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Analysis of internal gravity waves with GPS RO density profiles

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Abstract

GPS (Global Positioning System) radio occultation (RO) data proved to be a great tool for atmospheric monitoring and studies. In the recent decade, they were frequently used for analyses of the internal gravity waves in the upper troposphere lower strato-

- ⁵ sphere region. Atmospheric density is the first quantity of state gained in the retrieval process and is not burdened by any additional assumptions. However, there are no studies elaborating in details the utilization of GPS RO density profiles for gravity waves analyses. In the presented paper, we introduce a method for the density background separation and a methodology for internal gravity waves analysis using the density
- profiles. Various background choices are discussed and the correspondence between analytical forms of the density and temperature background profiles is examined. In the stratosphere, the comparison between the power spectrum of normalized density and normalized dry temperature fluctuations confirms the suitability of the density profiles utilization. In the height range of 8–40 km, results of the continuous wavelet transform
- ¹⁵ are presented and discussed. Finally, the limits of our approach are discussed and the advantages of the density usage are listed.

1 Introduction

Gravity waves in geophysical fluid dynamics are usually described as a group of wave motions in a fluid where the restoring force is the gravity (or so-called reduced grav-

ity). Internal gravity waves (IGWs) as a part of them are of special importance in the atmosphere for their effects on atmospheric composition, circulation and dynamics in general.

Internal gravity waves exist in continuously stratified fluids. They can be pictured as being composed of oscillating interacting particles with mutually connected phase.

²⁵ The waves can propagate both horizontally and vertically under a continuous interplay between gravity and inertia and an exchange between potential and kinetic energy



(Cushman-Roisin, 1994). A characteristic feature of their propagation is that the group and phase velocity are always perpendicular.

IGWs are very important for atmospheric dynamics. One of their crucial roles is that they transport angular momentum from the ground upwards. That helps to maintain the rotation of the upper parts of the atmosphere and also in general, they contribute to restore the equilibrium and to gain energetically more favourable conditions.

The importance of gravity waves for the dynamics of the middle atmosphere has been recognized for quite a while (Hines, 1960) and the research in this area is still very active and connected with some of the most challenging issues of Earth's climate and atmospheric science. Examples are: the role of the IGWs in driving the guasi-biennial

- ¹⁰ atmospheric science. Examples are: the role of the IGWs in driving the quasi-biennial oscillation (QBO) (eg. Marshall and Scaife, 2009), the downward coupling between atmospheric layers affecting the weather in the troposphere (e.g. Ern et al., 2011) or direct influence on the middle atmospheric climate and the acceleration of the Brewer Dobson (BD) circulation (e.g., Garcia and Randel, 2008) that implies the necessity for
- ¹⁵ incorporation of the IGWs in climate models. However, as the mesoscale waves have horizontal scales between tens and several thousand kilometres, they cannot be fully resolved by general circulation models and have to be parameterized. Therefore, it is necessary to gather as much information about the IGWs as possible.

A comprehensive review of the IGW's effects in the middle atmosphere up to the beginning of 21st century is given by Fritts and Alexander (2003). In discussion, they have also stated the future scientific goals of IGW research. Those are mainly observations and modelling studies of the gravity wave sources along with advanced studies of nonlinear processes influencing the wave's spectral evolution and propagation. Finally, it is necessary to incorporate all the knowledge into more accurate parameterizations.

²⁵ A variety of observation techniques has already been applied in the research of wave disturbances in the atmosphere. Those include radiosonde and rocketsonde measurements, balloon soundings, radar and lidar observations and other remote sensing measurements. In the last two decades, the remote sensing with occultation methods has undergone a remarkable development. Signals of the Global Positioning System (GPS)



are exploited by the radio occultation (GPS-RO) and are often utilized for studies of IGWs. In the future, the potential of these sounding techniques will most likely grow due to increasing numbers of transmitters and receiver platforms (Wickert et al., 2009).

- Wu et al. (2006) categorized the GPS-RO as a Line of Sight (LOS) sensor charac terized by excellent vertical and coarse horizontal (due to LOS-smearing) resolution. Therefore, the GPS-RO measurements are mostly sensitive to IGWs with a small ratio of vertical to horizontal wavelength. The observations are limited at high altitudes by ionospheric residual errors and at low altitudes by strong water vapour effects. Hence we focus on the height domain between 8 km and 40 km. The highest accuracy is found
- in the lower stratosphere where it is usually better than 1 K (Steiner and Kirchengast, 2000). The GPS-RO technique provides atmospheric profiles with global coverage under all weather and geographical conditions together with self-calibration ability and long term stability. That makes the GPS-RO an almost perfect tool for atmospheric monitoring (Foelsche et al., 2008).
- ¹⁵ Using GPS-RO, the IGWs description can be retrieved in a series of steps. At first, a height profile of atmospheric refractivity index is derived from bending angles. In this step local spherical symmetry is assumed and therefore a limited horizontal resolution of about 300 km is common to limb scanning methods (Preusse et al., 2008). Then, the refractivity index can be directly related to the density of dry air. Temperature profiles,
- which are usually used, are computed only subsequently from the density profiles using the hydrostatic balance and the state equation for dry air. This approach, termed "dry air retrieval", neglects the contribution of water vapour to radio refractivity, but in the height range of interest for our study this effect can be neglected (Foelsche et al., 2008).
- Research on atmospheric waves using GPS-RO data expanded since the papers of Tsuda et al. (2000) and Steiner and Kirchengast (2000). According to the linear theory of IGWs, a separation between a small wave-induced fluctuation and background field has to be performed, if vertical profiles of any state quantity are used for the detection of the gravity wave parameters. The choice of the background state significantly affects



the results and it is quite a complicated issue because an artificial model could never perfectly reflect the real state of atmosphere.

