Item-by-item response to Reviewer #1

The authors greatly acknowledge the anonymous reviewer for carefully reading the manuscript and providing constructive comments. This document contains the authors' responses to comments from reviewer #1. Each comment is discussed separately with the following typesetting:

***Reviewer's comment**

Author's response

Changes in the manuscript.

*The paper reports on error estimation related to data inversion. The data are acquired with socalled multi wavelength lidar that delivers backscattering at 3 wavelengths and extinction at 2 wavelengths. The authors study two cases, described by a monomodal lognormal distribution and a bimodal lognormal size distribution. The authors consider the case of statistical and systematic errors in the input data. The auhors treat the systematic errors as statistical errors and in this way develop a parameterization which allows them to estimate the error in their inversion products from the measurement errors. Main finding, according to the authors is that errors at different wavelengths are additive and that the measurement error transfers in a linear fashion to the output data products.

*The authors provide insight on the error propagation which to my knowledge is the first time that such work is available. I consider the results important in view of the fact that the underlying mathematical equations are nonlinear and that mathematical tools for error estimation in data inversion are needed.

*However, the manuscript does not show in a convincing way that a simple error propagation exists. The authors to my opinion wash away important stumbling blocks in their line of argumentation and the way they present their results.

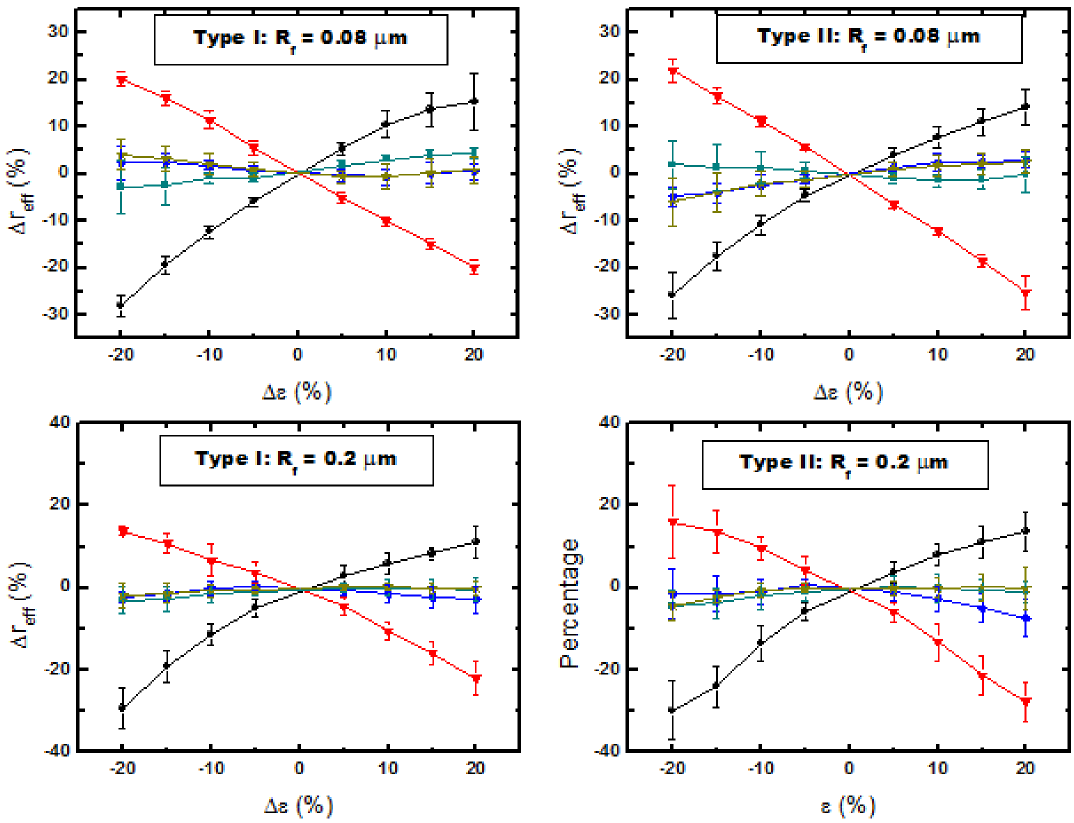
*The authors start out with two simple case studies. They use one monomodal and one bimodal distribution. The mode widths they use represent comparably narrow size distributions. I am wondering if this narrowness creates the linear behavior of error propagation and particularly the additive character of the errors of the different data products from lidar.

We should first point out that the selection of the mode widths for the study cases were based on the AERONET database provided by Dubovik et al., (2002). From this climatological database we observe that most of the mode widths that were used in the original version of the paper were within those measured by AERONET and therefore we consider they are representative of many aerosol conditions reported by AERONET. We agree that other size distributions with different mode widths can be studied. To that end, we have included an additional distribution in the revised version. We draw very similar conclusions from this third distribution as for the first two.

Also, to clarify this point we have added to the new manuscript (lines 188-191):

"…These mode radii and widths are representative of those provided by Dubovik et al., (2002) in the AERONET climatology database and are thus considered to represent a large fraction of naturally occurring aerosols.… "

Moreover, we clarify that the linearity presented here are for average values of the used size distributions. For the different aerosol size distributions, tests were done changing the fine mode radius to 0.08 µm and 0.20 µm. We attach here the graphs we obtained for the effective radius, both for aerosol type I and II.



As the referee can see, the linearity is essentially the same as that presented in Figure 2. Only changes in the absolute values of the slopes are observed but they are below ±20%. Similarly, we performed tests varying the width of the fine mode up to ±0.6 and we did not obtain remarkable departures either from the linearity or from the absolute values of the slopes. For all these reasons we believe that our results can be useful for many lidar applications. These points have been made in the revised manuscript (lines 399-406):

"…Finally, we remark that the values given in Table 1 are averaged for the particular size distributions used here. More simulations performed (graphs not shown for brevity) changing the fine mode radius between 0.08 µm and 0.20µm, both for aerosol type I, II and III, revealed the same average linear patterns as those shown in Figures 2 and 3 and in Table 1. The only differences observed were in the absolute values of the slopes with values between ±20%. On the other hand, no important departures from the linearity observed in Table 1 were found by changing the widths of the fine mode. Changes in the coarse mode were not tested because of the difficulty to assess retrievals of the coarse mode with the methodology used here …"

The authors use several refractive indices to compute the optical data that are used in the data inversion. The number of refractive indices is very limited, too, and thus may obscure a non-linear behavior of error propagation.

From previous works in the literature (Muller et al., 1999a; Veselovskii et al., 2002) the accuracy of the real part of refractive index was claimed as ±0.05, while the imaginary part possessed a 50% error. For the AERONET inversion scheme similar errors are reported. Later in the paper (Section 3.1), from the sensitivity study we obtain basically the similar errors. Thus, values of the real part (1.35, 1.45 and 1.55) and the imaginary part (0.005 and 0.01) used here are enough for our study. As the objective is to study those distributions previously reported in the bibliography (Dubovik et al., 2002), we believe that these sets of refractive indices, taking into account the error from the bibliography, cover the AERONET climatology database. Thus, to clarify this point we added to the manuscript (lines 216-220):

" … From previous studies (Muller et al., 1.999; Veselovskii et al., 2002) error in m_r was initially established as ±0.05 while error in m_i was approximately 50%. Moreover, the AERONET network provides refractive indices with very similar errors (Dubovik et al., 2000). Thus, the range of refractive indexes proposed for the size distribution is enough to cover most of the values obtained by AERONET (Dubovik et al., 2002) …"

The authors do not show in a convincing manner that the results they obtain can be generalized to the general case in which the size distribution may have any kind of mode radius and mode width. The weighting of the two modes (in the bimodal example) may also be simple a lucky shot. Yet, the authors present in figure 4 their results of the sensitivity study which is admittedly a highly attractive and elegant way.

