Influence of MSIS on lower altitudes

In response to the question about the influence of MSIS, a detailed analysis is given here. It is well known that the limb-viewing geometry of GPS RO allows a high vertical resolution for its data. This is due to the fact that a relatively thin atmospheric layer above a tangent point contributes most of ray's bending and phase delay to the measurement point; and, the contribution drops rapidly with increasing distance. The same applies to forward modeling. Instead of showing few examples, we prefer discussing the influence in an analytical manner so as to provide a rigorous explanation. Because the reviewer's questions are related to the vertical structure in the model atmosphere in high altitude, we think consideration of dry symmetric atmosphere is sufficient. In that case, the ray path differential ds in eq. (1) of our manuscript can be expressed as:

$$ds = \frac{nr}{\sqrt{(nr)^2 - a^2}}\,dr\ , \qquad \text{(R1)}$$

where n is the refractive index; r radius; and, a is the impact parameter. Substituting (R1) to eq. (1) of our previous manuscript and linearizing the equation lead to:

$$\Delta\Phi'(a) = \int n'\,ds = \int n' \frac{nr}{\sqrt{(nr)^2 - a^2}}\,dr = \int n'(r)\,d\mathrm{K}(a,n,r)\ , \qquad \text{(R2)}$$

where $n'(r) = 10^{-6}\,N'(r)$ is refractive index perturbation. This can be considered as the error of MSIS refractive index (i.e., difference from the true atmosphere or from ECMWF data). (There are few places refraction radius (nr) must be differentiated from radius. However, both are denoted as radius afterwards for the sake of simplicity.) Here, $\Delta\Phi'(a)$ is the response of $n'(r)$, and $\mathrm{K}(a,n,r)$ is the weighting kernel given to each layer. Eq.

(R2) is nothing but a standard form of Abel transform. The green line in Figure R1a shows $\mathrm{K}(a,n,r)$, normalized with the value at the lowest half level of a regular grid, for which dr=0.2 km and the local radius of the Earth (R_e) is 6360 km. As shown, $\mathrm{K}(a,n,r)$ decreases rapidly with height. In addition, refractivity $N(r)$ decreases exponentially with height and so does its perturbation $N'(r)$. Assuming that the ratio of $N'(r)$ to $N(r)$ (fractional error of MSIS) is constant in vertical, $N'(r)$ can be described as follows: $N'(r) = N'(r_0)\cdot\exp\left\{-(r-r_0)H_N^{-1}\right\}$, where r_0 is the reference height (= R_e+25 km for this discussion) and H_N is the scale height of refractivity. While the temperature and its perturbation increase almost linearly with height in the stratosphere, p and p' decrease exponentially. Also the pressure scale height H_p is comparable with H_N. Typical value of H_p can be glimpsed by considering an isothermal hydrostatic atmosphere. In dry atmosphere, the following relation holds (i.e., ideal gas law): $N'/N = \rho'/\rho = p'/p - T'/T$, where ρ is air density. If the atmosphere is in hydrostatic balance, $\rho'/\rho \approx p'/p$. Here, p' at an altitude is hydrostatic response to the accumulated effect of temperature perturbations above the height. Because both p and p' are supposed to be in hydrostatic balance,

$$p'(r) = p'(r_0)\ \exp\left\{-\frac{g\,dr}{R_d T}\right\} = p'(r_0)\exp\left\{-\frac{dr}{H_p}\right\}, \qquad \text{(R3)}$$

where g is gravity acceleration (held constant, 9.8 ms^{-2}, for this discussion only) and R_d is specific gas constant for dry air (=287 J kg^{-1} K^{-1}). For the isothermal atmosphere $T = 300$ K, H_p = 8.785 km. The blue line in Figure R1a shows N' normalized with the value at the lowest half level (i.e., 25.1 km). The actual contribution of individual layers to phase path perturbation $\Delta\Phi'(a)$ is the combined effects of $N'(r)$ and $\mathrm{K}(a,n,r)$, designated with red line. Contributions of atmospheric layers at 48 km and 65 km to $\Delta\Phi'(a)$ are 4.8×10^{-3} and 5.3×10^{-4}, respectively. Even the total contribution of the atmosphere above a certain height, $\int_{r}^{TOA} n'(r')\,d\mathrm{K}(a,n,r')$, is small: about 2.3 % and 0.265 % of $\int_{a}^{TOA} n'(r')\,d\mathrm{K}(a,n,r')$ for $r=R_e+48$ km and $r=R_e+65$ km, respectively (Figure R1b). Therefore, modeling error of excess phase at 25 km due to the systematic error of MSIS above 48 km and 65 km is negligibly small.

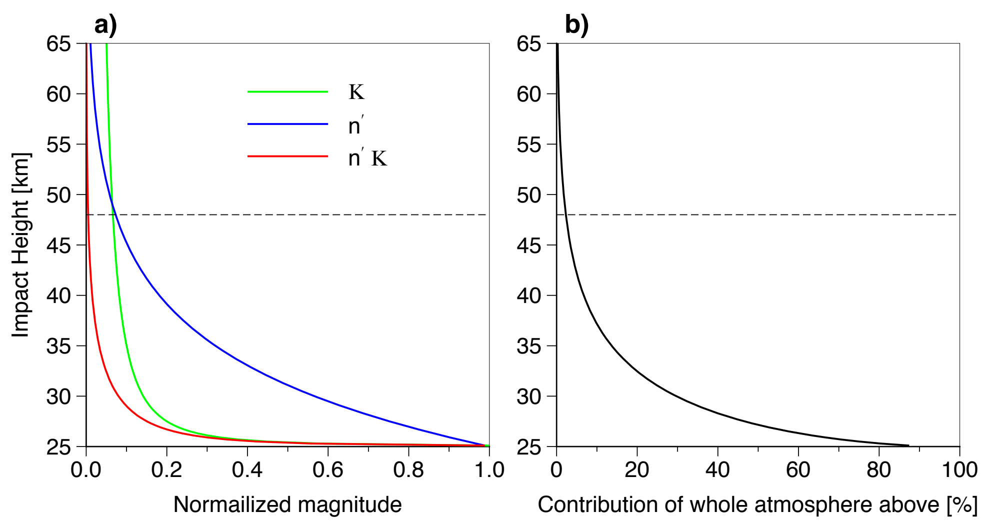

Figure R1: See text for details.