Lots of authors who have studied the IGWs retrieved from the GPS RO dry temperature profiles have utilized different methods for determining the background temperature. For example, Steiner and Kirchengast (2000) and many others have applied various sorts of band pass filters with cutoffs at some specified vertical wavelengths. E.g., Gubenko et al. (2012) or Vincent et al. (1997) have used approximations by low order polynomials for the stratospheric levels. The analysis of the tropopause region

is always connected with problems because of the artificial enhancement of the wave
 activity (even when using the advanced band-pass filter associated with different vertical wavelengths). Schmidt et al. (2012) suggest two possible approaches to solve this problem. Those are a separation of the profile into tropospheric and stratospheric parts and application of a filter for each region or, more appropriately, the double filtering method. The question of the background separation is an important part of our
 study too and it is discussed in details in the following sections.

In this paper, we present a new method for the analysis of IGWs using density profiles. Atmospheric density is the first quantity of state gained during the retrieval of GPS RO data and it is not burdened by any additional assumptions (e.g., hydrostatic balance). Moreover, we would like to stress out another advantage, which has not been discussed yet – unlike temperature profiles, the density evolution with the height is theo-

discussed yet – unlike temperature profiles, the density evolution with the height is theoretically inferable by means of statistical physics. Separation of the density background has not only a physical basis but it can also be computed partly analytically using our method presented in Sect. 2.

To our knowledge this is the first study where atmospheric density profiles from GPS RO are explicitly used for the study of IGWs. Liou et al. (2003, 2005) and Pavelyev et al. (2006, 2009), however, followed a methodologically similar approach by studying vertical refractivity gradients based on RO data. That is unexpected since the density data should be very accurate. For example, refractivity profile intercomparisons of nearly collocated profiles with radiosondes show differences of less than 0.7 % below



30 km altitude (Schreiner et al., 2007). Another instrument that gives direct information about density is the lidar. There are several observational studies of IGW induced density fluctuations (e.g. Wilson et al., 1991; Sica and Russel, 1999) and theoretical papers about the determination of density fluctuation from the returned power (Gardner, 1989),

⁵ and estimations of the response of neutral density layers to gravity waves perturbations (Gardner and Shelton, 1985; Chiu and Ching, 1978).

The next section introduces the methodology and the data sets used to retrieve the density perturbations profiles and to analyse the IGWs. The results are described in the third section and they are discussed with concluding remarks in Sect. 4.

10 2 Methods and data

2.1 Separation of the background density profiles

In deriving the separation method we will make use of two basic facts – the fact that the general form of the density decrease with height is known from theory and that the variations of the background Brunt–Vaissala frequency squared (N_0^2) are substantially small (Steiner and Kirchengast, 2000; Tsuda et al., 2000). The latter leads in various studies (e.g. Steiner and Kirchengast, 2000; Tsuda et al., 2011; Gubenko et al., 2012) to a simplification of equations where N_0^2 is replaced within the area of interest with one constant value. Care must be taken when the area includes more than one layer because there is a jump of N_0^2 at the boundaries (in our case at the tropopause).

Let us have a background density profile $\rho_0(z)$ and assume that all departures from the background density are due to the response to the IGW induced wind perturbations and are governed by the continuity equation. By assuming horizontal homogeneity and neglecting the diffusion and chemical effects, Gardner and Shelton (1985) have shown that the density response can be written as

²⁵
$$\rho(\mathbf{r},t) = e^{-\chi}\rho_0(z-\zeta).$$



(1)

Here $\rho(\mathbf{r},t)$ is the perturbed density at position \mathbf{r} and time t and χ , ζ are solutions of partial differential equations (for details, see Gardner, 1989) that are related to the wind divergence and to the vertical displacement, respectively.

Unlike continuous lidar measurements, the GPS RO data provide a snapshot of the ⁵ perturbed vertical density profile $\rho(z)$. Additionally, we will make an assumption that the background density profile could be expressed analytically in the form

 $\rho_0(z) = \hat{\rho}_0 \exp[-a(z)z],$

15

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where $\hat{\rho}_0$ is the background density generally at the lower boundary of our vertical profile, *z* is the vertical distance from the lower boundary and *a*(*z*) is a coefficient of the background density exponential decay. In the real atmosphere, the coefficient *a*(*z*) would be primarily influenced by the ambient temperature.

