Response to Anonymous Referee 2

We appreciate the reviewer's comments and suggestions. The review begins with some background discussion, and in the later paragraphs, the reviewer provides some specific comments. Here is our response to the reviewer's comments.

1. In the third paragraph the reviewer states: ***"However the problem with this method is the dependence of the AKs on the a priori covariance in the two retrievals..."***

   Regarding the influence of the a-priori covariance–In an optimal estimation framework, it is relatively simple to determine the impact of the a-priori covariance on the retrieval. In an optimal estimation framework [Rodgers, 2000] the following equations can be written:

$$\hat{\mathbf{x}}_1 = \mathbf{x}_{0,1} + \mathbf{A}_1(\mathbf{x} - \mathbf{x}_{0,1}) + \mathbf{G}_1\mathbf{n}_1 \quad (1)$$
$$\hat{\mathbf{x}}_2 = \mathbf{x}_{0,2} + \mathbf{A}_2(\mathbf{x} - \mathbf{x}_{0,2}) + \mathbf{G}_2\mathbf{n}_2 \quad (2)$$

   where $\mathbf{A}_i$, $\mathbf{x}$, $\hat{\mathbf{x}}_i$, $\mathbf{x}_{0,i}$, $\mathbf{G}_i$, and $\mathbf{n}_i$ are the averaging kernel, truth profile, retrieved profile, a-priori profile, gain matrix, and noise vector for the **ith** instrument (i = 1,2 for AIRS and MLS). The product $\mathbf{G}_i\mathbf{n}_i$ represents the errors introduced by the AIRS/MLS measurement and forward model. Note, we have not assumed that AIRS and MLS use the same a-priori profile (In principle, using the same a-priori profile would be ideal). Now, $\mathbf{G}_i$ can be written as

$$\mathbf{G}_i = (\mathbf{K}_i^T\,\mathbf{S}_{e,i}\,\mathbf{K}_i + \mathbf{S}_{a,i})^{-1}\mathbf{K}_i^T\,\mathbf{S}_{e,i}^{-1} \quad (3)$$

   where $\mathbf{K}_i$, $\mathbf{S}_{e,i}$, and $\mathbf{S}_{a,i}$ are the Jacobian, error covariance, and a-priori covariance matrices for each instrument.

   The reviewer is correct that if the a-priori uncertainties are very different between the two retrievals, i.e. both retrievals having very different $\mathbf{S}_a$, this would impact any comparisons made between the two retrievals.

   Unfortunately, because the AIRS retrieval process does not work in a optimal estimation framework, $\mathbf{S}_a$ does not exist, making it virtually impossible to characterize any errors introduced by the a-priori. The $\mathbf{S}_a$ term in equation 3 is a offset matrix in the AIRS retrieval that ensures the Jacobian's are inveritable. It does not incorporate any errors that arise from the a-priori.

2. In paragraph 4, the reviewer states that: ***"The correct method for setting the weights is to use the covariances of the two datasets or, in a level-by-level approach, just the variances..."***

   We agree with the reviewers statements. If we understand the reviewer correctly, the statement above is referring to a joint retrieval between AIRS and MLS. One can derive how to combine the two profiles in an optimal estimation framework by first defining the combined *a-posteriori* covariance:

$$\hat{\mathbf{S}}_t = (\hat{\mathbf{S}}_1^{-1}\mathbf{A}_1 + \hat{\mathbf{S}}_1^{-1}\mathbf{A}_2 + \mathbf{S}_a^{-1})^{-1} = (\mathbf{K}_1^T\mathbf{S}_{e,1}^{-1}\mathbf{K}_1 + \mathbf{K}_2^T\mathbf{S}_{e,2}^{-1}\mathbf{K}_2 + \mathbf{S}_a^{-1})^{-1} \quad (4)$$

   where $\mathbf{K}_i^T$ and $\mathbf{S}_{e,i}$ are the Jacobian and error covariance matrices for AIRS (i=1) and MLS (i=2); $\mathbf{x}_a$ and $\mathbf{S}_a$ are the a-priori profile and a-priori covariance which are assumed to be the same in a joint optimal estimation retrieval.

The joint retrieval can be written as:

$$\hat{\mathbf{x}}_t = \mathbf{x}_a + \hat{\mathbf{S}}_t\hat{\mathbf{S}}_1^{-1}(\hat{\mathbf{x}}_\mathbf{1} - \mathbf{x}_a) + \hat{\mathbf{S}}_t\hat{\mathbf{S}}_2^{-1}(\hat{\mathbf{x}}_\mathbf{2} - \mathbf{x}_a) \quad (5)$$

$$\hat{\mathbf{S}}_i = (\mathbf{K}_i^T\mathbf{S}_{e,i}^{-1}\mathbf{K}_i + \mathbf{S}_a^{-1})^{-1} \quad (6)$$

where $\hat{\mathbf{S}}_1$ and $\hat{\mathbf{S}}_2$ are the *a-posteriori* error covariances for AIRS and MLS respectively [Rodgers, 2000]. Here $\hat{\mathbf{S}}_t\hat{\mathbf{S}}_2^{-1}$ (AIRS) and $\hat{\mathbf{S}}_t\hat{\mathbf{S}}_2^{-1}$ (MLS) show up as weights in the joint retrieval equation 5. As discussed above, however, these terms are not available in the current retrieval framework of AIRS, so we must use a different approach to combine the datasets.

