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Impacts of spectroscopic errors on O₂ measurement requirements for the ASCENDS mission

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Abstract

Remotely sensed observations of atmospheric composition require an estimate of surface pressure. This estimate can either come from an instrument with sensitivity in an O₂ absorption feature in the spectrum, or it can be provided by a numerical ⁵ weather prediction (NWP) model. In this work, the authors outline a information-based methodology for setting measurement requirements for an active measurement of O₂ in the context of the Active Sensing of Carbon Emissions over Nights, Days and Seasons (ASCENDS) mission. The results indicate that the impacts of correlations in the spectroscopic errors between CO₂ and O₂ measurements are nontrivial, and actually ¹⁰ tighten the requirement for the O₂ measurement by 5–10%.

1 Introduction

The present surface based network of observing systems has been shown to be inadequate for reducing uncertainty in surface flux estimates of CO_2 at all but the coarsest spatial scales (Houweling et al., 2004). On the other hand, other experiments with pseudo-data (e.g. Rayner and O'Brien, 2001; Houweling et al., 2004; Hungershoefer et al., 2010) suggest that column integrated CO_2 mixing ratio, denoted as XCO_2 , as retrieved from radiances measured by satellites with instruments that are sensitive to CO_2 absorption features, can provide enough observations with suitable precision to both improve current surface flux estimates and reduce their associated uncertainties.

²⁰ Retrievals of XCO₂ require an estimate of surface pressure p^* , and at present, two options exist for providing it. The first is to use the collocated value of surface pressure derived from numerical weather prediction (NWP) models. The second is to employ a laser differential absorption spectrometer (LAS) to measure absorption in an O₂ band and utilize the near constant O₂ mixing ratio to retrieve a robust estimate of the local surface pressure. It is reasonable to ask whether the reduced errors in surface pressure





provide improvements to the retrieved mixing ratios and resulting flux estimates that justify the cost.

Here we present a methodology to partially answer that question, by determining which observable contains the most information on the model profile of CO_2 mixing ra-

- ⁵ tio, denoted $q_{CO_2}(p)$, and hence the most information on the surface fluxes in a transport model inversion. Specifically, we seek to provide an error constraint on the O_2 measurement, beyond which the cost of the measurement would not be justifiable. Section 2 defines the observations of interest and relevant terminology. In Sect. 3, the notion of Fisher information is introduced, and the relevant forms used in our method-
- ¹⁰ ology are discussed. The error components are defined in Sect. 4, and these together with the derivative of the observables with respect to the column concentration of CO_2 are used to calculate the information in each observable as a function of a few parameters related to observational error and NWP model surface pressure errors. Finally, a measurement requirement, in the form of an upper bound, is derived in Sect. 5 for
- the observational component of the O₂ error, and the value of this upper bound is computed for different magnitudes of surface pressure error.

2 Differential absorption lidar measurements

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LAS instruments measure the difference in transmitted/received energies at two or more wavelengths (which for the two-line case we call " T_{on} " and " T_{off} "). T_{on} is a wavelength absorbed by CO₂ while T_{off} is not subject to absorption. We denote the logarithm of the ratio as $\Delta \tau$. That is,

$$\Delta \tau_{\rm CO_2} := \log \left[\frac{T_{\rm on}}{T_{\rm off}} \right] = \frac{1}{m_{\rm a}g} \int_0^{\rho^*} q_{\rm CO_2}(\rho) \Delta \xi_{\rm CO_2}(\rho) d\rho \tag{1}$$

where m_a is the molar mass of air, g is the gravitational constant, and $\Delta \xi_{CO_2}$ is the differential absorption cross section at pressure altitude p. This definition follows



immediately from Beer's Law, which states that the transmission is given by

$$T = \exp\left[-\int_{R_1}^{R_2} n_{gas}(r)\Delta\xi(r)dr\right]$$

with n_{gas} being the number density of the gas, and r denoting the path traversed by the laser. Since the actual quantities measured by the instrument are T_{on} and T_{off} , the quantity $\Delta \tau_{\text{CO}_2}$ is the most direct link between the actual observation and the model profile $q_{\text{CO}_2}(p)$, assuming that p^* and $\Delta \xi_{\text{CO}_2}$ are known. The choice of observation determines an operator from the model state (q_{CO_2}) to the

