

1 Applying receptor models Unmix and PMF on real data set of elements in PM 2 for sources evaluation

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9

10 Abstract

11 Two advanced receptor modeling techniques Unmix and PMF were applied to a data set of daily
12 measurements of 11 elements in particulate matters (PM) of 252 samples. Samples were
13 collected every sixth day as a 24h sample in the 5 year period (1995 – 2000) in the coastal part of
14 the Herceg-Novi town (Montenegro) of the sea costal side region (Southeast Adriatic Sea). In the
15 vicinity of the sampling site road traffic is a permanent.

16 The application of the receptor models to find the emission sources in the reverse order, using
17 data set of pollutants concentrations measured on the receptor, is not enough to get satisfactory
18 real solution relying only on the results of the applied models even if used the state-of-the-art
19 models such as Unmix and PMF. In this work we applied Unmix and PMF on dataset which
20 already modeled by PCA and EF in order to show how many solutions could be find and how
21 many errors could be made as well as we harmonized these advanced models to find the most
22 realistic solution. The model Unmix has the ability to suggest the solution by self-modeling
23 while PMF model can be adjusted to calculate the solution for the number of emission sources
24 that we have set. Unmix found thirteen solutions in total for several combinations of species, but
25 four solutions satisfy its criteria: $\text{Min } R^2 > 0.8$ and $\text{Min } S/N > 2$. The PMF model has given 3
26 possible solutions and by further analysis the best solution of four sources was selected. F-peak
27 refinement enabled finding a more realistic solution. We noticed that for the species with many
28 missing values but, their presence is not desirable because of its harmfulness such as cadmium in
29 this work the knowledge of emission sources is very important. Due to their limitations Unmix

30 and PMF is not able to give the solution for such cases. Other simple model applied together
31 with advanced models could help to solve similar problems.

32

33 **Key words:** Modeling, Unmix, PMF, real data set

34 **Introduction**

35 A state-of-the-art multivariate receptor models are applied in the diverse fields of
36 environmentrics, chemometrics, geology and remote sensing. Multivariate receptor modeling is a
37 term applied in the field of air quality for the solution of the general linear mixture problem. For
38 conservative chemical species, i.e. those that do not undergo reactions in the atmosphere, the
39 principle of mass balance is applied. The mass balance for species i can be written as:

$$40 C_{ij} = \sum_{k=1}^N a_{ik} S_{kj} \quad i = 1, \dots, m, j = 1, \dots, n \quad (1)$$

41 In this equation, C_{ij} is the observed concentration of species i in sample k , S_{kj} is the total amount
42 of particulate mass from source k in sample j and a_{ik} it the composition fraction of species i from
43 the source k . In air quality studies, the units of C_{ij} are usually micrograms per cubic meter. Thus,
44 since a_{ik} is a dimensionless mass fraction, the units of S_k are also micrograms per cubic meter.
45 Eq. (1) is the physical basis of all receptor models. C_{ij} is subject to random error and a_{ik} to
46 random variations (Henry, 2002).

47 Unmix seeks to solve the general mixture problem where the data is assumed to be a linear
48 combination of an unknown number of sources of unknown composition, which contribute an
49 unknown amount to each sample. Unmix assumes that the data and the compositions of the
50 sources are all strictly positive (because of the effects of errors, small values less than zero are
51 allowed in order to reduce the bias in the results). Unmix further assumes that for each source
52 there are some samples that contain little or no contribution from that source. For a given
53 selection of species, Unmix estimates the number of sources, the source composition, and source
54 contributions to each sample. The usual analytical approach to fitting the model in Eq. (1) is to
55 find the values of a_{ik} and S_{kj} that minimize the weighted mean square error F (Henry, 2002) of
56 the model:

$$57 F = \sum_{i=1}^m \sum_{j=1}^n (w_{ij} C_{ij} - \sum_{k=1}^N a_{ik} S_{kj})^2 \quad i = 1, \dots, m, j = 1, \dots, n \quad (2)$$

58 Unmix diagnostic edges plots are used to show how well-defined one or more edge is by the
59 data. If the edge plots show that all the edges are straight and well defined, then the Unmix

60 results should be more reliable and should be preferred over the PMF results (Henry and
61 Christensen, 2010).

62 The General Mixture Problem and the special case of multivariate receptor modeling are ill
63 posed problems. There are simply more unknowns than equations and thus there are many wildly
64 different solutions that are all equally good in the least-squares sense. In a statistical way these
65 problems are not identifiable. One approach to ill-posed problems is to impose conditions that
66 add additional equations, which then define more realistic solutions to be closer to unique
67 solution. The non-negativity conditions as additional conditions are imposed by the physical
68 nature of the problem (Henry, 2001). Source composition and contributions must be non-
69 negative but non-negativity conditions alone are not sufficient to give a unique solution. More
70 constraints are needed (Henry, 1987). Under certain, rather mild conditions, the data themselves
71 can provide the needed constraints (Henry 1997). This is how Unmix works.