We agree that the distributions used here do not represent all the possible distributions that can be obtained. However, due to the complexity of lidar systems and the limitations for obtaining microphysical aerosol properties from these measurements by the regularization technique, we try to simplifythe procedure to study the effects of systematic errors in the optical data on the microphysical retrieval . However, as stated, we try to use those distributions that can be representative of many of those obtained by AERONET (although its inversion algorithm relies on irradiance and sky radiance measurements). The applicability and validation of the AERONET inversion algorithm is widely recognized internationally. The work of Dubovik et al., (2002) summarizes AERONET results where they obtained bimodal aerosol size distributions. This assumption of the size distribution is widely accepted in the literature. For example, the AERONET network provides fine and coarse mode optical depth based on a Spectral Deconvolution Algorithm (O'Neill et al., 2001a,b) whose assumption is a bimodal aerosol size distribution. Another example of the use of bimodal aerosol size distributions is the retrieval technique used by the MODIS sensor (e.g. Levy et al., 2013). For all these reasons we focus our study on bimodal aerosol size distributions. Although tri-modal size distributions can also be found in nature (e.g. Eck et al., 2010) its study is out of the scope of the present work.

To clarify all these points we have made changes in the new manuscript. In the abstract we have remarked that point (lines 30-31):

" Using bimodal aerosol size distributions,….."

Also in the introduction section (lines 107-108)

"… Particularly, we will focus on the study of bimodal size distributions widely found in nature (e.g. Dubovik et al., 2002)…."

And also in the conclusion section (lines 609-616)

"….Simulations have been done for different bimodal aerosol size distributions that are representative of AERONET climatologies. The values used for aerosol refractive indexes, as well as mode radius and widths were selected as representative of those climatologies as well. The selected aerosol bimodal size distributions include one with fine mode predominance (type I), another with predominance of coarse mode but with significant presence of fine mode (type II) and another with predominance of fine mode but with significant presence of coarse mode (type III). Optical data consistent with these bimodal size distributions were generated using Mie theory.…"

From a simple point of view, we deal with a size distribution where fine mode predominates, and this is what we called type I. These size distributions are found for pollution and biomass-burning aerosols. Size distributions where coarse mode predominates are expected for dust and marine aerosol. But as we mentioned in the manuscript we need to use kernel functions based on randomly-orientated spheroids to study this case. As this would imply the use of a more complex code, we decided to not

study this case in our work. Indeed, we studied a size distribution with slight predominance of coarse mode but also with an important contribution of fine mode. This is what we called study case II. In this sense, as we showed in Figures 2 for the effective radius and in Figure 3 for number concentration, the effects on the retrievals of systematic errors in the input optical data can be as average linearized. Table 1 summarized the slopes of the linear fits for every microphysical parameter and for every optical data. Both type I and type II presented the same sign of slopes in every case (only small changes when the slopes are very low and that not influence is expected on the retrievals). The only change is in the absolute value of the slopes, which was commented in the text, depends on the aerosol type. For other size distributions whose weight of fine/coarse mode is between both aerosols types the expected errors associated with systematic errors in the optical data would be between those stated in Table 1. However, to satisfy referee demands we have extended our analysis for a size distribution where fine mode predominates but still there is an important fraction of the coarse mode. This is what we call in the revised manuscript study case III and would represent a mixture of pollution/biomass with dust/marine. The different weights of size distributions are clearly seen in the new figure 1 (also required by the other referee to identify clearly both modes). The volume size distributions are normalized.

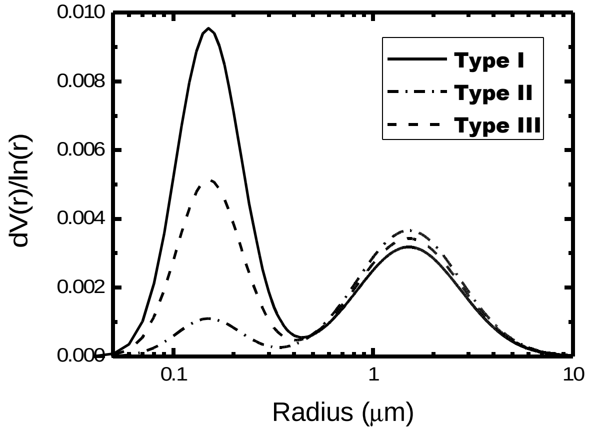


Figure 1: Normalized size distributions used for computing the simulated optical data. The ratio between the volume of fine and coarse mode, V_{tc}/V_{tc}, is 2 for type I, 0.2 for type II and 1 for type III.

The analysis of case III also reveals linear dependencies of the errors in the microphysical parameters to systematic errors in the optical data. We have shown this in the new figures 2 and 3. The results for the other parameters are included in the revised Table 2 as well.

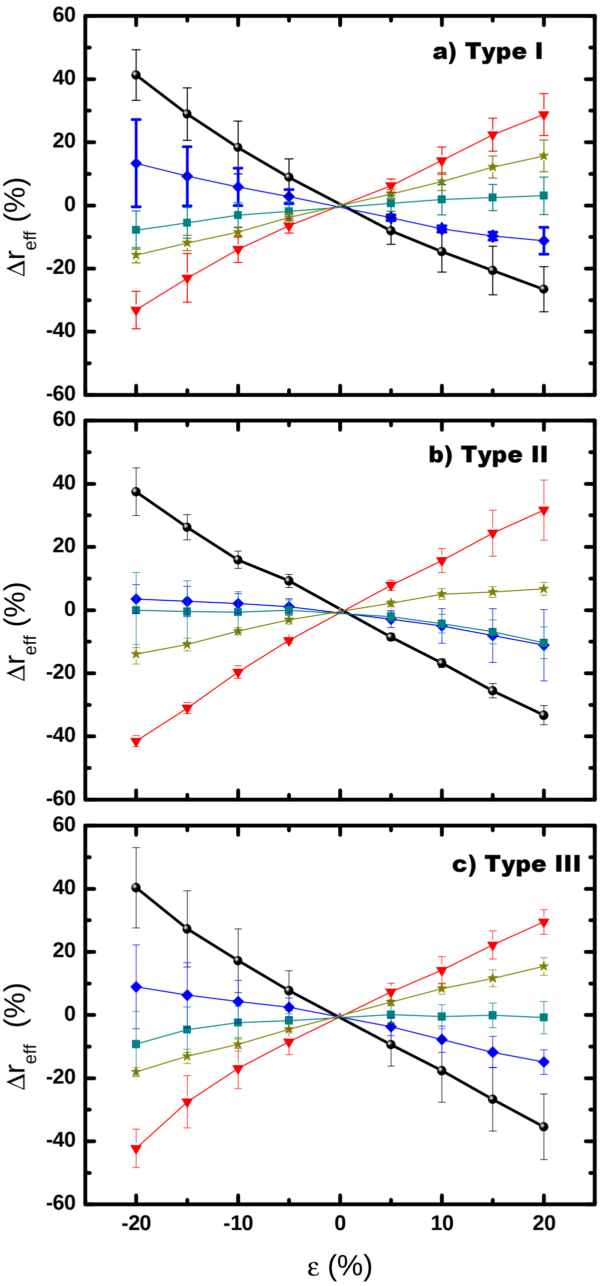


Figure 2: Percentage deviation of the effective radius as a function of systematic bias in the optical data (ε). a) Type I. b) Type II. C) Type III.