In general, the buoyancy frequency at any vertical level is

$$N^2 = -\frac{g}{\bar{\rho}} \frac{\mathrm{d}\bar{\rho}}{\mathrm{d}z}.$$
(3)

For our case, we will define our own slightly altered expression of the background Brunt–Vaissala frequency squared

$$N_0^2(z) = -\frac{g}{\rho_0(z)} \frac{d(\rho_0(z))}{dz}.$$
(4)

²⁰ By substituting the background density in the form defined by Eq. (2) and by omitting the notation of functional dependencies and introducing the notation $\frac{da(z)}{dz} = a'$ we gain

$$N_0^2 = -g(-a - za'). (5)$$

Now it comes to the previously mentioned fact that N_0^2 generally evolves very weakly inside layers and with moderate jumps on boundaries. Hence, the frequency should be



(2)

suitable for approximation by selected analytic functions. First, we must create a vertical profile of the perturbed local buoyancy frequency squared according to

$$N(z)^2 = -\frac{g}{\rho(z)}\frac{\mathsf{d}(\rho(z))}{\mathsf{d}z},$$

⁵ which is illustrated in Fig. 1. Then, by fitting the perturbed frequency, we have the formula for N_0^2 and we can substitute it back to Eq. (5).

Further steps will be illustrated on the simple example of fitting the perturbed local buoyancy frequency squared by a fourth order polynomial (see Fig. 1) in the form

$$A_4 z^4 + A_3 z^3 + A_2 z^2 + A_1 z + A_0.$$

After that we can substitute into Eq. (5) to get the first order differential equation, which can be (after making a substitution $y = a \cdot z$) directly solved to gain an expression for a(z). Then going back to Eq. (2) we have:

$$\rho_0(z) = \hat{\rho}_0 \exp(-\frac{\frac{A_4 z^5}{5} + \frac{A_3 z^4}{4} + \frac{A_2 z^3}{3} + \frac{A_1 z^2}{2} + A_0 z}{g} - C).$$
(8)

20

Where A_i and C are constants.

Consequently, we fit the perturbed profile $\rho(z)$ with the previously derived function of the background density supposed dependence on the height, using the specified curve fitting procedure MPFIT introduced by Markwardt (2009). For a better and faster convergence of iterations, the numerical values of A_4 , A_3 , A_2 , A_1 , A_0 can be used as the first guess from the previously done fit of the perturbed local buoyancy frequency squared. For $\hat{\rho}_0$, the density value at the lower boundary can be chosen and the integration constant *C* as a correction to ρ_0 can be assumed to be zero as a first guess.

2.2 Data description and methodology

²⁵ The method of the density profile separation and identification of the IGWs is illustrated using data from the Constellation Observing System for Meteorology, Ionosphere and



(6)

(7)

Climate (FORMOSAT3/COSMIC, Anthes et al., 2008). We have analysed a sample of 60 observational events spatiotemporally nearest (prior as well as subsequent) to the 2011 Tohoku earthquake. From this sample, one representative RO profile was chosen from 11 March 2011 04:08:29 UTC. This occultation event was used for illustration of the results where only a single profile is used. The dry temperature profile of this event is depicted in Fig. 2, note the inversion layer around 10 km altitude and the significantly

perturbed tropopause. The whole sample of 60 events was averaged for the analysis and results described in Sect. 3.2.1. Results are computed using only the "dry" profiles of density and temperature. The

vertical extent of our analysis is 8–40 km and it includes the areas of two basic inversion algorithms for GPS RO – geometrical optics (GO) and radio holography (RH). At the COSMIC Data Analysis and Archive Center (CDAAC) RH (Full spectrum inversion method in this case) is applied from ground to the upper troposphere and GO from there to the top of the profile. The vertical resolution is therefore variable across the height profile, but not less than 1.5 km (Melbourne, 2005). For details about the algorithms and the discussion of their usage possibilities see e.g., Tsuda et al. (2011).

The used methodology is limited because the tropopause is included in the investigated vertical range too. In the tropopause region assumptions of the Wentzel, Kramers and Brillouin (WKB or ray tracing) theory, that background velocity and Brunt–Vaissala

- frequency vary slowly over a wave cycle (Fritts and Alexander, 2003), are violated. Using this theory we are treating the wave packets as particles moving along rays (Sutherland, 2010) and using the WKB approximations we can relate the wave frequency to a wave's spatial characteristics and background atmosphere properties through a dispersion relation as shown by Fritts and Alexander (2003). Polarization relations used
- ²⁵ for the determination of the IGWs parameters from temperature or density profile as proposed by Gubenko et al. (2011) are also a product of WKB approximations.

In the region where the ray theory is not valid, we cannot consider the amplitude envelope as slowly evolving and the relative phase, amplitude and spatial extent of the wave packet may change significantly as it evolves over time. Therefore the results are



separated into the parts including the whole profile (8–40 km) and only the stratosphere (tropopause–40 km). In the whole vertical range, the typical analysis method of vertical wave number spectral density (e.g. Steiner and Kirchengast, 2000; Tsuda and Hocke, 2002; Tsuda et al., 2011) wouldn't give much sense as a consequence of the facts discussed above.

Relying only on the linear theory, we will present the results of our method for background separation in the extensive region using the continuous wavelet transform and its skeleton as used by Chane-Ming et al. (2000). The wavelet transform was computed using the Morlet wavelet and algorithms proposed by Torrence and Compo (1998).