The choice of observation determines an operator from the model state (q_{CO_2}) to the space where the observations lie. The candidate observables in this document are the differential optical depth for CO₂, $\Delta \tau_{CO_2}$, given in Eq. (1), and the ratio $\frac{\Delta \tau_{CO_2}}{\Delta \tau_{O_2}}$, which is written explicitly as

$$\frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = \frac{\int_0^{p^*} q_{\rm CO_2}(p) \Delta \xi_{\rm CO_2}(p) dp}{\int_0^{p^*} q_{\rm O_2} \Delta \xi_{\rm O_2}(p) dp}.$$
(2)

Note that the ratio $\frac{\Delta \tau_{CO_2}}{\Delta \tau_{O_2}}$ resembles (modulo the different weighting functions) the ratio $\frac{q_{CO_2}}{q_{O_2}}$, and so can be thought of as a surrogate for XCO₂, with appropriate scaling for the abundance of O₂ in the atmosphere.

3 Information content

From (Rodgers, 2000), the Fisher information for the linearized retrieval problem with Gaussian error statistics is given by

$$I = \mathbf{H}^{\mathsf{T}} \mathbf{R}^{-1} \mathbf{H} + \mathbf{B}^{-1},$$

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where **R** is the observation error covariance matrix, **B** is the prior error covariance matrix, and **H** is the Jacobian of the observation operator **h** that transforms model variables into observations. The Fisher information can be thought of as the information in the observation or retrieval about the input parameters, which compose the domain ⁵ of **h**.

For the observation operators in Eqs. (1)–(2), the predicted quantity is a scalar h, meaning that **H** is a column vector. Assuming that the prior information on q_{CO_2} is the same for each candidate observable (i.e. they assume the same error statistics), this suggests that the scalar quantity

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$$I_{q_{\text{CO}_2}} = \mathbf{H}_{q_{\text{CO}_2}}^{\mathsf{T}} \mathbf{R}^{-1} \mathbf{H}_{q_{\text{CO}_2}}$$

provides a measure of a particular observable's information content on the model profile of CO₂. Here, $\mathbf{H}_{q_{CO_2}}$ is the Jacobian of *h* with respect to the model layer mixing ratios $[q_{CO_2}^1, \dots, q_{CO_2}^{n_{layers}}]$, and so the quantity $\sqrt{I_{q_{CO_2}}}$ has units ppm⁻¹, regardless of the ¹⁵ units of *h*. This quantity thus provides a useful manner in which to compare the utility of very different observations. It should be noted that the Jacobian $\mathbf{H}_{q_{CO_2}}$ is expensive to estimate for passive measurements (e.g. using a finite difference approximation), but quite simple to compute analytically in the case of a lidar measurement, by simply differentiating Eqs. (A1) and (A2) with respect to the layer mixing ratios $q_{CO_2}^i$, to yield ²⁰ Eqs. (A3) and (A4). It remains to compute **R**.

Eqs. (A3) and (A4). It remains to compute I

4 Decomposition of errors

For the linear problem, the matrix **R** characterizes the uncertainty present in the model's predicted value of the observable, as well as the observed value itself (Tarantola, 2005). As such, **R** is a combination of observational uncertainty (in the signal to noise sense), the uncertainty in the simulated/retrieved external quantities (such as surface pressure,



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temperature and moisture), and the uncertainty from things that are not explicitly modeled. The first source is treated as random error with known statistics computed from the SNR. The second source includes surface pressure errors and errors in spectroscopy ($\Delta\xi_{CO_2}$) due to misspecification of local temperature and water vapor, on which the spectroscopy strongly depends. The third category of errors is assumed to be negligible since we assume that transmission in nearby wavenumbers in the spectra differs only in the gas absorption, and so all other effects cancel out when we take the difference.

R is treated as a diagonal matrix, whose entries are the sum of error variances ¹⁰ arising from the three components described above. Assuming a single grid point in the retrieval (or spatially uncorrelated errors), **R** can be considered to be a constant $\sigma^{2}(h)$, which is decomposed as

$$\sigma^{2}(h) = \sigma_{\rho^{*}}^{2}(h) + \sigma_{\Delta\xi}^{2}(h) + \sigma_{obs}^{2}(h)$$
(5)

¹⁵ The *h* dependence is included to emphasize the variability of the observable *h* due to errors in surface pressure, spectroscopy and instrument noise. Thus the information in a single measurement on the co-located model q_{CO_2} is $\sigma^{-2}(h)|\mathbf{H}_{q_{CO_2}}|^2$, which suggests that the *i*th component of $\sigma^{-1}(h)\mathbf{H}_{q_{CO_2}}$ will represent the information in the observable about the model's *i*th layer CO₂ mixing ratio.