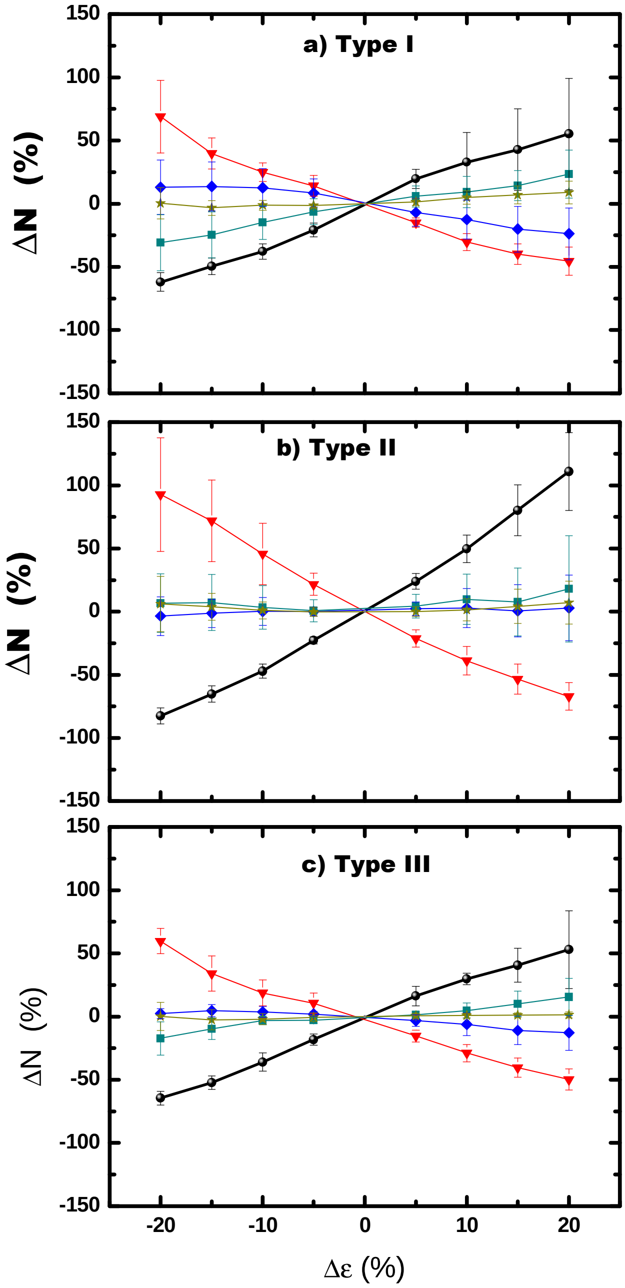


Figure 3: Percentage deviation of the number concentration as a function of systematic bias in the optical data (ε). a) Type I. b) Type II. c) Type III.

For case III, both for effective radius and number concentration, we observe that the slopes are within those obtained for case I and case II. Therefore, the results presented here for bimodal size distributions with different refractive indexes indicate that are not a "lucky shot".

Therefore, we have added the study case III to the new manuscript. The results from this study case are included in Table 1. In the introduction section now can be read (lines 109-112).

"…The study involves simulations based on three different bi-modal aerosol size distributions, one with a large predominance of fine mode, another with slight predominance of coarse mode and the last one with slight predominance of fine mode …"

In section 2.2 where we deal with the size distributions used for simulations (lines 197-200)

" … Finally, type III yields V_{tf}/V_{tc} = 1 and corresponds to a slight predominance of fine mode over the coarse mode [e.g. Xia et al., 2007: Ogunjobi et al., 2008; Yang and Wening, 2009; Eck et al., 2009]. This type is representative of predominance of pollution or biomass-burning but with considerable influence of dust particles…"

Which make us add the following references:

Yang, X., and Wenig, M. : Study of columnar aerosol size distribution in Hong Kong, Atmospheric Chemistry and Physics, 9, 6175-6189, 2009.

Ogunjobi, K.O., He, Z., and Simmer, C.: Spectral aerosol optical properties from AERONET sun-photometric measurements over West Africa, Atmospheric Research, 88, 89-107, 2008.

Eck, T.F., Holben, B.N., Reid, J.S., Sinyuk, A., Hyer, E.J., O'Neill, N.T., Shaw, G.E., Vande Castle, J.R., Chapin, F.S., Dubovik, O., Smirnov, A., Vermote, E., Schafer, J.S., Giles, D., Slutsker, I., Sorokine, M., and Newcomb, W.W.: Optical properties of boreal region biomass burning aerosols in central Alaska and seasonal variation of aerosol optical depth at an Arctic coastal site, Journal of Geophysical Research, 114, D11201, doi: 10.1029/2008JD010870, 2009.

Xia, X., Li, Z., Holben, B., Wang, P., Eck, T., Chen, H., Cribb, M., and Zhao, Y.: Aerosol optical properties and radiative effects in the Yangtze Delta region of China, Journal of Geophysical Research, 114, D22S12, doi:10.1029/2007JD008859, 2007.

We have also changed many parts of the paper where we discussed sensitivities of microphysical properties to systematic errors in the optical data and added the values obtained by type III.

In section 3.2, lines 273-277:

"…Considering the parameters to which the retrievals are most sensitive, the linear fit of α(355 nm) gives negative values of slope (a = -1.68 ± 0.12 for type I, a = -1.74 ± 0.03 for type II and a = -1.84 ± 0.04 for type III), while for α(532 nm) the slopes are positive (a = 1.51 ± 0.04 for type I, a = 1.82 ± 0.09 for type II and a = 1.71 ± 0.10 for type III) …"

between lines 298-299:

" … Finally, for β(1064 nm) we observe positive slopes (a = 0.791 ± 0.008 for type I, a = 0.54 ± 0.07 for type II and a = 0.84 ± 0.02 for type III) …"

Between lines 307-310:

"…positive values for α(355 nm) (a = 3.09 ± 0.12 for type I, a = 4.83 ± 0.22 for type II a = 3.05 ± 0.13 for type III) and negative values for α(532 nm) (a = -2.78 ± 0.17 for type I, a = -4.09 ± 0.23 for type II and a = -2.61 ± 0.12 for type III)…"

Between lines 319-321:

" … for r_{eff} as a function of biases in β(1064 nm) (0.79 ± 0.01 for aerosol type I, 0.54 ± 0.07 for aerosol type II and a = 0.84 ± 0.02 for type III) … ".