After the subtraction of the background density, the resulting density perturbations are further normalized and scaled by the square root of the background density to conserve the kinetic energy as suggested by Hines (1960) and discussed by Sica and Russell (1999). Nevertheless, for the results in the stratospheric region alone, we use only the normalized density perturbations. Thus, we are able to compare directly the vertical wavenumber spectra of the density and temperature perturbation and evaluate them with theoretical model spectra. This distinction, where needed, is emphasized in following text.

3 Results

The goal of our paper is to present our method for the background separation from GPS

RO density profiles. However, the issue of background determination is not satisfactory solvable and it is also not possible to identify with certainty the ideal background profile. Therefore, where possible, the results are shown for more types of the Brunt–Vaissala frequency background profile fits and then their qualities are discussed.



3.1 The background issue

In this subsection, the fits of the Brunt–Vaissala frequency squared are shown and discussed. Consequently, after application of our method, the resulting normalized and scaled density perturbation profiles are given. Additionally, the corresponding forms

of the temperature background are computed using the equation of state for dry air and the hydrostatic balance. Reversely, background density profiles are calculated for various forms of background temperature functional dependence on altitude.

In the stratospheric region (in our case from tropopause up to 40 km), the results are derived using polynomial fits of $N^2(z)$, starting with the second order polynomial and ending with the fourth order. The second order is the lowest appropriate one to capture the supposed nonlinear decay of N_0^2 . On the other hand, higher order than the fourth order polynomial can oscillate and even the fourth order polynomial may gain a wavelike form in this narrow region. Taking into consideration the slow changes of N_0^2 in the stratosphere and the limited vertical extend of our region, we must be careful not to exaggerate the polynomial order, which could result in filtering of waves with vertical wavelengths in the interval of the IGWs likely appearance.

Chane-Ming et al. (2000) introduced a review of studies focused on the vertical wavelengths of dominant modes. In the area of our vertical range, the dominant vertical wavelengths were smaller than 10 km. Considering the limited height range, Wang

- and Alexander (2010) suggested to limit the analysis to vertical scales up to 15 km. Steiner and Kirchengast (2000) and most of other relevant studies applied the analysis threshold of the IGWs around 10 km of the vertical wavelength. The lower boundary of analysed IGWs vertical wavelengths should be determined by the Nyquist frequency arising from the vertical resolution, but since the discussion about the analysis range
- ²⁵ for studying IGWs is still open (Luna et al., 2013) we will examine a broad range of vertical scales.

Having the N_0^2 profile in the form of a polynomial, the analytical form of the density background can be easily inferred from Eq. (8). An illustration of fits to the perturbed



local buoyancy frequency squared and corresponding density perturbations are shown in Figs. 1 and 3. In Fig. 3 we added also the density perturbations resulting from the subtraction of the background in the form

$$_{5} \rho(z) = \rho_{0} \exp(-az)$$

20

corresponding to an isothermal atmosphere. The other forms of the density background which we use are not so easily transferable in the temperature space.

For example, even when we consider the density background derived from linear N_0^2 profile

$$\rho(z) = \rho_0 \exp\left(-\frac{\frac{A_1 z^2}{2} + A_0 z}{g} - C\right).$$
(10)

The background temperature profile can be derived as follows. Assuming that the background state is the same for all quantities of state and that the background state is in hydrostatic balance, we use the equation of state for dry air and the hydrostatic balance ¹⁵ equation. After solving the differential equations, the resulting background temperature profile has a quite complicated form:

$$T_{0}(z) = \exp\left[\frac{(A_{1}z + A_{0})^{2}}{2A_{1}g}\right] K - \frac{\exp\left[\frac{(A_{1}z + A_{0})^{2}}{2A_{1}g}\right] g^{\frac{3}{2}} \sqrt{\frac{\pi}{2}} \operatorname{Erf}\left(\frac{A_{1}z + A_{0}}{\sqrt{2A_{1}g}}\right)}{\sqrt{A_{1}}R},$$
(11)

where K is an integration constant, g is the gravitational acceleration and R is the gas constant.

If we analyse an inverse problem and try to find out which background density profile corresponds to frequently used temperature backgrounds, we discover that the polynomial fits of temperature are leading to an unphysical background in density space,

(9)

generally looking like:

$$\rho_0(z) \approx \frac{\exp[-f(z)]}{\text{pol}},$$

where *f* is non specific function of *z* and pol denotes the original polynomial. Other-⁵ wise, if the temperature background is written using goniometric functions, the resulting density profile behaves like an exponential, but *f*(*z*) has a very complicated form. In the region from 8 km to 40 km, the task of the background separation is more complicated because we have to deal with the jump of N_0^2 across the tropopause. For demonstration of our method, we have chosen to fit the $N^2(z)$ profile with polynomials of fourth, fifth and sixth order. The corresponding density background forms can be easily

inferred from Eq. (8). The local buoyancy frequency squared fits and corresponding density perturbations are then illustrated in Fig. 1.

As we can see in Fig. 1a, the sixth order polynomial gains a wavelike pattern with vertical wavelength of approximately 20 km, which is out of the interval of vertical scales

¹⁵ normally considered to be gravity wave induced. Taking into account the temperature profile shown in Fig. 2, we can see in Fig. 1b that the tropopause (around 18 km) does not cause enhancement of the gravity wave activity unlike the inversion layer around 10 km altitude (which causes serious problems to our method). In general, the perturbations are not biased and are centred around zero and due to the scaling, the decrease of the wave activity in stratosphere is clearly visible.