²⁰ 4.1 The surface pressure error contribution $\sigma_{p^*}^2(h)$

In order to determine the error variance contribution due to surface pressure misspecification, we make use of the uncertainty propagation formula, where if y = h(x) and $\mathbf{H} = \nabla h$, then

$$\mathbf{R}_{y} = \mathbf{H}^{\mathsf{T}} \mathbf{R}_{x} \mathbf{H},$$



(6)

where \mathbf{R}_x and \mathbf{R}_y are the covariance matrices of *x* and *y*, respectively. In the context of surface pressure errors, this implies

$$\sigma_{p^*}^2(h) = \left(\frac{\partial h}{\partial p^*}\right)^2 \sigma^2(p^*$$

⁵ Note that $\frac{\partial h}{\partial p^*}$ has been computed in Eqs. (A5)–(A6).

An estimate of the range of uncertainties in surface pressure knowledge was computed based on an extensive set of matched pairs of observations and model pairs derived from surface weather observation station reports (METAR/SYNOP) (US DOC/NOAA OFCM, 2005) and NWP model fields collected both over the continental United States as well as on a global basis for representative periods between July 2011 and July 2012. The representative time periods were chosen to include data from all seasons as well as both daytime and nighttime observations. The surface observations were obtained from publicly available sources and the matching model data were extracted from both the 12 km North American Mesoscale Model (NAM) (Rogers et al.,

- 2009) and 0.5° Global Forecast System (GFS) (NCEP, 2003) analysis fields. The NAM was chosen to represent the uncertainty statistics associated with a high spatial resolution model for a well-instrumented area, and the GFS fields were chosen to illustrate the errors associated with a coarser global domain. Only 0 h forecasts or model analysis fields were selected in this work to describe the model error characteristics based
- ²⁰ on the assumption that any operational retrieval system would either acquire data from an external source or employ an *N*-dimensional variational data assimilation system to minimize the impact due to uncertainties in the atmospheric state. While METAR and SYNOPS are by no means an absolute representation of the atmospheric state at any point (Sun et al., 2010), they do provide a consistent measure that can be com-
- ²⁵ pared to NWP data for statistical purposes. The observed surface pressure values were extracted from 107 airport and/or permanent surface weather observation station reports for the same contiguous United States (CONUS) and global regions described above along with their corresponding NWP model values. The NWP model values were



(7)



corrected to the observed station height using a standard lapse rate relationship. The resulting 1σ value for the CONUS region was approximately 1.1 mbar and 2σ value was 2.1 mbar. The global region exhibited a 1σ value of 0.8 mbar and a 2σ value of 1.7 mbar respectively. Globally these observations showed no significant biases, and only slight seasonal variation in standard deviations. The standard deviation varied as expected as a function of hemispheric region.

For the calculations that follow, we keep these values in mind, but treat $\sigma^2(p^*)$ as a parameter whose value will partially determine the amount of O₂ error we deem cost-effective.

¹⁰ 4.2 The environmental contribution $\sigma^2_{\Delta\xi}(h)$

The differential absorption cross section $\Delta \xi$ is a function of the atmospheric state variables, and as such, the weighting functions in an automated retrieval will be dynamically estimated according to local temperature, water vapor and pressure. These atmospheric values will themselves be estimates, either taken from NWP models, satellite

- ¹⁵ soundings or other proxies. In order to quantify the uncertainty in the observable due to uncertainty in spectroscopy, $\sigma_{\Delta\xi}(h)$, the uncertainty in $\Delta\xi$ due to uncertainty in the atmospheric state must be quantified. To this end, sample sets of simulated weighting functions for both representative CO₂ absorption features at approximately 1.5711 and 2.0510 µm (6364.92203 and 4875.59 cm⁻¹), and the O₂ absorption lines at 0.76468 and 1.2625 µm (13 077.29386 and 7920.5976 cm⁻¹) were constructed from observed
- and modeled atmospheric profile data using a radiative transfer (RT) model to compute associated differential optical depths.