Given that AIRS and MLS do not work in the same retrieval framework, the only way we could merge the two datasets together is by weighting them by their respective local verticalities (LV) which is the sum of the averaging kernels in the narrow layer encompassing the retrieval level of interest. There are two options to verify whether our merge method produces acceptable profiles: (1) to inter-compare the merged profiles with in-situ profiles that have been smoothed through using equation 8 (in Maddy and Barnet [2008]), or (2) use existing error estimates, empirically derived from previous validation campaigns, and verify that the new profiles do not significantly deviate from these error estimates—we elected method (2) for its simplicity (our Figure 6 result). We show in Figure 6 that the profiles do not significantly deviate from the original profiles when accounting for the empirically derived errors of both instruments.

3. The reviewer states: "***There is no mention in this paper of the covariance matrices of either the retrievals or their a priori estimates...***" *(at the end of paragraph 4)*

   Since AIRS does not retrieve using in an optimal estimation framework it is difficult to compare the MLS and AIRS covariances matrices. As mentioned before, the a-priori influence in AIRS cannot be quantified. It is noted that we did not indicate this in our manuscript. We will add the following text to the indicate sections to indicate why we merged AIRS and MLS using our method versus an optimal estimation joint retrieval method.

Add Section "2.2. Optimal Estimation Retrievals" Section 2.2 will become section 2.3. The text in the "new" section 2.2 will read:

"The satellite retrievals of an atmospheric constituent such as water vapor are typically done using an optimal estimation framework. The retrievals can be linearly related to the actual water vapor distribution in the following manner:

$$\hat{\mathbf{x}}_\mathbf{i} = \mathbf{x}_{0,i} + \mathbf{A}_i(\mathbf{x} - \mathbf{x}_{0,i}) + \mathbf{G}_i\mathbf{n}_i \quad (\mathbf{1})$$

where $\mathbf{A}_i$, $\mathbf{x}$, $\hat{\mathbf{x}}_i$, $\mathbf{x}_{0,i}$, $\mathbf{G}_i$, *and* $\mathbf{n}_i$ *are the averaging kernel, truth profile, retrieved profile, a-priori profile, gain matrix, and noise vector for the* **ith** *instrument.* $\mathbf{A}_i$ *is an* N *x* N *matrix where* N *is the number of retrieval levels. The product* $\mathbf{G}_i\mathbf{n}_i$ *represents the errors introduced by the AIRS/MLS measurement and forward model. Now,* $\boldsymbol{G}_i$ *can be written as*

$$\mathbf{G}_i = (\mathbf{K}_i^T\,\mathbf{S}_{e,i}\,\mathbf{K}_i + \mathbf{S}_{a,i})^{-1}\mathbf{K}_i^T\,\mathbf{S}_{e,i}^{-1} \quad (\mathbf{2})$$

$$\mathbf{A}_i = \mathbf{G}_i\mathbf{K}_i \quad (\mathbf{3})$$

where $\mathbf{K}_i$, $\mathbf{S}_{e,i}$, *and* $\mathbf{S}_{a,i}$ *are the Jacobian, error covariance, and a-priori covariance matrices for* ith *instrument. Equations 1 and 2 are the basis for optimal estimation retrievals. Note* *the gain matrix in Eq. 2 takes into account the influence of the a-priori covariance on the final retrieved profile and also impacts the AK matrix (Eq. 3)*

In order to compare the AK's of AIRS and MLS (or between any two remote sensing systems) the influence of the a-priori should be accounted for in order to compare the AK's in a consistent manner. Unfortunately, the current AIRS retrieval does not work in an optimal estimation framework and does not incorporate any effects from the a-prior covariance matrix $\mathbf{S}_a$*, thus making it virtually impossible to account for any AK differences between the AIRS and MLS a-prioris. Although both the AIRS and MLS retrievals do use Eq. 3 in some form, the AIRS retrieval, however, uses a matrix* $\mathbf{H}$ *in place of* $\mathbf{S}_a$ *to prevent the Jacobian matrix inversion from failing. Since the gain matrices, which account for instrumental and retrieval errors, do not have the same information, i.e. only MLS accounts for the a-priori covariance, the interpretation of comparisons between the AIRS and MLS AK's needs to be handled with care. Nevertheless, as will be shown in Sect. 3.1 and 3.2, the AIRS and MLS AK's do provide consistent information that quantifies their relative sensitivity to UTLS* H_2O*".*