3.2 Comparison with temperature

3.2.1 In the stratosphere

In this section, we present a comparison between the normalised density perturbations resulting from the polynomial fits of $N^2(z)$ and the fit of the temperature profile with a cubic polynomial as used, e.g., by Gubenko et al. (2008). We have computed the mean vertical wavenumber power spectrum for the sample of sixty profiles. The

(12)

vertical wavenumber spectra of the normalized density and temperature perturbations are further compared with the theoretical shape of saturated spectra for density perturbations induced by the IGWs as derived by Senft and Gardner (1991).

- The mean power spectral densities (PSD) computed for various backgrounds and dry temperature for sixty profiles are depicted in Fig. 4. The presented range of vertical wavelengths is from 20 km to 400 m. The lower frequency limit was chosen due to the vertical extent of the examined area and the higher frequency limit was chosen as 2 times the Nyquist frequency of the interpolation of data to the height profile. According to Luna et al. (2013), the adequate choice of the wavelength cutoff for studying gravity waves through RO measurements is still open. For example, Wang and Alexander (2010) limited their analysis to vertical scales between 4 and 15 km, Tsuda and Hocke (2002) applied no minimum wavelength limit but Marquardt and Healy (2005) argued that in the altitude region below 30 km only temperature fluctuations with vertical
- 15 atmospheric waves.

20

In Fig. 4 in the lower wavenumber area, the influence of the background separation is dominant with higher order fits giving lower powers. The fourth order fit has the maximum shifted from roughly 12.5 to 7.5 km unlike the other density and temperature fits. That is a clear consequence of the higher possibility for this order fit to gain a wavelike form in this area (see Fig. 3a) and therefore the decrease of power in the longwave part. For determination if such a longwave mode is caused by IGW, more information (temperature, velocity and so on) would be needed and then the polarization relations would have to be examined.

wavelengths more than 2 km can be safely interpreted as originating from small-scale

Between the vertical wavelength of about 10 km and 2.5 km, the spectra of density from all three fits and the temperature spectrum are similar and their slopes are in good agreement with the theoretically predicted slopes. An important feature emerges at approximately 2.5 km, where the temperature fluctuation spectrum begins to decrease more rapidly than the density spectra regardless of the fit order. This should clearly be the consequence of the usage of hydrostatic balance in the temperature data retrieval,



which excludes nonhydrostatic waves and according to Steiner and Kirchengast (2000) acts to suppress wave amplitudes. The exponential fit (isothermal atmosphere) is failing to give correct orders of PSD values as well as to capture the feature of saturated theoretical spectrum.

5 3.2.2 Over the whole profile

Since we cannot rely on the theory (derived using WKB approximations) in the full vertical extend, we have chosen the continuous wavelet transform (CWT) (Torrence and Compo, 1998) method for analysis of the IGWs. Using CWT, we can study the behaviour of our method for the background separation in the tropopause region and track the gravity wave activity behaviour depending on altitude. Further, for the determination of dominant modes and for studying their development with height, we have applied a method of reducing CWT to its skeleton. We have used the same setting for drawing the spectral lines as Chane-Ming et al. (2000).

In Fig. 5a–c, the CWTs are shown for the scaled normalized density perturbations, resulting from different background fits. For comparison, CWTs of the unscaled normalized density perturbations resulting from 6th and 5th order polynomial fit are depicted in Fig. 5d, e. Finally, the normalized temperature perturbations resulting from the separation of the background in the form of a sixth order polynomial are shown in Fig. 5f.

The differences between CWTs of different fits as well as between the scaled and unscaled perturbations point to characteristic features. The differences between dif-

- ferent fits are, as expected, most pronounced in the region of the longer vertical wavelengths. This is even more evident comparing the skeletons of CWT in Fig. 6. In general, the difference between the scaled and unscaled perturbations is mainly connected with the theoretically predicted decrease of wave activity in the stratosphere
- and the shift of wave activity to longer wavelengths with height due to gradual filtering of small wavelength IGWs above the troposphere (Fritts and Alexander, 2003). That is captured better by the CWT of the scaled perturbations. Nevertheless, there are also unexpected differences between the scaled and unscaled CWTs in the region of



wavelengths around 16 km, where the amplitude is around one order higher and constant with height for the scaled case. The temperature CWT behaves similarly to that of the unscaled density perturbations.

