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Comment

# ***Interactive comment on “Sensitivity of large-aperture scintillometer measurements of area-average heat fluxes to uncertainties in topographic heights” by M. A. Gruber et al.***

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We thank the reviewers and Dr. Lysy for their time and effort to make this article better. Below are our responses to the comments and details on how we will address them in the updated manuscript.

Response to Reviewer #1:

We thank Reviewer #1 for their comments. We are indeed assuming that the friction velocity is measured independently. We feel that this is the first step in the expansion of

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uncertainty analyses from flat terrain to variable terrain since we can compare with the flat terrain study of Andreas (1989). The next step would be to include derived friction velocity measurements either via the Businger-Dyer relation for large-aperture scintillometers or with displaced-beam scintillometers. With displaced-beam scintillometers we could compare to the flat terrain study of Gruber & Fochesatto (2013), and indeed we have produced this study which we hope to submit to a journal shortly. You can view a preliminary version of this article at: <http://arxiv.org/abs/1405.2309>

Eqs. 2, 3, 5, and 7: We agree that we can re-write Eqs. 2 and 3 to introduce the effective beam height there and to have  $T^*$  on the left hand side. We can then get rid of Eqs. 5 and 7 since they originally were the same equations just with effective beam height instead of a flat terrain beam height.

P. 35 Line 14-22 - We have not included in our analysis the displacement distance since we are presenting the first step in taking into account variable terrain and we see this as a matter of future extension to this study, however we can include a short discussion on it. Also, the field site we use as an example is Alaskan tundra with very short and even vegetation.

P. 36 Line 11-16: "Heterogeneous terrain implies..." We included this paragraph starting at "In using the Monin-Obukhov similarity hypothesis..." to justify using the equations that we did, however heterogeneous terrain is indeed not relevant to this study so we could truncate this paragraph and start it at "Sensitivity studies have so far been restricted..." at line 14.

P. 36: Line 16-18: "Hartogensis et al. (2003) anticipated..." We can get rid of this sentence.

P. 39 Line 2-3. We are indeed neglecting humidity fluctuations to simplify the equations involved but we believe that the results for  $S_{H,z(u)}$  are the same if humidity were taken into account with a two wavelength system as in Andreas (1989). We can include a sentence stating that humidity fluctuations are being neglected.

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P. 40 Line 1. “To illustrate this we re-write for example Eq. 6 as...” Thanks to Dr. Lysy’s comments, we can simplify this whole section by just invoking functional derivatives to arrive at Eq. 15. Eqs. 12, 13 and 14 can be eliminated.

P. 42 Line 19 – P. 43 Line 5. If  $u^*$  is unknown, we need another equation to resolve it, and this is either the Businger-Dyer relation for large-aperture scintillometers, or the equation relating  $I_o$  to  $\epsilon$  to  $u^*$  for displaced-beam scintillometers. Without the addition of one of these equations there are more unknowns than equations and the system is undetermined. In the case of the Businger-Dyer equation, we have not yet performed this analysis so we are unsure if we can use fixed point recursion to solve. In the case of displaced-beam scintillometers, we can use fixed point recursion to isolate a single solution; see <http://arxiv.org/abs/1405.2309>

P. 43-46: We do need to solve Eq. 21 and 26. Above equation 21, we can get rid of “For example”. After equation 21, for stable conditions we indeed only need to implicitly differentiate for  $d\zeta/dz_{eff}$ , so we can re-write the paragraph at line 15 to get rid of “We will need some derivatives...”. For unstable conditions, Eq. 27 is an explicit derivative of Eq. 24 (using the tree diagram as guidance), while Eq. 31 is different. It is tricky, but you implicitly differentiate Eq. 29 (using Eq. 30) to arrive at:

$$\frac{1}{2} \hat{f}(-1/2) * (df/d\zeta * d\zeta/dz(u) + df/dz(u)_{\zeta}) = -2b d\zeta/dz(u)$$

to get:

$$df/d\zeta * d\zeta/dz(u) + df/dz(u)_{\zeta} = -4b\hat{f}(1/2) d\zeta/dz(u)$$

and then you isolate for  $d\zeta/dz(u)$  and substitute back in Eq. 29. We can write that more clearly at line 11 on page 45. We can also try to write in a parallel style for each stability section.