Between lines 342-344

" … at β(355 nm) are generally larger for type I than for type II (absolute values of slopes are larger) with type III being in the middle …"

Between lines 364-368

"…For type III aerosols the sensitivities to bias in the optical data are important both at β(355 nm) (slope of -1.04) and at α(532 nm) (slope up to 1.46). These differences among the aerosol types I, II and III demonstrate the different sensitivities of volume concentration retrievals when the PSD possesses different weights of fine and coarse mode …"

And between lines 381-383

"…for the fine mode volume concentration (V_{fine}), the largest sensitivities in the retrieval are found to systematic biases at α(355 nm), with slopes of 1.59 ± 0.05, 1.66 ± 0.17 and 1.56 ± 0.06 for types I, II and III, respectively …"

For section 3.2 we also made changes in the text to introduce the study case III (lines 474-475):

"… Furthermore, very similar additive properties were found for aerosol type III (graph not shown for brevity) …"

We have also introduces some changes in the text in section 3.3 to take into account the study case III. Now, between lines 521-524:

"…As an illustration, Figure 5 shows the frequency distribution of the differences in the microphysical parameters studied here, for all aerosol size distributions type I, II and III, where 15% random error is assumed in all the optical data. Those differences are in percentages and denoted as 'deviation' in the 'x' axis of the histograms …"

Lines 539-540:

" … Generally, there are many similarities in the standard deviations between aerosol types I, II and III … "

And between lines 543 and 544

“…while the retrieval of number concentration has the highest sensitivity, with 1-sigma values of 67.6% for type I, 95.2% for type II and 61.4% for type III …”

Moreover, the results from the study case III are now included in Figure 5 and tables 2 and 3.

The Gaus-like distributions indicate a rather simple relationship between input and output errors. However, these results are based on two size distribution and a few refractive indices only. Either the authors show in tables in figure that relationships indeed hold for a broader range of input parameters (of the size distributions), or the authors make it very clear in the paper that these error propagation rules hold for a very limited input data set, cannot be generalized yet and need further investigation.

We also agree that we have studied only, now, three size distributions. But as commented above, we believe that the use of bimodal size distributions used here is representative of a wide range of distributions found by AERONET. As commented before, we believe that the range of refractive indexes used here are appropriate due to the limitations of the regularization technique. We also agree that the statement that the results presented here cannot necessarily be generalized to multi-modal size distributions although the consistent behavior of the three distributions studied leads one to expect that similar results would be obtained for multi-modal distributions. We make a clear statement of the applicability of our results in the conclusion section. Now, in the revised manuscript, can be found (lines 649-659)

“…The results presented here cannot be generalized to every possible size distribution as we only focused on bimodal size distributions representative of those obtained by AERONET. Studies of the sensitivities of the microphysical retrieval to errors in the optical data for other size distributions such as, for example, one showing tri-modal behavior are still needed although the results presented here for three differing bi-modal distributions leads one to expect that similar results would be obtained for tri-modal distributions as well. The tests performed here showed that the average linearity of the sensitivities in the retrieval to random errors in the input data can be useful for a wide range of lidar applications, and thus can be used to establish acceptable error budgets in optical data if maximum permissible errors in the retrieved quantities can be established. Therefore, the values given here for the sensitivities of the microphysical properties to systematic errors in the optical data can be useful for many lidar applications …”

The authors also do not explain in detail how they treated the input errors. A few more details on the gauss error of the input would be valuable in this context.

We believe that the referee refers to how we treat the effects of random errors because here is where we use the gauss error of the input. We have given a more detailed description. Now, between lines 505 and 520 can be read:

"…To assess the sensitivity of the retrievals to random errors we use the additive properties of the systematic biases just described. The procedure used consists of generating random numbers distributed in a Gaussian way centered at zero with width according to the value of the random error to study. These random errors are applied to each optical channel of the 3β + 2α configuration. This procedure was repeated 50,000 times for each parameter studied. Also, the initiation of the random number generation is different for each channel to avoid the situation where all the random numbers are the same in every channel. Finally, we introduced for every optical data this random number and computed the corresponding error in the retrieved microphysical parameter using the slopes provided in Table 1. For every set of 3β + 2α values, the final error obtained in the microphysical parameter is the sum of the error obtained for each channel. The study of the frequency distributions of the final errors for this large number of simulations yields the effects of random errors. If the frequency distribution is a normal one, the standard deviation (Full-Width-at-Half-Maximum) provides the final error in the microphysical parameter. Moreover, if the normal distribution is not centered at zero it demonstrates an interesting property; that the presence of systematic errors in the retrieved microphysical property can be induced by random errors in the input optical data …"

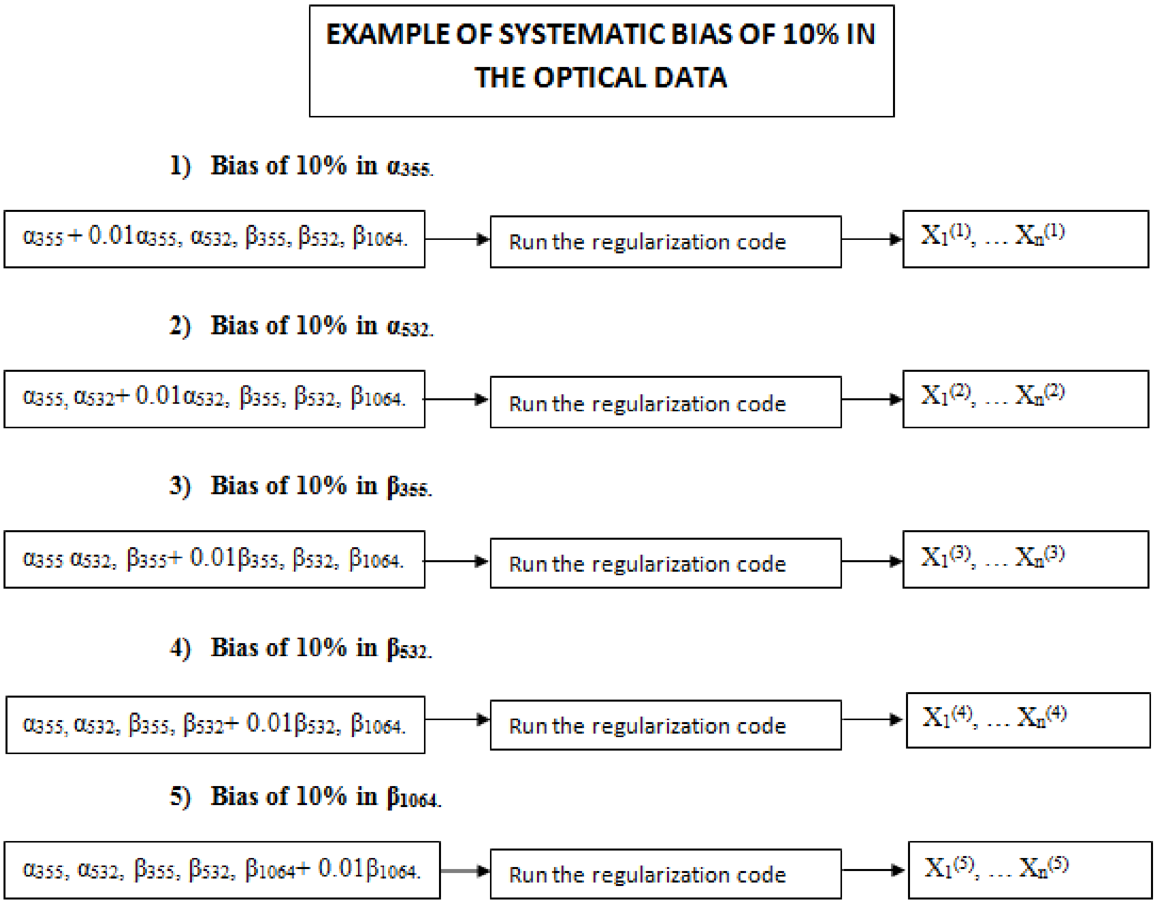
I am surprised that statistical errors can be treated like statistical error. To my opinion this concept is in contradiction to the theory of statistical mathematics, or it is at minimum an oversimplifying concept with respect to the definition of systematic errors in the presence of statistical error. I may be wrong on my opinion and would be happy to see more explanations and also illustrations that explain in more detail the validity of this elegant assumption.