Spectral lines of the CWTs maxima are plotted in Fig. 6a-f in the same order as in

- ⁵ Fig. 5a–f. From comparison of Fig. 6a–c, we can infer that the dominant mode with the wavelength around 16 km is probably caused by low quality separation of background. We can argue this, because in the cases of the temperature perturbations (Figs. 5f and 6f) and fourth order $N^2(z)$ fit (Figs. 5c and 6c) where this mode is the strongest, it has height-constant wavelength regardless of the variable local stability.
- ¹⁰ Other dominant modes occurring across the different fits and across the whole profile have scales around 13 km and approximately from 5 to 7 km. The skeletons should be compared with the N_0^2 profiles in Fig. 1a because according to, e.g., Chane-Ming et al. (2000) the change of the vertical wavelength of individual modes with height should be inversely proportional to the local stability.
- In our case, in the upper stratosphere, the decreasing wavelength of modes with height suggests the role of nonlinear wave processes acting to limit the amplitude which is reflected by the cessation of the wavelength grow. On the other hand, in the upper troposphere/lower stratosphere region where, in our case, the stability is growing, we can find increasing wavelength with height, which is to see for example for the mode with scale of 5–7 km in Fig. 6a. This suggests the possible enhancement of this mode's
- amplitude by means of wave-wave interactions or possible amplification resulting from some instabilities in the tropopause region.

4 Summary and conclusions

The aim of this study was to present and evaluate a method for the analysis of IGWs from GPS RO density profiles. After a description of the method for the density background separation, the methodology suitable for evaluation and comparison of resulting



IGWs analysis from density profiles with those from temperature profiles was discussed with regard to theory.

In the process of deriving our method in Sect. 2.1, we described the quantities defined by Eqs. (3)–(5) as the Brunt–Vaissala or buoyancy frequency. Although it helps

- to introduce and to understand the separation method, it might be also seen as phenomenologically inconsistent. Such a description would be correct for a fluid where the density is independent of pressure and the density of a fluid particle could be considered unchanged when displaced vertically (Sutherland, 2010). In our case this is not true. Therefore, a different naming convention should be used for these quantities in future works. For example, using the density scale height (H_{ρ}) definition (e.g. Sutherland,
 - 2010), Eq. (3) can be written like

 $\frac{g}{H_o}$

Nevertheless, the naming convention has not any influence on the results.

¹⁵ In Sect. 3.1, the problems of the background choice are discussed, correspondence between analytical forms of density and temperature background profiles is examined and the unphysicality of polynomial temperature fit is shown by solving related differential equation.

A very important result is shown in Sect. 3.2.1. In the high wavenumber region from ²⁰ approximately 0.4 cycle km⁻¹ (2.5 km vertical wavelength), the power spectrum of the normalized fluctuations derived from dry temperature GPS RO data has lower values than for normalized density fluctuations regardless of the background type. Steiner and Kirchengast (2000) argued that the likely causes of a tendency of increasing underestimation of spectral power toward higher wavenumbers are the GO technique, the local spherical symmetry assumption, and hydrostatic equilibrium assumption. The latter is

²⁵ spherical symmetry assumption, and hydrostatic equilibrium assumption. The latter is confirmed by our results.

Steiner and Kirchengast (2000) stated that above mentioned causes of underestimation are acting to suppress the wave amplitudes and therefore to lower the PSDs.



(13)

But this is not the only implication. Filtering the data with the hydrostatic balance in the standard temperature retrieval leads to the fact that in the equations of motions of the filtered field also the dynamic pressure and density fluctuations are in hydrostatic balance. That results in vanishing of vertical accelerations. The consequence is that the whole group of nonhydrostatic IGWs is filtered out. According to Sutherland

(2010), those are waves with frequency close to the buoyancy frequency and waves with phase line slopes significantly different from zero. It should be noted that these waves with smaller ratio of horizontal to vertical wavelength may be already filtered by the local spherical symmetry assumption.

5

- In the Sect. 3.2.2, the results of CWT and its skeleton do not reveal noticeable differences between the dry temperature and density based IGW analyses. Rather the differences between the scaled and unscaled cases are more important and the role of subjective background choice has visible influence on the longer modes. We do not discuss the propagation or sourcing of waves, which could be tempting especially in the
- ¹⁵ case of modes emerging or ending suddenly in the profile. The GPS RO measurement is a "snapshot" of the real atmosphere and no information from the cotangent space is included. Hence, using just a simple profile, we cannot say if or in which direction the mode propagates. Special care must be taken also when interpreting the results of the CWT skeleton because the vertical behaviour of modes could be also a consequence
- ²⁰ of fluctuation retrieval rather than of physical processes. That can be seen in Fig. 6 from comparison of the dominant modes between 4 and 8 km vertical wavelength in the upper stratosphere.

In summary, the analysis of IGWs with the GPS RO density profiles bears advantages over the analysis from dry temperature profile. Primarily, unlike the temperature,

the density background has a familiar form and the consequent analysis is not restricted to hydrostatic waves only.

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References

Anthes, R. A., Bernhardt, P. A., Chen, Y., Cucurull, L., Dymond, K. F., Ector, D., and Thomp-

