Appendix

A) Reformulation of B1999 correction

The filter transmission function for the B1999 correction without particle scattering is given by

$$f_{B1999}(\tau(t)) = \frac{\sigma_{ap}(t)}{\sigma(t)} = \frac{1}{c_1 \cdot \tau(t) + c_2}. \quad \text{(A1)}$$

First, the independent variable *t* is replaced by the length *l*, which is the column of air sucked through the filter. Using the basic relations $\delta(l) = -\ln(\tau(l))$ and $\sigma(l) = \frac{d}{dl}\delta(l)$ we get

$$\sigma_{ap}(l) = \frac{\frac{d}{dl}\delta(l)}{c_1 \cdot e^{-\delta(l)} + c_2}. \quad \text{(A2)}$$

Integration of Eq. (A2) in the interval *l*=[0,L] and using the relation $\int_0^L \sigma_{ap}(l)\, dl = \delta_{ap}(L) - \delta_{ap}(0)$ leads to

$$\delta_{ap}(L) - \delta_{ap}(0) = \int_0^L \left[\frac{\frac{d}{dl}\delta(l)}{c_1 \cdot e^{-\delta(l)} + c_2} \right] dl. \quad \text{(A3)}$$

The right hand side may be rewritten as

$$... = \int_0^L \left[\frac{e^{\delta(l)} \frac{d}{dl}\delta(l)}{c_1 + c_2 e^{\delta(l)}} \right] dl. \quad \text{(A4)}$$

Substituting $x(l) = c_1 + c_2 e^{\delta(l)}$ and $\frac{d}{dl}x(l) = c_2 e^{\delta(l)} \frac{d}{dl}\delta(l)$ leads to

$$... = \int_0^L \left[\frac{\frac{1}{c_2}\frac{d}{dl}x(l)}{x(l)} \right] dl = \frac{1}{c_2}\int_0^L \left[\frac{d}{dl}\ln(x(l)) \right] dl = \frac{\ln(x(L)) - \ln(x(0))}{c_2} \quad \text{(A5)}$$

With boundary condition for an initially unloaded filter $\delta_{ap}(0)=0$ and $\delta(0)=0$, Eq. (A3) can be written as

$$\delta_{ap}(L) = \frac{\ln\left(c_1 + c_2 e^{\delta(L)}\right) - \ln(c_1 + c_2)}{c_2}. \quad \text{(A4)}$$

Further reformulation to separate the filter optical depth yields:

$$\ln\left(c_1 + c_2 e^{\delta(L)}\right) = c_2\delta_{ap}(L) + \ln\left(c_1 + c_2\right) \quad , \tag{A5}$$

$$c_2 e^{\delta(L)} = e^{c_2\delta_{ap}(L)+\ln(c_1+c_2)} - c_1 \quad , \tag{A6}$$

$$\delta(L) = \ln\left[\frac{e^{c_2\,\delta_{ap}(L)+\ln(c_1+c_2)} - c_1}{c_2}\right] = \ln\left[\frac{(c_1+c_2)e^{c_2\,\delta_{ap}(L)} - c_1}{c_2}\right]. \tag{A7}$$

B) Reformulation of V2005 correction

The filter transmission function for non-scattering particles is given by

$$f_{V2005}(\tau(t)) := \frac{\sigma_{ap}(t)}{\sigma(t)} = c_1 + c_2 h_0 \ln(\tau(t)) . \tag{B1}$$

The independent variable *t* is replaced by the length *l*, and with relations $\delta = -\ln(\tau)$ and $\sigma(l) = \frac{d}{dl}\delta(l)$. Eq. (B1) may be written as

$$c_1 \frac{d}{dl}\delta(l) - c_2 h_0\, \delta(l) \frac{d}{dl}\delta(l) = \sigma_{ap}(l) . \tag{B2}$$

Eq. (B2) is solved by integration over the interval *l*=[0,L]

$$\int_0^L \left(c_1 \frac{d}{dl}\delta(l) - c_2 h_0\, \delta(l) \frac{d}{dl}\delta(l) \right) dl = \int_0^L \sigma_{ap}(l)\, dl . \tag{B3}$$

The solution of the integral equation is

$$c_1\, \delta(L) - \frac{1}{2} c_2 h_0\, \delta(L)^2 - c_1\, \delta(0) + \frac{1}{2} c_2\, h_0 \delta(0)^2 = \delta_{ap}(L) - \delta_{ap}(0) . \tag{B4}$$

With the boundary conditions for an initially unloaded filter $\delta(0) = 0$ and $\delta_{ap}(0) = 0$, rearranging Eq. (B4) yields

$$c_1\, \delta(L) - \frac{1}{2} c_2 h_0\, \delta(L)^2 = \delta_{ap}(L) \quad . \tag{B5}$$

The solution of the quadratic equation is:

$$\delta(L) = \sqrt{\left(\frac{c_1}{c_2 h_2}\right)^2 - \frac{2\delta_{ap}(L)}{c_2 h_0}} + \frac{c_1}{c_2 h_2} \quad . \tag{B6}$$