The procedure used here to study the effects of systematic errors in the input optical data does not assume any statistical errors. We just compute the theoretical optical data for the input aerosol size distributions commented before (types I, II and III).

First we define the systematic errors to study. Those are -20, -15, -10, -5, 5, 10, 15 and 20%. Later, we introduce a systematic bias to one optical datum at a time. But again, we remark that we are not dealing here with a statistical error. Later, we run the regularization technique, compute the size distributions and the microphysical parameters, and compared with those not affected by this systematic bias. The procedure implies that only one channel is affected by the systematic error, while the others remain constant. Once we have computed the effect of the systematic error in one optical channel, we apply the same procedure to others optical channel. This process is repeated until we end up with all the channels of the 3β + 2α configuration. Later it is shown that these individual effects can

be added resulting in the additivity property that was used to assess the influence of random errors on the retrievals.

In the following graph we illustrate how we study the effect of 10% bias in the optical data. Let's assume that we initially have a set of microphysical parameters α_{355}, α_{532}, β_{355}, β_{532}, β_{1064}. After running the code we obtain the initial microphysical properties $X_1^{(0)}$, … $X_n^{(0)}$. Later, we applied the bias to the optical data.



We do not believe that introducing this graph in the revised manuscript is necessary. Indeed, we would prefer to make the text more clear. Thus, in the revised manuscript we can read between lines 221 and 227:

"…The regularization inversion is then performed on these data and we obtain the retrieved microphysical parameters 'M_{ret}'. The next step consists of applying a systematic bias, denoted as $\Delta\varepsilon$, to one optical datum at a time. The bias varies from -20% to +20% in 8 intervals.

For each of these induced biases, the inversion is performed and a new size distribution and set of microphysical parameters, M_{bias}, are then obtained. The comparisons to be performed are expressed as the percentage difference 100* (M_{bias} - M_{ret})/M_{ret}. This procedure is applied to each of the 5 optical data used in the 3β + 2α lidar configuration …”

Finally, the errors of the output data products with regard to statistical errors are basically known from previously literature. Even though the authors point out to this fact referring to their own work, they should consider a more precise description of the novelty of their work, which is the treatment of systematic errors, which brings me back to my doubt that systematic errors can be treated like statistic error. Or to put it into a simple procedure: why not subtract the systematic error from the mean optical data and do the inversion according to the statistical error only?

Again, as previously discussed, our procedure to estimate the errors in the microphysical parameters due to systematic errors in the optical data is not based on a statistical treatment of the errors. We just alter one channel by a certain bias and study how the microphysical parameters are affected.

In the previous works done and reported by the literature they estimated the errors in the microphysical parameters by introducing random errors in the optical data and running the code. This was done only for a very situation where all optical data were considered to possess 10% random noise. The novelty we present here is that our results for random error sensitivity are based on the additivity of systematic errors in the optical data. Such an approach allows the estimation of the effects of random errors in the microphysical retrievals in an easy and straightforward way, avoiding running the code thousands of times. In this sense, we have been able to estimate the effects on the microphysical retrievals when more noise in the optical data is allowed (15 or 20%) and also for more accurate systems (5%) and where the amount of noise is different in the various optical channels. Our results are in general agreement with those previously obtained by Muller et al., (1999) and Veselovskii et al., (2002) for 10% random errors in the optical data. Moreover, a direct application is shown for the case of the upcoming ACE-mission where a 15% error in the backscatter and extinction is desired.

Following referee suggestions we have remarked the novelty of the procedure we proposed here for studying the effects of random errors in the optical data. In Section 3.3, between lines 565 and 567, can be read:

”… Müller et al., [1999a,b] and Veselovskii et al., [2002, 2004] studied 10% random uncertainties in the optical data in the 3β + 2α lidar configurations by introducing random errors in the optical data and running the regularization code repeatedly. These studies reported that …”

And between lines 571 and 575:

"… The method shown here for assessing the sensitivity of retrievals to random errors is generally consistent with these earlier results but permits the influence of varying amounts of random error to be studied. It also permits the influence of random errors in different input optical channels to be quantified. We will now apply this capability to the problem of instrument specification …"

Finally, the suggested procedure of substracting the systematic error from the optical data and doing the inversion according to the statistical error only is a more complicated procedure. It assumes that one knows the value of the systematic error which often is not the case. To understand how systematic error affects lidar measurements we had already introduced in our manuscript (lines 92-100):

"Systematic errors in lidar systems come from many different sources and need to be considered. From the hardware point of view, systematic errors can be due to, for example, non-linearity of a photodetector or errors in calibration of the optical data. From the methodological point of view, systematic errors can be caused by, for example, errors in the assumed atmospheric molecule density profile, the selection of the reference level (an "aerosol-free" region that may actually contain a small amount of particles), the effect of depolarization due to optical imperfections in channels that are sensitive to polarized light or the use of an incorrect extinction-to-backscatter ratio to convert backscatter lidar measurements to extinction. "

Some general comments:

Page numbers and line numbers would be very helpful in the revised version. It would make it easier to refer points that need to be revised.

We are sorry about that. This has been a misunderstanding of the authors. We thought that with line numbers given by Atmospheric Measurement Technique Discussions was enough. We will be more careful in the revised version.

Check your reference list: I do not find Tesche et al., 2013 and Wanger et al., 2013.

We are sorry for the mistake. We have already corrected the references in the revised manuscript.

Tesche, M., Müller, D., Gross, S., Ansmann, A., Althausen, D., Freudenthaler, V., Weinzierl, B., Veira, A., and Petzold, A.: Optical and microphysical properties of smoke over Cape Verde inferred from multiwavelength lidar measurements, Tellus B, 63B, 677-694, 2011.

Wagner, J., Ansmann, A., Wandinger, U., Seifert, P., Schwarz, A., Tesche, M., Chaikovsky, A., and Dubovik, O.: Evaluation of Lidar/Radiometer Inversion Code (LIRIC) to determine microphysical properties of volcanic and desert dust, Atmospheric Measurement Techniques, 6, 1707-1724, 2013.

Sentence: " From an instrumental point of view …" I am missing references to Veselovskii et al., 2002 and Muller et al., 2001.

This has been corrected. Now, between lines 81 and 83:

"…Müller et al., [2001, 2004, 2005] and Veselovskii et al., [2002, 2004] demonstrated the capability of the regularization technique …"

We have added the reference Müller et al., 2001:

Müller, D., Wandinger, U., Althausen, D., and Fiebig, M.: Comprehensive particle characterizations from three-wavelength Raman-lidar observations: case study, Applied Optics, 40, 4863-4869, 2001.

Sentence: " … data are affected by small random errors" : Please quantify what you mean by "small"?

We believe that this is a misunderstanding as this sentence is when we briefly describe the regularization technique. We have deleted "small" and now can be read between lines 88-90:

"…This averaging procedure increases the reliability of the inversions even when the input optical data are affected by random errors [e.g. Veselovskii et al., 2002] ..."