72 Based on the multivariate factor analysis and the results in factor profiles and contributions,
73 Paatero and Tapper (Paatero and Tapper, 1993; Paatero and Tapper, 1994; Paatero, 1997)
74 established the advanced factor analysis method - positive matrix factorization (PMF). Several
75 features are incorporated in this model:

- 76 - weights data points by their analytical uncertainties,
- 77 - constrains factor loadings and factor scores to non-negative values and thereby minimizes
78 the ambiguity caused by rotating the factors,
- 79 - uses weighted least-squares fits for data,
- 80 - expresses factor loadings in mass units, which allows factors to be used directly as source
81 signatures,
- 82 - provides uncertainties for factor loadings and factor scores.

83 In PMF, the matrix \mathbf{X} ($n \times m$) includes measured mass concentrations, and is represented
84 as the sum of the product of \mathbf{G} ($n \times p$) and \mathbf{F} ($p \times m$) matrices and the residual matrix \mathbf{E} ($n \times m$),
85 where \mathbf{n} is the number of samples, \mathbf{m} is the number of chemical species, and \mathbf{p} is the number of
86 independent source types. This model can give a solution that can be displayed in matrix form:

$$87 \quad \mathbf{X} = \mathbf{G} \cdot \mathbf{F} + \mathbf{E} \quad (3)$$

88 The object function Q that is to be minimized is defined as:

$$89 \quad Q = \sum_{i=1}^n \sum_{j=1}^m (\varepsilon_{ij}/u_{ij})^2 \quad (4)$$

90 where u_{ij} is the uncertainty of the species j in a sample i and residuals ϵ_{ij} i.e. the portion of the
91 measured concentration.

92 In addition, non-negativity constraints should be fulfilled, meaning that all the elements
93 in G and F are to be non-negative. The main process of the PMF is minimizing the Q -value
94 which is defined in the Eq. (4) as the sum of square of the residuals (ϵ_{ij}) weighted inversely with
95 uncertainty (u_{ij}) of the data point (Polissar et al., 1998; Lee and Hopke, 2006).

96 The solution of Eq. (4) is obtained by iteration until convergence is reached.

97 Bootstrapping is an advanced analysis that examines the stability of solutions of the
98 tested models. The bootstrap method is essentially based on resampling methods in which “new”
99 data sets that are consistent with the original data are generated. Each “new” data set (which is
100 essentially a subset of the original database), is decomposed into profile and contribution
101 matrices, and the resulting profile and contribution matrices are compared with the base run
102 (Eberly, 2005), giving the distribution for each species to evaluate the stability of the solution.

103 Numerous studies employing both the PMF and Unmix models have been done in recent years
104 (Pekney et al., 2006; Poirot et al., 2001; Kim et al., 2004; Chen et al., 2007 in: Hegg et al., 2010).

105 Paatero’s positive matrix factorization (PMF) approach weights the data by the inverse of the
106 measurement error for each observation. A major advantage of this approach is that the missing
107 data can be included as observations with a large error. However, the minimization of F is still
108 an ill-posed problem, or in other words, the model is not identifiable. Even the inclusion of the
109 non-negative constraints does not provide an identifiable model. Paatero addresses this problem,
110 which he named rotationally indeterminacy, by adding one or more user-selected parameters.
111 Park et al., (2002) have used modern constrained minimization methods on F along with specific
112 conditions, e.g. each source composition must have at least one species absent from that source.
113 Finally, Paatero has generalized F in a natural way to include the estimation of even more
114 unknown parameters associated with spatial variations (Henry, 2002).

115 Multivariate source apportionment models, Unmix and positive matrix factorization (PMF),
116 often produce nearly the same source apportionment, however some investigations have shown
117 that this is not always the case (Henry and Christensen, 2010). These models do not specify a
118 minimum number of samples, but the stability of their solutions increases with the number of
119 samples (Chen et al., 2007). In this study, we calculated sources composition and sources
120 contributions of elements in PM using real data base.

121 The main aim of this study is to show that a simple application of the most advanced
122 mathematical models may leads to erroneous conclusions because each of these models can
123 provide a larger number of mathematically correct solutions. Which solutions are really true
124 cannot be known only on the basis of the results obtained by modeling, even using models such
125 as Unmix and PMF. Our goal was to apply these state-of-the-art models, respecting their criteria,
126 on data-base previously submitted to other models; Principal Component Analysis (PCA) and
127 Enrichment Factors (EA) to compare, to be able to finding the most accurate solution relying on
128 Unmix self-modeling and PMF application to adjust and confirm the solutions found by Unmix.

