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Interactive comment on "Sensitivity of large-aperture scintillometer measurements of area-average heat fluxes to uncertainties in topographic heights" *by* M. A. Gruber et al.

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1 General Comments

The authors present a method of determining the sensitivity of a heat flux measurement to measurement errors of scintillometer beam height over heterogeneous terrain. Previous analyses of this type having assumed a uniform beam height, the contribution is indeed novel. I also believe that it is scientifically relevant, though I would have appreciated a more substantive effort by the authors to underline this point. For example, in the abstract it is claimed that "uncertainty may be greatly reduced by focusing precise topographic measurements in these areas". I would like to see this claim somewhere translated into numbers, e.g., "by focusing xx% more measurements at the following locations, uncertainty would be decreased by yy%."

2 Technical Comments

I have a couple of comments on the mathematical formulation in this article.

1. Equation (9). This equation is incorrect. The proper way to go about this is as follows. Assume that measurements $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_N)$ are independent stochastic measurements of the source variables $x = (x_1, \ldots, x_N)$, having systematic error
$$\sigma^2_{x_{s_i}} = (E[\hat{x}_i] - x_i)^2, \quad 1 \leq i \leq N$$
and random error
$$\sigma^2_{x_{r_i}} = \mathrm{var}(\hat{x}_i), \quad 1 \leq i \leq N.$$
Then by using a Taylor expansion about x, the mean-square error of estimating the derived variable $f(x)$ by $f(\hat{x})$ becomes
$$\sigma^2_f = E[(f(\hat{x}) - f(x))^2] = \sum_{i=1}^{N} \left[\frac{\partial f(x)}{\partial x_i}\right]^2 (\sigma^2_{x_{s_i}} + \sigma^2_{x_{r_i}}).$$
Computational error σ_{f_c} aside, the expression above is at odds with equation (9). It does, however, still lead to the sensitivity functions in equation (11).

2. Equation (13). The introduction of a "new" differential operator is unwarranted, as the usual notion of a functional derivative is sufficient to cover this case. In my opinion, the confusion stems from having defined the sensitivity function in equation (11) as
$$S_{f,x} = \frac{x}{f}\left(\frac{\partial f}{\partial x}\right).$$

A more appropriate notation would have been

$$S_{f,x}(i) = \frac{x_i}{f}\left(\frac{\partial f}{\partial x_i}\right).$$

The important distinction is that, in this case, the interest lies in the contribution to the error from variable i – upon considering the values of $x = x_0$ to be *fixed*. By the same token, the sensitivity function $S_{H_S,z}(u)$ considers the entire path $z(u) = z_0(u)$ of height measurements to be *fixed*: the interest lies in the contribution to the overall error $\sigma^2_{H_S}$ by the height measurement error at each location u. To this end, $S_{H_S,z} \cdot H_S/z$ coincides exactly with the functional derivative of $H_S : z(u) \to \mathbb{R}$ with respect to $z(u)$, *evaluated at* $z_0(u)$ (e.g., Courant & Hilbert, 1953):

$$\left.\frac{\delta H_S}{\delta z}\right|_{z=z_0(u)} = \lim_{\epsilon\to 0}\frac{H_S[z_0(u)+\epsilon\phi(u)] - H_S[z_0(u)]}{\epsilon},$$

where $\phi(u)$ is an arbitrary function. It should also be noted that functional derivatives has a long history of application to error analysis, e.g., Fernholz (1983), Beutner & Zähle (2010), and many references therein.

References

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