Sentence: "We will show that the results obtained can also be used to assess the sensitivity of the retrievals to random errors in a new way": I find this part a major weakness of the paper, as the proof of concept is insufficiently described (two size distributions and a few refractive indices only). Particularly: why don't you simply correct for the systematic error and treat the error source of statistical error only?

We have already discussed these questions. Please see the answer above on the main comments.

You use the real part from 1.33 –1.65 and the imaginary part from 0 … 0.01. In how far does this constraint force the error propagation to be linear, and in how far does the refractive index constraint naturally lead to the nice parameterizations of error propagation? Some words on this would be helpful to evaluate the merit of this study.

Those values of refractive indexes were used to run the regularization code. In Section 3.1 called “Uncertainties in the retrieval of particle refractive index“ we discuss the effects that choosing different ranges of refractive indexes have on the retrieval. For the real part (m_r), with the stepsize of 0.025 we cover almost all type of aerosol particles (See references of Veselovskii et al., 2002). But for imaginary part we can only retrieve its value under certain constraints.

The test we performed revealed that allowing large values of m_i of 0.1 (very absorbing particles) forced the retrieved m_i to very large values of 0.3 when the expected value of input size distributions is between 0.01-0.005. This is because the retrieval is under-determined. However, allowing values of m_i of 0.01 did not have significant influence on the retrieval of m_r. Therefore, to clarify more how the range of refractive indexes used here affect the retrieval of the refractive index we have added between lines 248 and 250:

“… For example, computations allowing m_i to range up to 0.1 provides retrieved values of m_i of approximately 0.03, when the values of the input size distributions where 0.01-0.005 …”

You use the fine mode radius of 140 nm and the coarse mode radius of 1500 nm for the volume distribution. What are these values for the number concentration? Are these numbers realistic values for size distributions? 1.5 for the fine mode and 1.8 for the coarse mode are at the lower range of numbers for natural size distributions. I suspect that your linear error propagation is in part the result of this serious constraint. If you use broader size distributions you may completely lose the linear error propagation.
Please comment on my assumption.

The fine and coarse mode radius we give in the paper are for volume size distributions. The corresponding values for a number size distribution are 0.08663 µm for the fine mode and 0.50939 µm for the coarse mode. Those numbers can be easily obtained from equation 5 given in the manuscript.

We checked the AERONET climatology published by Dubovik et al., (2002). For the fine mode all the values obtained by Dubovik et al., (2002) are between $r_f^v - \sigma$ and $r_f^v + \sigma$, where r_f^v is the one we propose in our simulations. For the coarse most of the values obtained in the bibliography also are between $r_c^v - \sigma$ and $r_c^v + \sigma$, being r_c^v the one we use. Here we want to point out that due to the limitations of Kernel functions for the $3\beta + 2\alpha$ configuration we can invert particles up to 5 µm while AERONET using sky radiances the retreival can reach up to 10 µm and thus the comparison of coarse mode must be done carefully. This explains the slightly lower value we choose for coarse mode radius.

Moreover, as stated in the manuscript and in the above comments, the study of predominance of coarse mode is out of the scope of the manuscript due to the necessity of using non-spherical Kernel functions. Thus we believe that the values of fine and coarse mode we selected are representative and useful.

As we just commented, the fine and coarse mode radius and their corresponding widths selected in this work are representative of many aerosol size distributions obtained by AERONET. We do not claim that our results are representative of all the size distributions that can exist in nature. But we believe that our results are representative of the vast majority of naturally occurring aerosols and provide useful insight into error propagation of the MW lidar technique.

In the paper we did our inversions for size distribution of different refractive indices. Thus, the linearization of errors is the results of averaging different sets of size distributions under different refractive index. Also we apply two different predominance of fine and coarse mode, and now in the extended manuscript there is an extra size distribution. For every optical data, we obtain the same linear patterns for each size distribution, lending greater generality to the results.

And again, we would like to emphasize that the linearization that we present are average results for the different size distributions used. Those size distributions are used to include most of the features found by AERONET network. We do not claim that our results can be used for any type of size distribution which as referee says, would need further studies. However, we claim that for the bimodal distributions used our results show how the different channels respond to errors in the optical data for a large majority of real size distributions.

You did not test the imaginary part of 0 in your sensitivity analysis? How would you go around this problem in practical application in which imaginary parts are considerably less than 0.005?

For the lidar configuration of 3β+2α errors in the imaginary parts are known to be high, approximately 50%. This is already known in the literature (e.g. Veselovskii et al., 2002, 2004). Moreover, in section 3.1 called *"Uncertainties in the retrieval of particle refractive index"* we describe the effects of errors in the refractive index. Basically, with the limitations of the regularization technique for the 3β + 2α we cannot provide imaginary part of refractive index below 0.005. More details can be consulted in Veselovskii et al., (2013).

Sentence: "A more depth discussion about limitations of the averaging procedure used here to retrieve accurate values of particle effective radius is in Veselovskii et al. 2013": this paper is not published. I tried to find more information and I see that the manuscript is in the discussion status. Please provide a short summary here.

We have already sent the revised version of this paper to the journal and actually that paper is already accepted for publication. But in the online version there is no criticism to the discussion presented about refractive index. We believe that including a discussion about it in the paper would be more confusing for the reader.

Sentence "The lowest sensitivities are to biases …. and 532 nm". I am raising once more the point that the choice of size distribution may lead to this conclusion.

Again, we point out that our results are for bimodal aerosol size distributions that cover most of the cases found by AERONET. The changes introduced in the manuscript according to this question have been included (see above comments).

Moreover, many works ... found an inverse relationship between the ... for low values of the Angstrom exponent". This sentence does not really need references from 2003 and 2009. These findings are considerably older.

We agree with the referee that this relationship is even older. We just introduce these references to document that this finding is already known.

Sentence: "but the generality of the results needs to be examined": it is certainly this sentence that bears proof in this paper.

The sentence is included in the beginning of section 3.2.1. called *"Effects of the constraints of the retrievals on the sensitivity test results"*. As the title of the section suggest, the objective is to see the influence of the different constraints in the inversion on the linearity found out in the previous sections. The objective was not to claim that our results are applicable for any kind of aerosol size distributions. To clarify this point we have deleted this sentence and introduced the new one (lines 411 an 412 of the new manuscript).

"… But the generality of the results for different constraints in the inversion code needs to be examined…"

Sentence "The values used as the baseline on … with no induced systematic errors" this means you did not use any errors at all? So again; how representative are your results?