Similarly to the method described in Sect. 4.1, modeled and observed atmospheric state vectors were obtained for both surface and upper air temperature and moisture.

The observed profiles were derived from RAwinsonde OBservation (RAOB) observations, while model data were taken from NWP model fields. The RAOBs were obtained from publicly available sources and the matching model data were extracted from GFS





(NCEP, 2003) analysis fields. RAOBs provide a consistent measure that can be compared to NWP data for statistical purposes (Sun et al., 2010). The matching NWP profiles were selected using a nearest neighbor approach based on the RAOB station location, and contained vertical temperature/moisture (RH) profiles on a fixed pressure

⁵ grid, and surface parameters (temperature, RH, surface pressure and station height). A conservative quality control scheme was used to screen out RAOB with missing data, and those in cloudy conditions based on the model cloud fraction and RAOB upper-air water vapor.

The associated optical depths for the desired wave numbers, at standard layer heights between the surface and 100 km above the surface, were computed by combining the atmospheric state vectors with a nominal CO₂ profile with a constant vertical mixing ratio of 385 ppm to construct appropriate input parameters for the Line-By-Line Radiative Transfer Model (LBLRTM) (Clough et al., 2005). LBLRTM computes optical depths from Voigt line shape functions and a continuum model that includes self-15 and foreign-broadened water vapor as well as continua for carbon dioxide, oxygen,

¹⁵ and foreign-broadened water vapor as well as continua for carbon dioxide, oxygen, nitrogen, ozone and extinction due to Rayleigh scattering. The version employed in this study included 2012 updates to the CO_2 line parameters and coupling coefficients based on the work of Devi et al. (Devi et al., 2007a, b) the O_2 line parameters based on HITRAN (Rothman et al., 2009) and additional quadrupole parameters between 20 7571–8171 cm⁻¹.

Each of 2500+ pairs of observation and model-based weighting functions were constructed by dividing the discrete optical depth for each layer by the layer thickness given as a function of layer height or difference in atmospheric pressure. The top panel of Fig. 1 shows the average for the ensemble set of weighting functions at each selected ²⁵ wavelength, and the bottom panel illustrates the ensemble variance due to variations in vertical temperature and moisture. The weighting functions were created for -10 pmoffset for the CO₂ absorption features at 1.571 µm and the off-line center position for the 2.051 µm CO₂ feature and the two selected O₂ absorption lines at 0.76 µm and 1.26 µm.





Assuming that the differences in optical depths derived from NWP and observed environments have the same distribution as the true errors, the sample error covariance $\mathbf{R}_{\Delta\xi}$ is computed, and transformed into $\sigma_{\Delta\xi}^2(h)$ using the error propagation formula Eq. (6), with the relevant partial derivatives are given by Eqs. (A7)–(A9). This matrix has dimensions $(2n_{\text{layers}}) \times (2n_{\text{layers}})$, where n_{layers} is the number of atmospheric layers in the model. The differences are binned into layers, and the variance for CO₂ and O₂ is computed, in addition to the in-layer covariance between CO₂ and O₂. Between-layer error correlations are assumed to be zero.

Sample error variances for the $1.571 \,\mu\text{m} + 10 \,\text{pm}$ and $2.051 \,\mu\text{m}$ CO₂ instruments and the 0.76468 μm and $1.2625 \,\mu\text{m}$ O₂ instruments as a function of height are shown in Fig. 2, and the covariances are shown in Fig. 3. Note that the sign of the correlation for each CO₂-O₂ varies with height and by instrument. For example, the covariance $1.571 \,\mu\text{m} + 10 \,\text{pm}$ and $1.26 \,\mu\text{m}$ instruments is negative near the surface and positive above the boundary layer, while the opposite pattern is observed for the 2.0510 μm and $1.2625 \,\mu\text{m}$ instruments. The layer thickness for the displayed error covariances is 25 mb, and varying this parameter showed that the impact of this choice on the covariances was minimal.