- son, D. C.: The COSMIC/FORMOSAT-3 mission early results, B. Am. Meteorol. Soc., 89, 313–333, doi:10.1175/BAMS-89-3-313, 2008.
 - Chane-Ming, F., Molinaro, F., Leveau, J., Keckhut, P., and Hauchecorne, A.: Analysis of gravity waves in the tropical middle atmosphere over La Reunion Island (21° S, 55° E) with lidar using wavelet techniques, Ann. Geophys., 18, 485–498, doi:10.1007/s00585-000-0485-0, 2000.
- ¹⁰ Chiu, Y. T. and Ching, B. K.: The response of atmospheric and lower ionospheric layer structures to gravity waves, Geophys. Res. Lett., 5–6, 539–542, 1978.
 - Cushman-Roisin, B.: Introduction to Geophysical Fluid Dynamics, Prentice Hall, Englewood Cliff, New Jersey 07632, 1994.
 - Ern, M., Preusse, P., Gille, J. C., Hepplewhite, C. L., Mlynczak, M. G., Russell, J. M., and
- Riese, M.: Implications for atmospheric dynamics derived from global observations of gravity wave momentum flux in stratosphere and mesosphere, J. Geophys. Res.-Atmos., 116, D19107, doi:10.1029/2011JD015821, 2011.
 - Foelsche, U., Borsche, M., Steiner, A. K., Gobiet, A., Pirscher, B., Kirchengast, G., and Schmidt, T.: Observing upper troposphere–lower stratosphere climate with radio occultation data from the CHAMP satellite, Clim. Dynam., 31, 49–65, 2008.
- data from the CHAMP satellite, Clim. Dynam., 31, 49–65, 2008.
 Fritts, D. C. and Alexander, M. J.: Gravity wave dynamics and effects in the middle atmosphere, Rev. Geophys., 41, 1003, doi:10.1029/2001RG000106, 2003.
 - Garcia, R. R. and Randel, W. J.: Acceleration of the Brewer-Dobson circulation due to increases in greenhouse gases, J. Atmos. Sci., 65, 2731–2739, 2008.
- Gardner, C. S.: Rayleigh and sodium lidar techniques for measuring middle atmosphere density, temperature and wind perturbations and their spectra, University of Illinois, Urbana-Champaign, 1989.
 - Gardner, C. S. and Shelton, J. D.: Density response of neutral atmospheric layers to gravity wave perturbations, J. Geophys. Res.-Space, 90, 1745–1754, 1985.



- Gubenko, V. N., Pavelyev, A. G., and Andreev, V. E.: Determination of the intrinsic frequency and other wave parameters from a single vertical temperature or density profile measurement, J. Geophys. Res.-Atmos., 113, D08109, doi:10.1029/2007JD008920, 2008.
- Gubenko, V. N., Pavelyev, A. G., Salimzyanov, R. R., and Pavelyev, A. A.: Reconstruction of
- internal gravity wave parameters from radio occultation retrievals of vertical temperature profiles in the Earth's atmosphere, Atmos. Meas. Tech., 4, 2153–2162, doi:10.5194/amt-4-2153-2011, 2011.
 - Gubenko, V. N., Pavelyev, A. G., Salimzyanov, R. R., and Andreev, V. E.: A method for determination of internal gravity wave parameters from a vertical temperature or density profile measurement in the Earth's atmosphere, Cosmic Res., 50, 21–31, 2012.
 - Hines, C. O.: Internal atmospheric gravity waves at ionospheric heights, Can. J. Phys., 38, 1441–1481, 1960.
 - Liou, Y. A., Pavelyev, A. G., Huang, C. Y., Igarashi, K., Hocke, K., and Yan, S. K.: Analytic method for observation of the gravity waves using radio occultation data, Geophys. Res. Lett. 20, 2021, doi:10.1020/2002CL017818.2002
- Lett., 30, 2021, doi:10.1029/2003GL017818, 2003.

10

20

- Liou, Y. A., Pavelyev, A. G., Wickert, J.: Observation of the gravity waves from GPS/MET radio occultation data, J. Atmos. Sol.-Terr. Phy., 67, 219–228, 2005.
 - Luna, D., Alexander, P., and de la Torre, A.: Evaluation of uncertainty in gravity wave potential energy calculations through GPS radio occultation measurements, Adv. Space Res., 52, 879–882, doi:10.1016/j.asr.2013.05.015, 2013.
- Markwardt, C. B.: Non-linear least squares fitting in IDL with MPFIT, arXiv preprint arXiv:0902.2850, 2009.
- Marshall, A. G. and Scaife, A. A.: Impact of the QBO on surface winter climate, J. Geophys. Res.-Atmos., 114, D18110, doi:10.1029/2009JD011737, 2009.
- Melbourne, W. G.: Radio occultations using Earth satellites: a wave theory treatment, Vol. 5, Wiley-Blackwell, ISBN 0-471-71222-1, 2005.
 - Pavelyev, A. G., Wickert, J., Liou, Y. A., Pavelyev, A. A., and Jacobi, J.: Analysis of atmospheric and ionospheric wave structures using the CHAMP and GPS/MET radio occultation database, in: Atmosphere and Climate Studies by Occultation Methods, edited by:
- ³⁰ Foelsche, U., Kirchengast, G., Steiner, A., Springer Verlag, Springer Berlin Heidelberg, doi:10.1007/3-540-34121-8_19, 225–242, 2006.
 - Pavelyev, A. G., Liou, Y. A., Wickert, J., Gubenko, V. N., Pavelyev, A. A., and Matyugov, S. S.: New applications and advances of the GPS radio occultation technology as recovered by



analysis of the FORMOSAT-3/COSMIC and CHAMP data-base, Springer, Berlin Heidelberg, 165-178, 2009.