This sentence was used to introduce the reader to the case of evaluating how the different constraints in the code (e.g. allowing larger maximum radius or larger maximum values of m_i) can affect to the average linear patterns of systematic errors we found for the bimodal size distributions studied. The values used as reference to compute the effects of systematic errors in the input optical data were those with no errors in the optical data, and, moreover, with the constraints of maximum radius r_{MAX} of 5 µm and maximum m_i of 0.01. Later in this section, we discuss the effects of systematic errors in the optical data

when running the code with different r_{max} and m_i. To clarify this point, we delete the sentence that confused the referee and introduced in the revised manuscript (lines 420 - 423):

"…The results of these studies were compared with a baseline retrieval obtained with r_{max} = 5 μm and with maximum value of m_i of 0.01. To compute the baseline microphysical parameters, no induced systematic errors were included. We also computed the retrievals using the new constraints and introducing systematic errors in the optical data as done before …"

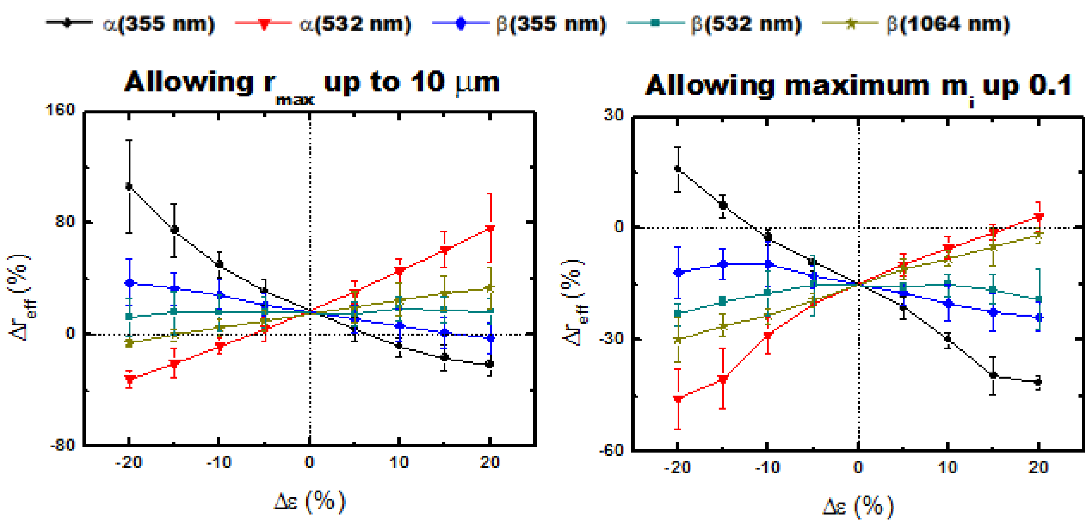
And again, those results are representative for bimodal aerosol size distributions selected (types I, II and III) that cover most AERONET inversions. Section 3.2.1 discusses the effects of the constraints in the inversion code.

Last paragraph before section 3.2.2: I am a bit confused about this very generalizing comment. I do not find proof in the paper that this is the case.

Again, section 3.2.1 is devoted to a discussion of the effects of the constraints in the average linear patterns observed on the bimodal size distributions studied. We have added that these graphs are not shown for brevity (lines 424-425 of the revised manuscript).

"…The new simulations performed after changing the constraints for r_{max} and maximum m_i also reveal linear patterns (graphs not shown for brevity) …"

Here, we show to the referee some of these plots for aerosol type I. Particularly, for the effective radius, we observe the same linear patterns both allowing r_{max} up to 10μm and m_i up to 0.1. The only remarkable effect is that when there is no error in the optical data the linear fits do not pass through zero. This is expected due to the constraints for the values not affected by errors and taken as reference used the constraints of r_{max} up to 10μm and m_i up to 0.1.



We believe that including these graphs in the manuscript is not necessary, as the discussions of the effects of the constraints are done in section 3.2.1. Including these graphs would make the manuscript larger and confusing.

Sentence: "although different combinations of over/under estimations are allowed" it remains unclear which combinations you really used. It also remains unclear in how far the spectral slopes became distorted to a degree that renders the input data set useless, as they do not represent the real situation anymore.

We agree with the referee that this sentence does not clarify what we want to communicate. The sentence is included in section 3.2.2 where we study the additivity of systematic errors in the optical data. Originally, we wanted to communicate that we did simulations allowing systematic errors in at least two optical data, but both of the same magnitude. The magnitude of these errors can lead to an overestimation or an underestimation. For example, for an absolute magnitude of 10% it can be -10% or 10%. Therefore, we have clarified this point in the new manuscript. Now, between lines 445 and 450 we can read:

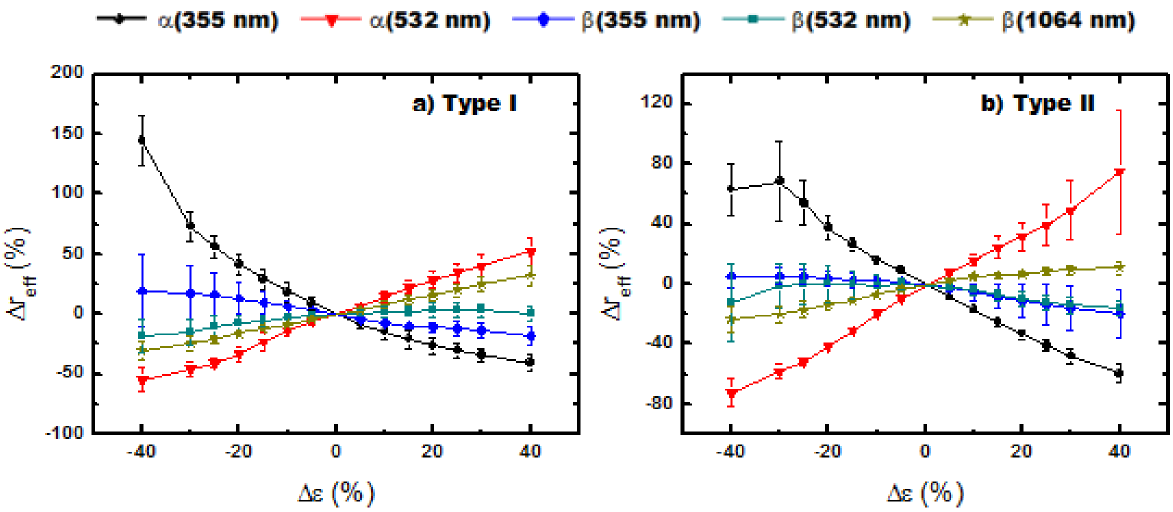
"….we performed a set of simulations where two or more optical channels were perturbed simultaneously by biases of the same magnitude, but allowing different signs (over/under estimation). For example, let's assume that we have systematic errors of absolute magnitude of 5%. Then different combinations of ±5% are allowed, as for example at α_{355} and α_{532}, at α_{355} and β_{532} or at β_{355}, β_{532} and β_{1064}. This procedure was repeated for different sets of biases of magnitude up to 10%. ..."

We also decided to cut the effect of systematic errors up to ±20%. Larger error in the optical data could make them useless. Actually, the linear fits showed in Figure 2 and Figure 3 reveals that the errors at ±20% are the largest. This agrees with referee statement that as we increase the systematic errors we are further from the original solution. Thus, very large errors (e.g. larger than ±20%) can yield a very different solution from the original one and can affect the error propagation in a different way such that the results are not linear. As an example, we show here Figure 2 (for aerosol types I and II) of the paper but extending the systematic biases in the optical data until ±40%. As commented, we sometimes lose the linearity for biases in the optical data larger than ±20%. However, we did not claim in any statement of the paper that the linearity presented is valid for those very large biases in the optical data. Actually, in the last paragraph of section 3.2.2 we already said:

"The results here indicate, therefore, that for biases in the input data of up to 20%, whether for a single channel or multiple ones simultaneously, the solution space possesses linear properties and an additive behavior can be assumed…"

But to clarify more this point we have added in the new manuscript between lines 380 and 390:

