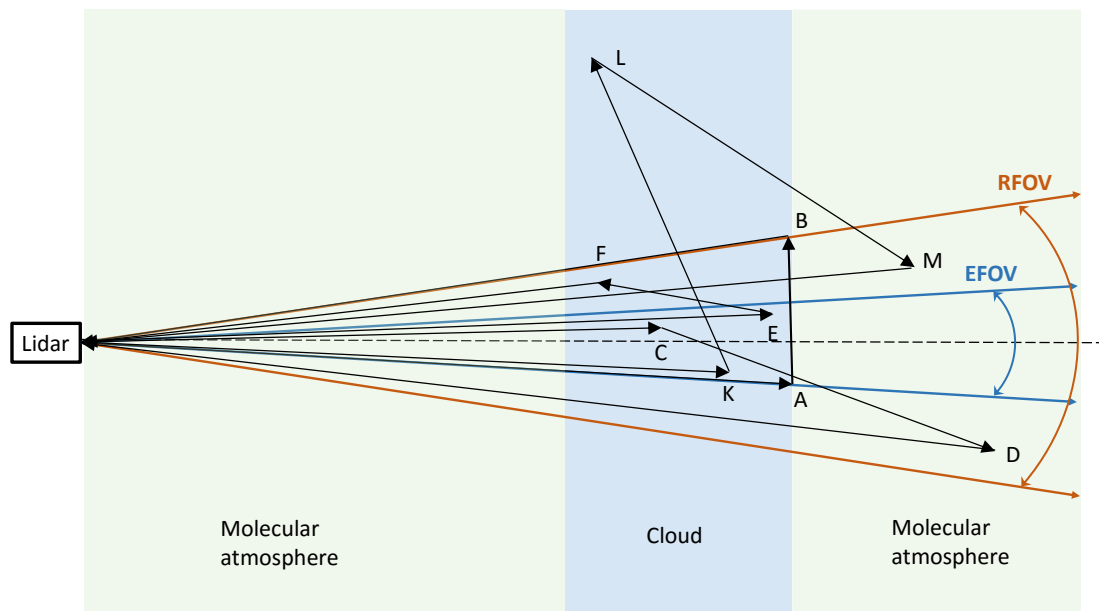
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Figure S1 shows some example of scattering events for cases of the double and the triple scattering approximations. It has to be underlined the following. The first scattering must always be within the volume bounded by the emitter field of view (EFOV). When  $n$  scattering orders are dealt with, the  $n^{th}$  scattering must be within the volume bounded by the receiver field of view (RFOV). Generally, it is not necessary that intermediate scatterings, i.e. 2, 3, ...  $n - 1$ , are within the RFOV. The following examples are shown in Fig. S1.

The double scattering: (i) “lidar  $\rightarrow$  A  $\rightarrow$  B  $\rightarrow$  lidar” is one of the cases when the round-trip distance of a double-scattered photon has the maximal gain; (ii) “lidar  $\rightarrow$  C  $\rightarrow$  D  $\rightarrow$  lidar” is the case when the first scattering is within the cloud and the second one is within the molecular atmosphere; (iii) “lidar  $\rightarrow$  E  $\rightarrow$  F  $\rightarrow$  lidar” is the case when both scattering are within the cloud.

The triple scattering: “lidar  $\rightarrow$  K  $\rightarrow$  L  $\rightarrow$  M  $\rightarrow$  lidar” is a case when the second scattering is outside the RFOV.



**Figure S1.** Examples of scattering events (see the text); EFOV is the emitter field of view, RFOV is the receiver field of view.

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The ground-based lidar in Section 3 has the EFOV 0.14 mrad, the distance to the cloud is 8 km, the cloud depth is 3km, i.e. the distance to the far edge of the cloud is 11 km. If the RFOV is 1.0 mrad (Fig. 2), the maximal round-trip distance of the double scattering, i.e. “lidar  $\rightarrow$  A  $\rightarrow$  B  $\rightarrow$  lidar” is 22.00628 km. A double-scattered photon can gain  $(22. - 22.00628)/2=0.00314$  km Therefore, only the range  $d \in ]3., 3.02]$  km can be