129

130 **Materials and Methods**

131 The sampling site is situated only 10 meters away from the coast of the Adriatic Sea. Samples of
132 PM were subjected to gravimetric analysis for determination of total mass concentrations and
133 subsequently to elemental analysis for Fe, Mn, Ti, Pb, Cr, Cu, Cd, Co, Ni, Hg and Se. Suspended
134 particles were collected using a high-volume Aerosol Sampler, AQUERO model 400XT
135 sampling system, on boron-silicate fiberglass filters every sixth day as a 24h sample in the period
136 of 1995 - 2000. The sampler was located in the town of Herceg-Novi (Fig. 8) 18⁰33" N, 42⁰27",
137 Montenegro (Fig. 1). The meteorological station is part of the MED POL program. The nearest
138 road is located about 100 m north of the meteorological station. There are no significant grassy
139 areas around the meteorological station, and there is no considerable construction work in
140 progress. The terrain surrounding the receptor is rocky with some small areas of soil (Đorđević et
141 al., 2004). Filters were digested with HNO₃ (ultra pure). A Flame Atomic Absorption
142 Spectrometry (F-AAS), Varian AAS–Spectr AA 55 instrument, was used to measure the
143 concentrations of Cd, Co, Cr, Cu, Ni, Pb, Ti, Fe and Mn. The concentrations of Hg and Se were
144 determined by the hydride vapor AAS method (HV-AAS) (Đorđević et al., 2005). The maximum
145 expanded uncertainty of measurements for all elements was about 5%.

146 The real data set of 11 trace elements in particulate matter (PM) obtained in 252 observations
147 was analyzed by Unmix 6.0 and PMF 3.0. The applied Unmix and PMF models were available
148 on the EPA Internet site (www.epa.gov).

149 Unmix and PMF used in this study do not limit the number of factors. The following initial
150 operations were subjected to the Unmix model data: *Suggest Exclusion*, *Initial Species*,

151 *Additional Species* including SAFER and Initial Points. PM was chosen for the total and for the
152 normalization. The data was screened using the signal-to-noise ratio (Min S/N ratio) criteria
153 higher than 2, estimated by Unmix. Only the component with S/N value greater than 2 will be
154 used for sources estimation. The agreement between the true and estimated source contribution
155 (Min R² greater than 0.8) was considered as well (Henry, 2003, EPA/600/R-07/089).

156 Applying of PMF model the procedure of Polissar et al. (1998) was used in this study to
157 calculate uncertainties in the species concentrations. Briefly, for the data below detection limit
158 (DL), the concentrations were replaced with the value DL/2 and the uncertainty was set as $\frac{5}{6}DL$.

159 For the missing data, concentrations were replaced by the geometric mean and the respective
160 uncertainty was set at four times of this mean concentration. At the first set up all elements are
161 labeled as Strong, since (the signal/noise ratio) $S/N > 2$ for all of them. Based on input data
162 statistics, residuals show bimodal distribution in the case of Ni, Mn, and Hg, so their
163 uncertainties are increased labeling them as Weak. Selenium is excluded from the model because
164 of a very small contribution and the correlation factor, while for cadmium more than 50% of
165 samples are below the detection limit. The Q value represents the goodness-of-fit and assesses
166 how the model fits the experimental data. Q_{true} is calculated taking into account all data points
167 while Q_{robust} is calculated accounting for outlier points. Data with scaled residuals above 4 are
168 regarded as outlier points. Evaluation of the validity of a solution is possible by using the G-
169 space scatter plot. Scatter plot of one versus the other factor may indicate the existence of a
170 rotational ambiguity. Namely, if the points on this graph fill the entire solution space evenly then
171 the edges of the Scatter Plot correspond to axes. If this is not a case it is indication that there is
172 rotational ambiguity and should be considered the possible rotation of the solution, using the
173 function Fpeak. The F-peak functions is used to rotate the data set, make fine tuning and
174 improvement of the model in the case of data with high noise (positive values F-peak) or clean
175 data (negative values F-peak). Normally, the default settings give satisfactory results, but in
176 some cases subsequent adjustments are needed. To ensure the robustness of statistics, 300
177 bootstrap runs were performed, while the default value of the minimum correlation (R-Value) of
178 0.60 was used.

179

180

181 Results and Discussion

182 We applied the Unmix and PMF models on dataset from our previous work regarding trace
183 elements in the PM (Đorđević et al., 2005). Fig. 2 shows the comparison of measured and the
184 predicted concentrations of trace elements in PM through time series and Min R^2 . Model Unmix
185 did not calculate R^2 values for Cd, Co, Hg and Se and neither satisfactory solution included these
186 elements since these variables contain a large number of missing values and outliers. Min R^2
187 values are given in table 1.