Preusse, P., Eckermann, S. D., and Ern, M.: Transparency of the atmosphere to short horizontal wavelength gravity waves, J. Geophys. Res.-Atmos., 113, D24104, doi:10.1029/2007JD009682, 2008.

5

- Schmidt, T., Wickert, J., De la Torre, A., Alexander, P., Faber, A., Llamedo, P., and Heise, S.: The effect of different background separation methods on gravity wave parameters in the upper troposphere and lower stratosphere region derived from GPS radio occultation data, in: 39th COSPAR Scientific Assembly, Vol. 39, July, p. 1721, 2012.
- Schreiner, W., Rocken, C., Sokolovskiy, S., Syndergaard, S., and Hunt, D.: Estimates of the pre-10 cision of GPS radio occultations from the COSMIC/FORMOSAT-3 mission, Geophys. Res. Lett., 34, L04808, doi:10.1029/2006GL027557, 2007,

Senft, D. C. and Gardner, C. S.: Seasonal variability of gravity wave activity and spectra in the mesopause region at Urbana, J. Geophys. Res.-Atmos., 96, 17229–17264, 1991.

Sica, R, and Russell, A.: Measurements of the effects of gravity waves in the middle atmosphere 15 using parametric models of density fluctuations. Part I: vertical wavenumber and temporal, J. Atmos. Sci., 56, 1308–1329, 1999.

Steiner, A. K. and Kirchengast, G.: Gravity Wave Spectra from GPS/MET Occultation Observations, J. Atmos. Ocean. Tech., 17, 495-503, 2000.

- Sutherland, B.: Internal Gravity Waves, Cambridge University Press, ISBN:9780521839150, 20 2010.
 - Tsuda, T. and Hocke, K .: Vertical wave number spectrum of temperature fluctuations in the stratosphere using GPS occultation data, J. Meteorol. Soc. Jpn., 80, 925–938, doi:10.2151/jmsj.80.925, 2002.
- Tsuda, T., Nishida, M., Rocken, C., and Ware, R. H.: A global morphology of gravity wave activity in the stratosphere revealed by the GPS occultation data (GPS/MET), J. Geophys. Res.-Atmos., 105, 7257-7273, 2000.

Tsuda, T., Lin, X., Hayashi, H., and Noersomadi: Analysis of vertical wave number spectrum of atmospheric gravity waves in the stratosphere using COSMIC GPS radio occultation data,

Atmos. Meas. Tech., 4, 1627-1636, doi:10.5194/amt-4-1627-2011, 2011. 30

Torrence, C. and Compo, G. P.: A practical guide to wavelet analysis, B. Am. Meteorol. Soc., 79.61-78.1998.



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- Vincent, R. A., Allen, S. J., and Eckermann, S. D.: Gravity-wave parameters in the lower stratosphere, in: Gravity Wave Processes, Springer, Berlin Heidelberg, 7–25, 1997.
- Wang, L. and Alexander, M. J.: Global estimates of gravity wave parameters from GPS radio occultation temperature data, J. Geophys. Res.-Atmos., 115, D21122, doi:10.1029/2010JD013860, 2010.

5

- Wickert, J., Schmidt, T., Michalak, G., Heise, S., Arras, C., Beyerle, G., and Rothacher, M.: GPS Radio Occultation with CHAMP, GRACE-A, SAC-C, TerraSAR-X, and FORMOSAT-3/COSMIC: Brief Review of Results from GFZ. New Horizons in Occultation Research, Springer, Berlin Heidelberg, 3–15, 2009.
- ¹⁰ Wilson, R., Chanin, M. L., and Hauchecorne, A.: Gravity waves in the middle atmosphere observed by Rayleigh lidars. Part 2: Climatology, J. Geophys. Res., 96, 5169–5183, 1991.
 - Wu, D. L., Preusse, P., Eckermann, S. D., Jiang, J. H., Juarez, M. D. L. T., Coy, L., and Wang, D. Y.: Remote sounding of atmospheric gravity waves with satellite limb and nadir techniques, Adv. Space Res., 37, 2269–2277, 2006.



Figure 1. (a) Various (polynomial) fits of the local buoyancy frequency squared $(N^2(z))$, **(b)** corresponding scaled normalized density perturbations.





Figure 2. The dry temperature profile corresponding to the occultation event 11 March 2011 04:08:29 UTC.



Introduction

References

Figures



Figure 3. (a) Various (polynomial) fits of the local buoyancy frequency squared $(N^2(z))$ in the stratosphere, **(b)** corresponding normalized density perturbations.











Figure 5. Wavelet power spectra of various perturbations resulting from different background states: scaled normalized density perturbations resulting from the 6th (a), 5th (b) and 4th (c) order polynomial fits; normalized density perturbations resulting from the 6th (d) and 5th (e) order polynomial fits; normalized dry temperature perturbations resulting from the 6th (f) order polynomial fit.





Figure 6. Wavelet power spectra skeletons of various perturbations resulting from different background states: scaled normalized density perturbations resulting from the 6th (a), 5th (b) and 4th (c) order polynomial fits; normalized density perturbations resulting from the 6th (d) and 5th (e) order polynomial fits; normalized dry temperature perturbations resulting from the 6th (f) order polynomial fit.