Applying Eq. (6) with the profiles in Figs. 2 and 3 and Eqs. (A7)–(A9) yields Table 1. The values between the $\Delta \tau_{CO_2}$ and $\frac{\Delta \tau_{CO_2}}{\Delta \tau_{O_2}}$ observables are not directly comparable, since they represent different quantities, but converting these values to percentages of a nominal measurement with a constant CO₂ mixing ratio of 400 ppm yields a variability of about 0.04 % for the $\Delta \tau_{CO_2}$ measurements, and 0.06 % for the ratio observables.

It is important to note that these computations assume a perfect radiative transfer model, and as such do not contain systematic errors in the spectroscopic characterizations themselves. Quantifying the impact of such errors is beyond the scope of this

²⁵ zations themselves. Quantifying the impact of such errors is beyond the scope of this paper, and we assume that the state of the art radiative transfer models will be used in any operational retrievals.





4.3 The observational error variance $\sigma_{obs}^2(h)$

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The observed differential absorptions $\Delta \tau$ contain an instrument specific level of precision, which we refer to as $\sigma_{obs}^2(h)$. This is typically quantified by the scientists that build and test the instrument in a controlled environment. The ratio observable has "noise" defined as the propagation of the noise from the CO₂ and O₂ instruments:

$$\sigma_{\rm obs}^2 \left(\frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}}\right) = \Delta \tau_{\rm O_2}^{-2} \sigma_{\rm obs}^2 (\Delta \tau_{\rm CO_2}) + \Delta \tau_{\rm CO_2}^2 \Delta \tau_{\rm O_2}^{-4} \sigma_{\rm obs}^2 (\Delta \tau_{\rm O_2}) \tag{8}$$

By controlling the size of the individual lidar instruments' noise, we control the contribution to the overall error, and thus we would expect that a smaller error in one of the instruments would allow a more relaxed requirement on the other, keeping the error budget for XCO_2 fixed.

5 An information-based O₂ requirement

In Sect. 4, an algorithm for computing $\sigma^{-1}(h)$ for two particular observation operators *h* from errors in temperature, water vapor and pressure is described. In the case of differential absorption lidar observables, we have simple analytical expressions for *h*, assuming knowledge of the surface pressure p^* and the weighting function, and so the Jacobian $\mathbf{H}_{q_{CO_2}}$ can be calculated directly using Eqs. (A3) and (A4). Using these two pieces, we can now compute the aforementioned quantity $I_{q_{CO_2}}$ for assumed values of the three parameters: $\sigma^2(p^*)$, $\sigma^2_{obs}(\Delta \tau_{CO_2})$ and $\sigma^2_{obs}(\Delta \tau_{O_2})$. A minimum requirement for the O₂ lidar investment to be cost effective is that the information in the ratio observable in Eq. (2) is larger than the CO₂ only observable, stated as:

$$\sigma^{-1}(\Delta \tau_{\rm CO_2}) \frac{\partial \Delta \tau_{\rm CO_2}}{\partial q_{\rm CO_2}^i} \le \sigma^{-1} \left(\frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} \right) \frac{\partial}{\partial q_{\rm CO_2}^i} \frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}}$$
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(9)

for every layer i. Squaring both sides of Eq. (9) and expanding the error variances yields the requirement

$$\sigma_{obs}^{2}(\Delta\tau_{O_{2}}) \leq \frac{\Delta\tau_{O_{2}}^{2}}{\Delta\tau_{CO_{2}}^{2}} \left[\sigma_{\rho^{*}}^{2}(\Delta\tau_{CO_{2}}) + \sigma_{\Delta\xi}^{2}(\Delta\tau_{CO_{2}}) - \sigma_{\rho^{*}}^{2} \left(\frac{\Delta\tau_{CO_{2}}}{\Delta\tau_{O_{2}}} \right) \Delta\tau_{O_{2}}^{2} - \sigma_{\Delta\xi}^{2} \left(\frac{\Delta\tau_{CO_{2}}}{\Delta\tau_{O_{2}}} \right) \Delta\tau_{O_{2}}^{2} \right]$$

$$(10)$$

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Note the lack of the observational error term for $\Delta \tau_{CO_2}$ on the right hand side of Eq. (10). The remaining terms depend solely on the expected error in surface pressure that arises from using an NWP estimate of p^* in place of an observed value, denoted $\sigma^2(p^*)$. The upper bound given by Eq. (10) was computed for different values of $\sigma(p^*)$, and the results are displayed as percentages of the corresponding $\Delta \tau_{O_2}$ observations in Table 2.