After this we will include a paragraph in section 3.2 after the first paragraph in the section. The paragraph will read:

"We briefly note that in an optimal estimation framework the AIRS and MLS AK's are influenced by their respective covariances and a-priori covariances. These influences, in principle, need to be accounted for before one can even use the AK's from both instruments to quantify their sensitivites in a consistent manner. Thus, the a-priori differences need to be considered in order to merge the data between the two systems. However, because AIRS does not work in a optimal estimation framework, quantifying any differences that result from their a-priori and covariances is virtually impossible. However, the results of Sect. 3.1 indicate that the AK's of both instruments do capture their sensitivities reasonably well. Below, we describe a simple method using the AIRS and MLS AK's that is able to produce merged profiles which, within their error estimates, do not deviate from either instrument's interpretation of the atmospheric state.

In order to merge the profiles the AIRS and MLS AK's ..."

4. In paragraph 8 the reviewer notes that; "***a significant omission in this paper is does not explain the (non-trivial) process of how averaging kernels are remapped from the AIRS pressure grid to the MLS grid...***"

   The reviewer is correct in that we did not explain in detail the process of remapping the AIRS averaging kernels to the MLS pressure grid. We did cite Maddy and Barnet [2008] (which I will call "MB2008" here-in) since their method is employed in our work. More specifically, in MB2008 the averging kernel $\Phi_{jj'}$ (a 11 x 11 matrix from the level 2 product) is converted to an "effective" averaging kernel, i.e. $\mathbf{A_{AIRS}} = \mathbf{F\Phi F^{+}}$ (where F represents the 11 trapezoidal basis functions, each defined for the 100 support product levels such that $\mathbf{F}$ is a 100 x 11 matrix). In order to remap Φ to the 47 MLS levels we construct the basis function $\mathbf{F}$ by merging the AIRS and MLS pressure grids which produces a $\mathbf{F}$ with 147 levels rather than 100. From $\mathbf{F}$ we extract only the levels that are relevant to the MLS levels creating a $\mathbf{F}_{MLS}$ matrix. This now serves as the smoothing functions to construct the AIRS "effective" averaging kernel $\mathbf{A'_{AIRS}} = \mathbf{F_{MLS}\Phi F^{+}_{MLS}}$, now on the MLS pressure levels Now that we have the newly constructed averaging kernel $\mathbf{A'_{AIRS}}$, the new kernels can now be compared directly with the kernels provided by the MLS product. We also note that

$Tr(\mathbf{\Phi}) = Tr(\mathbf{A_{AIRS}}) = Tr(\mathbf{A'_{AIRS}})$, demonstrating a successful remapping of $\mathbf{\Phi}$ onto the MLS levels (Tr is the trace of a matrix, i.e. the sum of the diagonal elements of a matrix). This explanation will also be encoporated into the revised manuscript. This will be done by replacing paragraph three in section 3.2 with the following text:

"The AIRS and MLS data products are provided on different pressure grids, therefore one needs to compute the AIRS AK on the MLS levels in order to produce comparable AK. To merge the profiles the AIRS AK's are computed on the MLS levels using Eq. 6. This is necessary as the 12 levels per decade for MLS leads to a coarser pressure grid than the AIRS L2 support product levels in the UTLS. In this case, instead of defining the trapezoidal basis functions on the 100 retrieval levels of AIRS, we combined the MLS pressure levels with the pressure grids defined by the AIRS retrieval trapezoidal basis functions [Maddy and Barnet, 2008] (these are a subset of the 100 support product levels) to compute the basis function matrix $\mathbf{F}$ *in Eq. 6. Then the basis functions only on the MLS pressure levels are extracted and inserted into Eq. 6 to compute a 47 × 47 "effective" AK matrix. This procedure, in effect, redistributes the information content in the* 100×100 *AIRS AK's matrix onto a* 47×47 *matrix corresponding to the 47 MLS pressure levels, resulting in two AK matrices,* $\boldsymbol{A}$ *and* $\boldsymbol{M}$*, that represent the AIRS and MLS AK's, respectively. A successful remapping of the AK matrix to the MLS levels satisfies the following condition:* $Tr(\mathbf{\Phi}) = Tr(\boldsymbol{A})$ *where* Tr *is the trace of the matrices. The AIRS* H_2O *are also interpolated (*$\log(P)$ *vs.* $\log(H_2O)$*) to the MLS levels."*

Eq. 5 will be a generalized version of $\mathbf{A'_{AIRS}} = \mathbf{F_{MLS}} \mathbf{\Phi} \mathbf{F^{+}_{MLS}}$.

References

Maddy, E. S. and Barnet, C. D.: Vertical resolution estimates in version 5 of AIRS operational retrievals, IEEE T. Geosci. Remote, 46, 2375–2384, 2008.

Rodgers, C. D.: Inverse Methods For Atmospheric Sounding, Theory and Practice, vol. 2 of Series on Atmospheric, Oceanic and Planetary Physics, World Scientific, 2000.