188 From statistical parameters displayed for each species, after input data and the following
189 operations: *Suggest exclusion*, *Influential points*, *Initial species*, *Additional species* and *SAFER*,
190 Unmix finds six combinations of species that give any kind of solution (Table 2). Min S/N for
191 each principal component and Min R^2 of all combinations of elements estimated by Unmix was
192 selected as good solutions that are in accordance with the Unmix criteria (Henry, 2003). Thirteen
193 solutions in total were found, but four solutions satisfy the above criteria, signed in bold in Table
194 2. The standard deviation of variable (sigma) is the criterion for evaluation whether the variable
195 eligible for modeling or not. The sigma-based parameters (Significant/Strong Species in Sources)
196 for each of satisfactory solution are also given in Table 2.

197 Taking into account the calculated good solutions presented in Table 2, the Edges plots were
198 done for these solutions (Fig. 3). The source profile of the solutions chosen according to the
199 criteria $S/N > 2$ and $R^2 > 2$ are given in Fig. 4.

200 In the first solution (combination of species Mn-Ti-Pb-Cr-Cu, 3 Sources Solution) the second
201 and third source are well defined by many points on the y-axis while source 1 has just a few
202 points on the x-axis. Pb is strong in the first source and this source can be attributed to traffic. In
203 the second source Cr and Cu are strong and Ti and Mn are significant. This source can be re-
204 suspension of elements previously settled from anthropogenic sources. In the third source neither
205 element is strong or significant.

206 The second satisfactory solution is for Fe-Mn-Ti-Pb combination of elements it also found 3
207 sources (Table 2, Fig. 4b) and does not show good accumulation of points on the x and y axes
208 (Fig. 3). This solution has the best values of Min R^2 and Min S/N compared to all combinations.
209 The first and the third source contain Pb which is a tracer for traffic. In the third source Pb is
210 strong, and it is reasonable to associate this source with traffic, while the first source could be

211 local re-suspension. The second source in this combination could be a long range transport of
212 Saharan dust since it contains crustal elements.

213 The third solution (combination of species Fe-Mn-Ti-Pb-Cr-Cu-Ni, 4 Sources Solution) shows
214 the edges on the y-axis defined by many points for the third and the fourth source, but the x-axis
215 has just a few points (Fig. 3).

216 In the fourth solution (combination of species Fe-Mn-Ti-Pb-Cr-Cu, 3 Sources Solution) good
217 accumulation of points are on the y-axis, for sources 2 and 3 while the x-axis has just a few
218 points for source 1 (Fig. 3).

219 In the third and the fourth solution, the sources where Pb is strong can be attributed to traffic;
220 namely, source 3 for Fe-Mn-Ti-Pb-Cr-Cu-Ni combination (Table 2, Fig. 4c) and source 1 for Fe-
221 Mn-Ti-Pb-Cr-Cu combination (Table 2, Fig. 4d). Another source in which Pb is present as
222 significant but not strong could be re-suspension. Source 4 for Fe-Mn-Ti-Pb-Cr-Cu-Ni
223 combination and source 2 for Fe-Mn-Ti-Pb-Cr-Cu combination could be attributed to re-
224 suspension, probably from various locations depending on wind directions. Factors containing
225 Cr and Ni indicate the existence of an anthropogenic emission source in the region (Đorđević et
226 al., 2005).

227 In our previous work (Đorđević et al., 2005) we applied the PCA method on this data set and 4
228 significant groups of sources contributions were found. The following contribution sources were
229 identified: re-suspension combined with re-suspended Saharan dust that had previously settled
230 (Fe, Mn, Ti) and settled combustion products mostly from traffic, and probably some local
231 stationary source (Cu, Pb). The remaining three factors represent the following combinations F2
232 by Cr and Ni, F3 by Cd and Se and F4 by Hg and Co.

233 The EF model revealed that in the region of the investigated receptor, the main contribution
234 source of Fe, Mn and Ti is the process of local re-suspension and that local re-suspension has no
235 influence on the content of Se in the atmospheric aerosol. The re-suspension is the dominant
236 emission source of Cd from the south-southeast direction from the nearby peninsula (Luštica) but
237 this source is not permanent (Đorđević et al., 2005).

238 The application of positive matrix factorization (PMF) to solve the number and profile of the
239 sources applied to the same database resulted in obtaining possible solutions for 3, 4 and 5
240 sources. For 6 or more sources the model does not find the convergence of the functions Q,
241 which implies that the model did not find any minima. Varying simulation conditions did not

242 contribute to significant improvement, even when the uncertainty is significantly increased.
243 Therefore possible solution should be sought among three possible cases.

244 Table 3 shows the categories of elements and the R^2 values for each of the three possible
245 solutions.

246 Each of the possible solutions obtained by PMF analysis will be considered. Fig. 5 shows F peak
247 strengths for 3 sources solution (Fig. 5a), 4 sources solution (Fig. 5b) and 5 sources solution (Fig.
248 5c).