To first order, one would expect that the O_2 measurement error requirement should match the pressure requirement, e.g. for a pressure error of 1 mb we should have an O_2 measurement error of about 0.1 %. Table 2 shows that the impact of including the spectroscopic errors as part of the calculation is to make the measurement requirements more stringent. For example, in order for the O_2 measurement to add information in the presence of a 1 mb NWP pressure error, the O_2 measurement must be made with a higher degree of precision, about 0.09%. This is quite different from the case of passive measurements where the joint measurement of CO_2 and O_2 can remove joint

²⁰ scattering effects (O'Brien and Rayner, 2002). Such effects are already handled by the DIAL measurement itself. This impact, while seemingly small, adds an extra burden to the design and implementation of an O₂ instrument. As higher values of $\sigma(p^*)$ become acceptable, the upper bound on $\frac{\sigma_{obs}(\Delta \tau_{O_2})}{\Delta \tau_{O_2}}$ approaches the value of $\frac{\sigma(p^*)}{p^*}$, which indicates the diminishing importance of spectroscopic errors as surface pressure errors get large. However, in the range discussed in Sect. 4.1, spectroscopic errors are non





trivial, and comprise between 5–10% of the upper bound value. Thus when stating measurement requirements, the simple scaling of surface pressure will not be enough to ensure that the O₂ measurement is cost effective.

Conclusions 6

The preceding work defines an information based requirement for an O₂ instrument to 5 provide additional information on column CO₂ above and beyond a CO₂ measurement taken together with an NWP prediction of surface pressure. The requirement includes the impacts of environmentally induced spectroscopic error correlations between the O₂ and CO₂ measurements, as well as the expected variability of each due to surface pressure errors. Tests were performed using proxies for errors in the atmospheric 10 state taken from NWP predictions and RAOBs for two different candidate CO₂ and O₂ spectral lines. The major result is that for reasonable surface pressure errors a measurement of O₂ needs to be better (in percentage terms) than the surface pressure estimate to contribute useful information to CO₂ retrievals.

Appendix A: Discretized operators and their derivatives 15

Equations (1)–(2) above are two examples of **h**. In the discrete case (i.e. in a numerical model), these observation operators are expressed using sums:

$$\Delta \tau_{\rm CO_2} = \frac{1}{m_{\rm a}} g \sum_{i=1}^{n_{\rm layers}} q_{\rm CO_2}^i \Delta \xi_{\rm CO_2}^i \Delta p^i$$
$$\frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = \frac{\sum_{i=1}^{n_{\rm layers}} q_{\rm CO_2}^i \Delta \xi_{\rm CO_2}^i \Delta p^i}{\sum_{i=1}^{n_{\rm layers}} q_{\rm O_2}^i \Delta \xi_{\rm O_2}^i \Delta p^i}$$



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(A2)

The derivatives of the discrete observation operators with respect to $q_{CO_2}^i$ are given by

$$\frac{\partial \Delta \tau_{\rm CO_2}}{\partial q_{\rm CO_2}^i} = \frac{\Delta \xi_{\rm CO_2}^i \Delta p^i}{m_{\rm a} g}$$
(A3)
$$\frac{\partial}{\partial q_{\rm CO_2}^i} \frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = \frac{\Delta \xi_{\rm CO_2}^i \Delta p^i}{\sum_{j=1}^{n_{\rm layers}} q_{\rm O_2}^j \Delta \xi_{\rm O_2}^j \Delta p^j}.$$

⁵ Assuming sigma coordinates, i.e. $p = bp^*$, the derivatives with respect to p^* are

$$\frac{\partial \Delta \tau_{\rm CO_2}}{\partial p^*} = \frac{1}{m_{\rm a} g} \sum_{i=1}^{n_{\rm layers}} q_{\rm CO_2}^i \Delta \xi_{\rm CO_2}^i \Delta b^i, \qquad (A5)$$

$$\frac{\partial}{\partial p^*} \frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = 0. \qquad (A6)$$