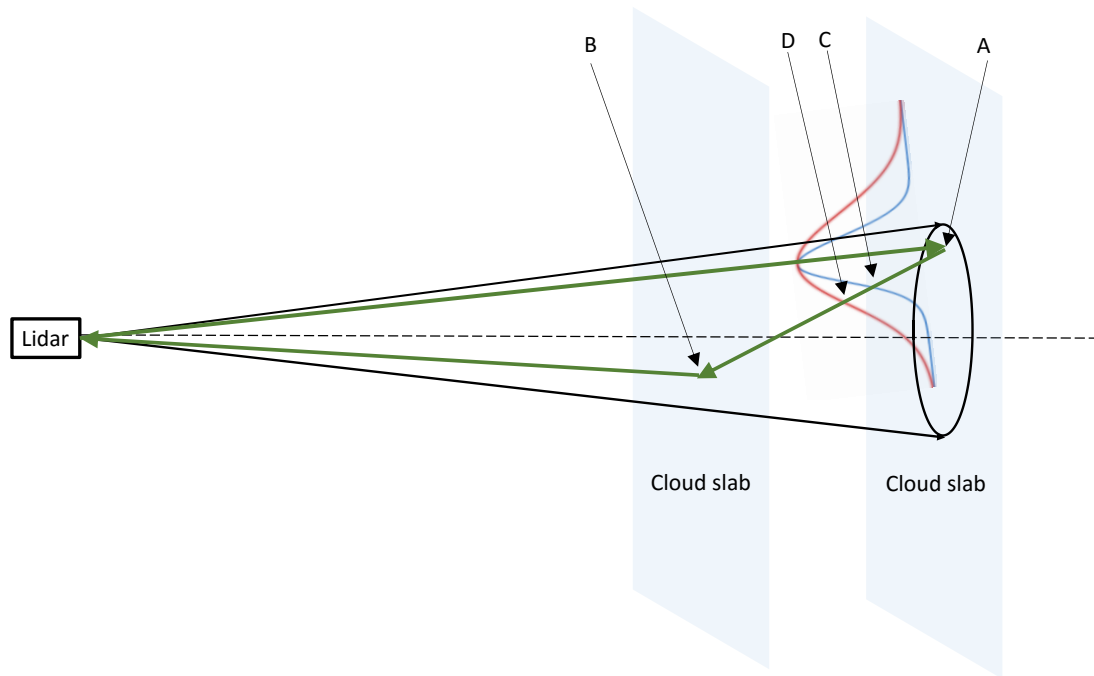
somewhat affected by the pulse stretching. (The range gate of our Monte-Carlo simulations is 0.02 km). If the RFOV is 110.0 mrad (Figs. 3c and 3d), the maximal round-trip distance of the double scattering, i.e. “lidar  $\rightarrow$  A  $\rightarrow$  B  $\rightarrow$  lidar” is 22.623 km. A double-scattered photon can gain 0.3115 km. Therefore, the range  $d \in ]3., 3.32]$  km can be somewhat affected by the pulse stretching.

The objective of Fig. S2 is to provide some intuitive understanding of the idea developed in the work by Eloranta (Eloranta, 1998). Namely, one of the key parameters that govern MS effects on lidar signals is a weighted average of a phase function near the backscatter direction (see Eq. (10) of Eloranta, (1998)). That parameter depends on the width of the backward peak of the phase function (other factors being the same). It is considered using as multiplying coefficients the ratios  $\mathcal{P}_n(\pi, h)/\mathcal{P}_1(\pi, h)$  in the Eloranta model, where  $n$  is the order of scattering.

In what follows, we provide an answer to the question why the ratio  $\mathcal{P}_2(\pi, h)/\mathcal{P}_1(\pi, h)$  depends on the phase function properties around the backward direction. (Our explanation corresponds to Fig. 3a and 3b of Section 3.1.1.) In Fig. S2, it is shown a double-scattering case “lidar  $\rightarrow$  A  $\rightarrow$  B  $\rightarrow$  lidar” when both scattering are within a cloud. The cloud is shown by two slabs. The blue curve illustrates the backscatter peak of the phase function  $f_{ch1}(\theta)$ , the red curve illustrates the backscatter peak of the phase function  $f_{ch2}(\theta)$ . The illustrations only show that  $f_{ch2}(\theta)$  is wider without taking into account the real width of the functions. The both functions have the maximum at  $\theta = \pi$  (the backward direction). The value of the maximum is the same for the both function because the ratio  $\mathcal{P}_2(\pi, h)/\mathcal{P}_1(\pi, h)$  is normalized by the  $\mathcal{P}_1(\pi, h)$ , i.e. by the value of the corresponding phase function at  $\theta = \pi$ .

After the first scattering, a photon goes from “A” to “B”. The intersections of the line “AB” with the curves, i.e. the points “C” and “D”, illustrate the values of the ratio  $\mathcal{P}_2(\pi, h)/\mathcal{P}_1(\pi, h)$  for the phase functions  $f_{ch1}(\theta)$  and  $f_{ch2}(\theta)$ , respectively. One can make a set of such illustrations, which take into account different possibilities/geometries of the double scattering within the cloud. The conclusion is always the same, i.e. the ratio  $\mathcal{P}_2(\pi, h)/\mathcal{P}_1(\pi, h)$  for the phase functions  $f_{ch1}(\theta)$  is lower than for  $f_{ch2}(\theta)$ . The ratios are equal only in the cases when the first scattering is at  $\theta = \pi$ , i.e. exactly at the backward direction.

In our opinion, the illustration we showed provides an insight into the property that MS effects on lidar signals depends on a weighted average of a phase function near the backscatter direction. At the same time, it is only an insight. The physical and mathematical grounds can be found in (Eloranta, 1998).



**Figure S2.** Double scattering within a cloud (see the text); The blue curve illustrates the backscatter peak of the phase function  $f_{ch1}(\theta)$ , the red curve illustrates the backscatter peak of the phase function  $f_{ch2}(\theta)$ .

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## References

Eloranta, E.: Practical model for the calculation of multiply scattered lidar returns, Appl. Opt., 37, 2464–2472, <https://doi.org/10.1364/AO.37.002464>, 1998.