249 3 Sources Solution: The relatively good correlation was obtained only for Cr and Pb, while
250 bimodal distribution is still present in the case of Co, Ti and Fe. Also, significant outliers are
251 present in the model. In addition, G-Space plots show considerable rotational ambiguity between
252 the sources 1 and 3.

253 Rotational ambiguity, which was found between the sources 1 and 3, decreases when the value
254 of Strength factor reaches -1.2 (Fig. 5a). This is mostly reflected in the increase of Ti
255 concentration in the source 2. However, such large values for F_{peak} is unlikely because the
256 quality of the fit decreases rapidly. The usually dataset rotations are generally much smaller and
257 they are close to the basic solution.

258 However, a small degree of correlation between the model and database indicates that the model
259 with three sources is insufficient to adequately describe a number of sources.

260 In this case, only Co, Ni and Fe show relatively good interquartile range of about 20%, while
261 other species show considerable variation and therefore represent a less stable solution. This is
262 especially pronounced in the case of Hg, Cr and Mn. Also, in some cases (Hg, Ti) base run
263 values are not within the interquartile range in the bootstrapping of results. This is probably a
264 consequence of assuming the model with only three sources. Profiles of sources are given in Fig.
265 6

266 4 Sources Solution: The model with four sources shows a significantly better correlation with
267 measured concentrations of elements. Although the agreement of time series for Ti and Cr is
268 excellent ($R^2 > 0.95$), and for Pb satisfactory ($R^2 = 0.70$), in the case of other elements there are
269 still episodes with very high concentrations that this model cannot fit. It should be noted that Cu
270 shows very good agreement between the predicted and observed concentrations, but the
271 existence of outliers have reduced the correlation to 0.34. A small degree of correlation in the
272 case of Co is the result of a significant number of measurements below the detection limit.

273 Bimodal distribution is still present in the case of Ni and Hg. G-Space plot shows that there is a
274 rotational ambiguity between sources 1 - 3 2 - 3, 3 - 4.

275 For a model with four sources, rotational ambiguity disappears when the F-Peak strength reaches
276 -0.8 (Fig. 5b). This rotation is mostly reflected in the increase of Ti content in source 1, and
277 largely in sources 2 and 3. On the other hand, this may just mean that the content of titanium in
278 this solution is divided among several sources. As in the case of a solution with three sources, a
279 significant rotation of the dataset ($F_{peak} = -0.8$) is less likely. It is necessary to consider these
280 results carefully and determine whether there is justification for it to be included in further
281 solving of the composition of the sources.

282 Interquartile range of solutions obtained by bootstrapping in the case of Fe, Pb, Cu and Cr are
283 about 20%, while in the case of other species this range is much higher indicating the instability
284 of the solution. Base run values which are not within the interquartile range in the bootstrapping
285 of the results are, in the case of Cu, Mn, Pb and especially Hg, calculated by the model only in
286 the fourth source.

287 5 Sources Solution: The model and data from the database show agreement (R^2) over 90% for
288 Cr, Ti, Fe and Pb, while just over 50% for Mn. The model also fits Cu real data very well, and
289 the correlation of 0.47 is caused by significant outliers that are related to individual episodes of
290 Cu emissions. In spite of the increased uncertainty Mn, Ni, Co and Hg show a lack of fit.

291 G-Space plot only shows some rotational ambiguity in the case of sources: 2 - 5, 3 - 5 and 4 - 5.
292 The F-Peak in the range -2 to 2 (Fig. 5c) showed the most impact on the sources of Cu,
293 especially at higher strength values, while the ambiguity between the sources mentioned above
294 still exist. The Peak F-curve is generally symmetrical in the examined interval.

295 In the case of five sources there are also unmapped results, suggesting a reduced stability of the
296 solution. In general, the most stable solutions are obtained for those elements that are present in
297 the source with the highest percentage. In these cases, the distribution of solutions obtained by
298 bootstrapping lie in the range of 15% of the concentration calculated in the base run. This is the
299 case for Fe, Pb, Ti. A slightly worse result of the bootstrap analysis is obtained for Cu, Mn, Cr,
300 Co and Ni (bootstrapping distribution of solutions equal or higher than 20%). The least stable
301 solution is for Hg with considerable dispersion in the bootstrap analysis solutions.

302 When discussing the number and origin of pollution sources, it is preferred to take into account
303 the real situation on the field. In this case the following sources that contribute to the overall PM

304 deposition can clearly be predicted: marine aerosols, traffic, re-suspension from the ground,
305 probably some local stationary source, as a shipyard located in the vicinity. Based on these
306 obvious sources, PMF analysis solution with only three sources is exempt from further
307 consideration.

308 In the case of PMF solution of five sources it may be noted that source No. 5 (Fig. 8), in which
309 Co, Cu, Ni and Mn are present, can be described rather as a splitting factor than as a separate
310 source. The most realistic solution that is imposed upon a detailed analysis is the solution with
311 four sources (Fig. 7).