"… At this point we would like to mention that our simulations (graphs not shown for brevity) showed some departures from the linearity shown in figures 2 and 3 and Table 1 for systematic errors larger than approximately ±30%, mainly when the absolute values of the slopes is larger than 1. We take this to be an indication that biases of approximately ±30% and larger can cause the regularization routine to choose a different solution space than the original retrieval based on data with no errors. On the other hand, up to errors of ±20%, we find that the same minimum in the solution space is generally found by the routine so the linear behavior seen in Figures 2 and 3 is taken to be a characteristic of a stable system that is displaced from its minimum point. Therefore, we selected a threshold value of ±20% where these results are applicable and stress that larger errors in the input data can cause significant and unpredictable deviations in the retrieved results …"



Sentence "simultaneous biases in the optical data of 1, 2, 5, and 10%": there remains the question in how far the use of 1% and 2 % is already having that much influence on your results that the linear error propagations results from using such small errors naturally occurs. You to force the linear behavior of error propagation by using unrealistically small errors. If I understood your final plots you use all results, i.e. 1, 2, 5, and 10% in one plot? If that is the case, then any non-linearity that appears in error propagation, let's say at 10% error might be masked by the results for the other error cases at 1% and 2%? Please comment on this and show the parameterizations for the different error levels separately.

Actually the complete sentence is "… Box-Whisker plots are used for multiple simultaneous biases in the optical data of 1, 2, 5 and 10%…". We are sorry that the referee did not understand the final plot (Figure 4 of the manuscript). Every Box-Whisker diagram is devoted only for a fixed absolute value of systematic errors. Let's say, the one

that is for 10% means that only systematic errors of ±10% are allowed. Those errors affected at least two channels. For example, we run the code with an overestimation of 10% at α_{355} and an underestimation of -10% at α_{532}. After running many of these combinations at different optical channels, we obtain a database for these absolute biases of 10% and later we plot the Box-Whisker. We did the same procedure for the cases of 1, 2 and 5%. Therefore, the parameterizations of the different errors are already shown separately in Figure 4.

To clarify this point, we have changed this expression by (lines 455-458):

"…Using this procedure, we generated for each absolute value of bias a statistical dataset that includes many different configurations of the different optical channels. Those datasets are analyzed using Box-Whisker diagrams as shown in Figure 4 for the effective radius …"

We would like also to mention that allowing different combinations of errors in the optical data yield to deal the problem as the effects of random errors in the optical data, which is done in section 3.3 of the manuscript.

Sentence: "Therefore, we conclude that the results of Table 1 can be reliably used to … biased input data". It may be the case that you can transfer the numbers from table one into parameterizations. Nevertheless, you use many assumptions in your retrievals. You use two size distributions, a limited set of error bars which favor low error cases. This to my opinion does not justify that you show the parameterizations without pointing out that this scheme cannot be generalized, if more size distributions are used.

We agree with the referee and as previously discussed our results are only for bimodal aerosol size distributions that try to be representative of most AERONET inversions. Thus, we have modified the manuscript and now can be read (lines 475-478):

"… Therefore, for the bimodal size distributions used here that cover most of those size distributions obtained by AERONET, we conclude that the results of Table 1 can be reliably used to calculate the deviations in retrieved quantities due to multiple simultaneously biased input data …"

Sentence "We take this result to be an indication that the solutions…" part for last paragraph before section 3.3. Please show a plot that exemplifies this "local minimum in the multidimensional solution space.

We believe that adding more graphs would make the paper more complex. Indeed, we give references where these plots can be found. Thus, now between lines 479 and 481 can be read:

"…We take this result to be an indication that, as mentioned earlier, the solutions found by the inversion technique generally define a local minimum in the multi-dimensional solution space (e.g. see Figures 1 of Veselovskii et al., 2002, 2012) ..."

Results of figure 4: you used 50000 biased optical data sets: what did this mean in reality? 1, 2, 5, 10% errors and then you parameterized? As you show your parameterization for up to 100%: What was the maximum ADDED error of the backscatter and extinction values (5 error values) that, for example led to, e.g. 50% and 100% error in the microphysics (as shown in the plots in figure 4)? Did you also test a large error for one input set and no error for the other four sets? Does this parameterization describe this case, too?

We guess that the referee question is about Figure 5 that is where we apply 50000 biased to the optical data. Also, we believe that the questions regarding Figure 4 have been already answered previously.

What we did in Figure 4 is, for each of the size distribution used in this work, generate 50000 random numbers that follows a normal distribution centered at zero and with the standard deviation equal to random errors in the optical data. We did this for every channel of the 3β + 2α lidar configuration. To clarify this point, we have added the following description in the new manuscript (lines 506-520):

"…The procedure used consists of generating random numbers distributed in a Gaussian way centered at zero with width according to the value of the random error to study. These random errors are applied to each optical channel of the 3β + 2α configuration. This procedure was repeated 50,000 times for each parameter studied. Also, the initiation of the random number generation is different for each channel to avoid the situation where all the random numbers are the same in every channel. Finally, we introduced for every optical data this random number and computed the corresponding error in the retrieved microphysical parameter using the slopes provided in Table 1. For every set of 3β + 2α values, the final error obtained in the microphysical parameter is the sum of the error obtained for each channel. The study of the frequency distributions of the final errors for this large number of simulations yields the effects of random errors. If the frequency distribution is a normal one, the standard deviation (Full-Width-at-Half-Maximum) provides the final error in the microphysical parameter. Moreover, if the normal distribution is not centered at zero it demonstrates an interesting property; that the presence of systematic errors in the retrieved microphysical property can be induced by random errors in the input optical data …"

We are also sorry because we did not express well what the 'x' axis represents in Figure 5. They represent the differences in the microphysical parameter between the values obtained with no errors in the input optical data and those obtained after applying random errors to the input optical data. To clarify this point, we have changed the manuscript between lines 520 and 524 to:

"… As an illustration, Figure 5 shows the frequency distribution of the differences in the microphysical parameters studied here, for all aerosol size distributions type I, II and III, where 15% random error is assumed in all the optical data. Those differences are in percentages and denoted as 'deviation' in the 'x' axis of the histograms …"

And we also make changes in the caption of Figure 5, which now can read as:

"... Frequency distributions of the different microphysical parameters for 15% random errors in the optical data using 50000 random samplings of the systematic error sensitivities shown in Table 1. The 'x' axis represents the difference between microphysical parameters with no errors in the input optical data and those affected by random errors in the optical data. Random errors were simulated by a normal distribution centred at zero and with standard deviation of 15%. The random number generator is initialized at different values for each of the 5 optical data used in the $3\beta + 2\alpha$ lidar configuration. The mean value of the deviation between the microphysical parameter affected by random error and that unaffected by random error is included in the legend..."

We decided to study the effects of both systematic and random errors up to ±20%. As shown before, very large errors would yield to no-linear patterns because of the solutions are out of the space of the original solutions. We show in the graph above that maybe the linearity can be extended up to ±30%, but we believe that establishing a threshold at ±20% is appropriate for practical applications.

When we compute the effects of random errors some values can be affected by errors larger than the established threshold of ±20%. For the normal distribution we used for generating the random errors, 75% of the data are within -1 and 1, which are after multiplied by the corresponding error. The other 25% of cases can produce deviation larger than the established ±20%, although their important only when we study random errors near to ±20%.But due to the large dataset used, these departures from linearity are expected not to have important influence in the retrieval of the effects of random errors. For random errors of 15, 10 and 5% these effects are even less important.