The derivatives of the observation operators with respect to the differential optical depth Δξⁱ are

$$\frac{\partial \Delta \tau_{\rm CO_2}}{\partial \Delta \xi^i_{CO_2}} = \frac{1}{m_{\rm a} g} q^i_{\rm CO_2} \Delta p^i,$$

$$\frac{\partial}{\partial \Delta \xi^i_{CO_2}} \frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = \frac{q^i_{\rm CO_2} \Delta p^i}{\sum_{j=1}^{n_{\rm layers}} q^j_{\rm O_2} \Delta \xi^j_{\rm O_2} \Delta p^j},$$

$$\frac{\partial}{\partial \Delta \xi^i_{\rm O_2}} \frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = -\frac{q_{\rm O_2} \Delta p^i \sum_{j=1}^{n_{\rm layers}} q^j_{\rm O_2} \Delta \xi^j_{\rm CO_2} \Delta \xi^j_{\rm CO_2} \Delta p^j}{\left(\sum_{j=1}^{n_{\rm layers}} q_{\rm O_2} \Delta \xi^j_{\rm O_2} \Delta p^j\right)^2}.$$

$$\frac{\partial}{\partial \Delta \xi^i_{\rm O_2}} \frac{\Delta \tau_{\rm CO_2}}{\Delta \tau_{\rm O_2}} = -\frac{q_{\rm O_2} \Delta p^i \sum_{j=1}^{n_{\rm layers}} q^j_{\rm O_2} \Delta \xi^j_{\rm O_2} \Delta p^j}{\left(\sum_{j=1}^{n_{\rm layers}} q_{\rm O_2} \Delta \xi^j_{\rm O_2} \Delta p^j\right)^2}.$$

$$\frac{\partial}{\partial \Delta \xi^i_{\rm O_2}} \frac{\partial \tau_{\rm O_2}}{\partial \tau_{\rm O_2}} = -\frac{\partial}{\partial \tau_{\rm O_2}} \frac{\partial \tau_{\rm O_2}}{\partial \tau_{\rm O_2}} \frac{\partial \tau_{\rm O_2}}{$$

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Table 1. $\sigma_{\Delta\xi}(h)$ for $h = \Delta \tau_{CO_2}$ (first row) and $h = \frac{\Delta \tau_{CO_2}}{\Delta \tau_{O_2}}$ (second and third rows) from spectro-
scopic errors computed using Eqs. (A7)–(A9) and the profiles in Figs. 2 and 3.

	1.571 µm	2.051 µm	
	1.646×10^{-4}	3.827×10^{-4}	$\Delta \tau_{\rm CO_2}$
0.76 μm 1.2625 μm	4.127×10^{-8} 5.390×10^{-7}	9.213×10^{-8} 1.279×10^{-6}	$\frac{\Delta\tau_{\rm CO_2}}{\Delta\tau_{\rm O_2}}$



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Table 2. Upper bounds on $\sigma_{obs}(\Delta \tau_{O_2})$ expressed as percentages of $\Delta \tau_{O_2}$, computed as a function of the expected surface pressure error $\sigma(p^*)$.

	1.571 µm		2.051 µm	
$\sigma(p^*)$	0.76 µm	1.2625 µm	0.76 µm	1.2625 µm
0.5000	0.0293	0.0200	0.0203	0.0190
1.0000	0.0914	0.0889	0.0889	0.0887
1.5000	0.1444	0.1428	0.1429	0.1427
2.0000	0.1959	0.1947	0.1947	0.1946
2.5000	0.2467	0.2458	0.2458	0.2457
5.0000	0.4984	0.4979	0.4979	0.4979
10.0000	0.9992	0.9989	0.9990	0.9989



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Figure 1. Normalized ensemble weighting functions derived from global vertical temperature and moisture profiles. The upper panel shows the average weighting function for each selected wavelength, and the lower plot illustrates the variability in the weighting functions due to differences in temperature and moisture as a function of vertical height. The variability shown in the lower plot is given as the standard deviations of the difference between the average weighting function values and the ensemble members.







Figure 2. The variance of the differences between differential optical depths derived from observed and modeled atmospheric soundings, as a function of pressure. The $0.76 \,\mu m$ instrumental variance is approximately 2 orders of magnitude larger at its largest value (not shown).



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Figure 3. The in-layer covariance of the differences between differential optical depths derived from observed and modeled atmospheric soundings, as a function of pressure.



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