312 Identification of sources was carried out and it agrees with the results of the Enrichment Factors
313 analysis well (Đorđević et al., 2005). The F-peak profiles shown in Fig. 7 in rotation of data set
314 for -0.8, increase the contents of Ti, in the case of sources 2 and 4.

315 Source 1 has been identified as re-suspension in combination with the long-range transport of
316 Saharan dust. The prevailing wind directions are over open sea (Đorđević et al., 2005).

317 Source 2 is attributed to the re-suspension, indicated in our previous work. Titanium found by F-
318 peak is in better accordance with the EF analysis (Đorđević et al., 2005).

319 Source 3 corresponds to the composition of the particles that come from some anthropogenic
320 source.

321 Source 4 with the highest content of Pb, is characteristic for urban traffic. F-peak is increasing
322 the value for Ti which is in better agreement with the traffic profile.

323

324 **Conclusion**

325 In this work we applied state-of-the-art mathematical models Unmix and PMF on database
326 previously modeled using more simple models (PCA and EF) to be compared. In this study we
327 have shown that only application Unmix and PMF for sources apportionment is not guarantee to
328 obtain the unique realistic solution. Thirteen solutions in total were found, but four solutions
329 satisfy the Unmix criteria: three solutions with three sources and one with four sources. In terms
330 of modeling all four solutions found by Unmix are satisfactory. PMF model has given three
331 possible solutions: one with three sources, one with four sources and one with five sources. By
332 further analysis of the results of PMF model the best solution with four sources was selected. F-
333 peak refinement was enabled to find a more realistic solution. Also we have shown that due to
334 their limitations Unmix and PMF were unable to calculate Cd and Se in used database, due to

335 large number of missing values. For example, although the presence of cadmium in terms of
336 concentration is negligible and there are many missing values the knowledge of emission sources
337 is very important regarding its harmfulness. The simple model of EF applied could help to solve
338 similar problem. For obtaining the best results using Unmix and PMF models our
339 recommendation is to start modeling by Unmix relying on its self-modeling to estimate all
340 possible types of sources and then apply PMF for confirmation. For the species that are
341 important and that cannot be modeled by advanced models like Unmix and PMF should be apply
342 other, even, the simple model.

343

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348

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405

406 **Tables**

407 Table 1. R² values obtained by Unmix of measured and the predicted concentrations of PM and
408 trace elements in PM

	PM	Fe	Mn	Ti	Pb	Cr	Cu	Ni
R ²	0.46	0.75	0.92	0.66	0.83	0.40	0.99	0.00

Table 2. All combination of elements for solutions obtained by calculation by Unmix

Combination of species	Number of sources	Min R ²	Min S/N	Significant/Strong Species in Sources (sigma-based)
Mn-Ti-Pb-Cr-Cu	3	0.84	2.49	Source 1: *Strong – Pb; Source 2: *Strong - Cr, Cu, **Significant - Ti, Mn; Source 3: *Strong – None, **Significant - None
	4	0.89	1.94	
	5	0.90	1.59	
Cr-Cu-Pb-Ti-Mn-Se-Cd-Co-Fe	3	0.68	2.41	
	4	0.76	2.13	
Cu-Ti-Fe-Mn-Pb-Cr-Hg-Se	2	0.56	2.29	
Fe-Mn-Ti-Pb	3	0.90	2.85	Source 1: *Strong – None, **Significant - PM, Pb, Fe Source 2: *Strong – None **Significant - PM, Fe, Mn; Source 3: *Strong - Pb, Mn, **Significant - Ti, Fe
	3	0.76	2.20	
Fe-Mn-Ti-Pb-Cr-Cu-Ni	4	0.83	2.18	Source 1: *Strong - Cr, Ni, **Significant – None Source 2: *Strong – None, **Significant – Cu; Source 3: *Strong – Pb, **Significant - PM, Cr, Cu, Ti, Fe, Mn; Source 4: *Strong – Ti, **Significant - PM, Cr, Pb, Fe
	5	0.89	1.67	
	3	0.83	2.57	Source 1: *Strong - Pb, **Significant – Cu; Source 2: *Strong – None, **Significant - PM, Cr, Pb, Ti, Fe; Source 3: *Strong – None, **Significant - PM, Cr, Cu, Ti, Fe, Mn
Fe-Mn-Ti-Pb-Cr-Cu	4	0.88	1.97	
	5	0.90	1.62	

* Source Composition ≥ 1 sigma

**Source Composition ≥ 2 sigma

Table 3. R^2 values obtained by PMF of measured and the predicted concentrations

Species	Category	R^2		
		3 sources	4 sources	5 sources
Cr	Strong	0.617	0.980	0.998
Ti	Strong	0.381	0.962	0.959
Fe	Strong	0.401	0.472	0.942
Pb	Strong	0.695	0.701	0.905
Mn	Weak	0.394	0.340	0.528
Cu	Strong	0.345	0.337	0.473
Ni	Weak	0.027	0.031	0.025
Co	Weak	0.006	0.013	0.018
Hg	Weak	0.002	0.005	0.003

Figure Legends

Fig.1 Sampling site and prevailing wind directions

Fig. 2 Predicted and measured concentrations

Fig. 3 Edge plots for chosen solutions that satisfy the conditions of Min S/N and Min R^2

Fig. 4 Source profiles for selected solutions that are in accordance with the Unmix criteria

Fig. 5 F-peak analysis for three a), four b) and five c) source solutions. The red mark represents the value of F-peak Strength, at which the rotational ambiguity disappears.

Fig. 6 Profiles in the case of three sources solutions. Comparison of base run profile and F-peak run profile with the strength of -1.2 (disappearance of rotational ambiguity).

Fig. 7 Profiles in the case of three sources solutions. Comparison of base run profile and F-peak run profile with the strength of -0.8 (disappearance of rotational ambiguity).

Fig. 8 Profiles in the case of three sources solutions. Comparison of base run profile and F-peak run profile with the strength of -2.0 where it can be seen that F-peak Strength does not affect the existing rotational ambiguity.

Answers to the **Reviewers 1 comments**

General Comments: In the present manuscript, Dordevic et al. apply two advanced popular receptor modelling techniques (Unmix and PMF) to a dataset of concentrations of 11 trace metals in PM samples obtained from a coastal site close to the Adriatic Sea. The dataset of metal concentrations is re-used from an earlier study (Dordevic et al., 2005), where two other receptor models have been applied to the data. My major problem with the manuscript is that I do not understand what it is actually aiming at:

- Is it aiming at a better understanding of the sources of trace elements at the receptor site, as the manuscript title would suggest? The major part of the discussion section actually reads like a source apportionment study attributing the different solutions of the models to physical PM/metal sources. However, as the authors indicate several times in the manuscript, all the conclusions regarding the different possible metal sources agree with the results from the previous Dordevic et al., 2005, study. And I have to agree: I do not see any scientific progress in the present data analysis beyond what has already been understood/proposed in the previous study (on exactly the same dataset). In addition, if the goal really was a refined or more detailed source apportionment then AMTD might not be the most suitable journal choice.

We corrected the Manuscript according to reviewers complying. We rewritten the Abstract, Conclusion and we more precisely rewritten the goal and discussion of the Manuscript. In yellow we highlighted our changes.

- Or is the main goal of the manuscript an evaluation of the (more complex) Unmix and PMF models as compared to the (more simple) approaches of the previous study, as is stated in P4948L15-19? Such evaluation would certainly fit better into the journal scopes, but I do not see which part of the manuscript would really critically evaluate the models. To me, the whole study seems to be a sheer application of two (further) re-ceptor models to an existing (and published) dataset, which does not represent enough scientific significance to justify publication in AMT.

The Manuscript's title has changed.

It is known that simple modeling even by state-of-the-art mathematical models is not enough to get satisfactory real solution. Our intention were not to only calculate different possible sources of metals in the PM but to use two state-of-the-art models (Unmix and PMF) on data set which already modeled in order to show how many solutions could be find and how many errors could be made as well as adjust these advanced models to find the most realistic solution. These solutions we compared with our previously work.

We made an effort to adjust our manuscript according to this goal.

If at all, the manuscript can only be considered for publication in AMT after major revisions, which would actually include a complete rewrite of several manuscript sections (abstract, discussion, conclusions).

We have rewritten the abstract, discussion and conclusions.

Specific comments: P4942L2-13: The abstract is poorly written. **The Abstract has rewritten.** It lacks important details on the dataset (location of the sampling site, time of sampling, size of dataset, etc. **It is corrected in Abstract and Materials and Methods**), it only lists the number of possible solutions of the models without really relating them to each other or concluding on their quality, it does not give the physical meanings for all of the mathematically suggested sources, and is in part even contradictory: "traffic is not a significant anthropogenic source at the sampling site" (L5) vs. "...more realistic solution that includes ... traffic as dominant source contribution" (L11-12).

The contradictory part in the Abstract is corrected.

P4942L17 and L24: I am sceptical, whether these references are really suitable to support the rather general statements/explanations in these two sentences. Be sure to avoid referencing secondary sources.

It is corrected. P4942L15 to L24 removed. References: *Lee, D., Balachandran, S., Pachon, J., Shankaran, R., Lee, S., Mulholland, J. A., and Russell, A. G., 2009. Ensemble-Trained PM2.5 Source Apportionment Approach for Health Studies, Environ. Sci. Technol., 43, 7023-7031* **and** *Lee, J. H., Lim, J. M., Kim, K. H., Chung, Y.S., and Lee, K.Y.,2003. Trace element levels of aerosols at an urban area of Korea by instrumental neutron activation analysis, J. Radioanal. Nucl. Ch., 256, 553–560* **was excluded.**

P4942L15-P4947L19: In the introduction, the Unmix and PMF models are described in much detail. The authors might want to check whether such level of detail is really necessary to understand the results and discussion sections of the manuscript (which actually depends on what the latter sections are aiming at).

P4945L7 to L9 removed

P4946L6 to L10 removed

P4946L13 to L21 removed

P4947L3 to L14 removed

P4945L16-P4946L2: This section would be more appropriately placed in the experimental section

It is moved in experimental section

P4947L4-14: This section seems a bit out of place here.

It is excluded from the Manuscript

P4947L15-19: The main goal of the study has to be more clearly defined (see above).

It is corrected

P4947L20-P4948L17: The Materials and methods section is poorly written. It contains

paragraphs which have to be moved to other sections (e.g. the first 7 lines are rather introductory than experimental – **It is corrected**), the information on sampling and analysis is incomplete (I miss details on the time of sampling (in which months/years?- **It is corrected**), the country the site belongs to (Google maps did help here but it might be worth to mention Montenegro in the manuscript - **It is corrected**), and data on the manufacturers of the applied instruments and materials – **in P4948L4 to L6 and P4948L12 to L17 have given**), and – most importantly – it does not give the experimental details of the two models applied. These are actually spread among the introduction (see above) and results sections, but they should be brought together here in this section - **It is corrected**.

P4948L19-27: These are experimental details, not results

It is moved to Material and Methods part

P4949L3: Why is R2 not available for these elements?

Model Unmix did not calculate R² values for Cd, Co, Hg and Se since these variables contain a large number of missing values and outliers. It is corrected in the manuscript.

P4949L12-13: How do the authors distinguish between a “strong” and a “significant” species and what does it actually mean?

Distinguish between a “strong” and a “significant” species have done according to criteria Source Composition > 1*sigma for Strong species and Source Composition > 2*sigma for Significant species. The criterion for evaluation whether the variable eligible for modeling or not is standard deviation of variable (sigma) that Unmix model is calculating among the first criteria for involving the variables in the further calculation.

It is corrected in the Manuscript

P4949L14-15: The authors might want to explain the benefits of “edge plots” in the introduction or experimental section.

It is added in the Introduction

P4950L2-22: Not results, move to experimental and/or introduction

It is moved to Introduction

P4951L3-7: I do not understand this paragraph. Please give more explanations.

It is excluded from the Manuscript

P4951L15: Please explain G-Space plots in introduction/experimental

It is added in the Material and Methods part

Evaluation of the validity of a solution is possible by using the G-space scatter plot. Scatter plot of one versus the other factor may indicate the existence of a rotational ambiguity. Namely, if the points on this graph fill the entire solution space evenly then the edges of the Scatter Plot correspond to axes. If this is not a case it is indication that there is rotational ambiguity and should be considered the possible rotation of the solution, using the function F-peak.

P4951L24 and elsewhere: In my understanding of descriptive statistics, the interquartile range (IQR) is always the 25% trimmed mid-range of a data distribution, i.e. representing 50% of the data points. How can it be 20% only and why would that be a “relatively good IQR”?

It is corrected.

Interquartile range of solutions obtained by bootstrapping in the case of Fe, Pb, Cu and Cr are about 20%, while in the case of other species this range is much higher indicating the instability of the solution.

In these cases, the distribution of solutions obtained by bootstrapping lie in the range of 15% of the concentration calculated in the base run.

(bootstrapping distribution of solutions equal or higher than 20%)

P4953L14-4955L24: As stated above, the discussion section does not fit into the scopes of AMT as it is focused too much on the source apportionment aspect of the study and hardly at all on the evaluation of the models.

Now is Results and discussion section. Part P4953L14 to P4954L28 has moved behind results of Unmix in the section Results and discussion to make is easier to follow and to better see why it is important to compare with previously work. The discussion is shortened to necessary level.

P4951L4 to L7 have excluded.

P4955L25-P4956L7: The conclusions are not conclusive. What are the main findings of the study? And why is it important for them to be published?

It is corrected

Figures and Tables: - Fig. 1 is not readable. Way too small font size. –

The Fig. 1 in the corrected Manuscript is Fig 2. This figure is original results from the Unmix model and this is reason of pure quality. We made an effort to improve as much as possible the quality.

Figure Numbering does not correspond to the appearance of the figures in the manuscript (e.g. Fig. 8 is the first one mentioned in the text).

We corrected the numbering of the figures. The Fig. 8 now is Fig. 1. In the text of the corrected Manuscript in the red color is the number of figures.