Abstract lines 17-18, Conclusions line 9-10, P51 line 20, P 52 line 2:

We are indeed using an independent  $u^*$  measurement. For path averaged  $u^*$  measurements over variable terrain, we have not yet done the analysis including the Businger-

Dyer equation so it is not yet known whether we can use fixed point recursion in this case. For displaced-beam scintillometers, we have done the analysis and you can use fixed point recursion, as seen at <http://arxiv.org/abs/1405.2309>. In Gruber & Fochesatto (2013) we just applied the already established technique of fixed point recursion to displaced-beam scintillometers over flat terrain. In this paper, we apply it for the first time to large-aperture scintillometers over variable terrain. Fig. 5 is an example of a single equation, not two equations, for which recursion will converge to a solution. It is our understanding that the iteration procedure used widely in the scintillometer community is to take a set of multiple equations, and to iterate through the entire set of equations while refining one variable. This may not converge, as we illustrated by quoting Press et. Al in Gruber & Fochesatto (2013). We prefer to convert this large system of nonlinear equations into a single nonlinear equation in a single unknown, in the form of  $\zeta = f(\zeta)$ , which is the fixed-point form. Not all functions in the fixed-point form converge, but the functions which we arrive at do. Iterating recursively over equations 20 and 25 is what we are arguing for as an alternative to iterating over a large set of equations as in for example Solignac et al (2009) and many other scintillometer studies. The advantage over iterating with a large set of equations is assured monotonic convergence and compact and simple computer code. Note that in the literature, fixed-point recursion is often called fixed-point iteration, however we have called it fixed-point recursion in our papers to differentiate it from the standard scintillometer iteration technique over a large set of equations.

P. 52 lines 12-13. Yes this is true we can re-write this sentence.

P. 53 lines 11-13. From Dr. Lysy's comment and a review of several sources, we can confirm that the Dirac-Leibniz derivative is indeed a functional derivative which have a long history of use. We have not managed to find, however, any studies using functional derivatives in error propagation over functionals such as the effective beam height. We are unsure if it has been done before in the way we have done it.

P. 34 Line 22: We can re-word here.

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P. 35 Line 10: Yes this is true, we can modify this.

P. 36 Line 9: We can say: “While we are considering topography which is not flat...”

P. 37 the definition of T: Yes this is true, it should be defined at line 20 on page 37.

P. 39 Line 12: We can change this to “resulting from a relative error”.

Eqs. 25, 32 and 34: Perhaps in Eq. 25, the multiplication dot should appear after the first line in the term under the square root, instead of at the beginning of the second line. In Eq. 32, maybe there should be braces around the entire denominator. Similarly, in Eq. 34, perhaps there should be braces around the entire denominator and the multiplication dots should be one line above each.

Eqs. 31 and 32: This is true we can keep just Eq. 32’s reference.

P. 48 Line 1: We can change the title of the section to “Application of the results for the sensitivity function  $S_{H,z}(u)$ ”

P. 48 Line 3: We can include the co-ordinates in the text here.

Fig. 3: We can try to plot in an aspect ratio that gives the same angle for Fig. 3A and Fig. 3B. We can change the x and y axes to better illustrate a 1km by 100m path. We can change the z axis to “height above 927m above sea level”. Fig. 3B: The brackets being different like they are is standard mathematical notation for a range of values including or not including the end point. We can change the caption of the figure to illustrate that the “survey” is the GPS measurements. The x axis is “Elevation difference: Survey – DEM” ; note the colon, and the “–” is a minus symbol (we can increase the size of this figure).

Fig. 4: We can try to put the units on the left and right sides of the graph (Matlab is notoriously finicky to do this properly).

Fig. 5: This figure is to illustrate the method of convergence. Not all fixed-point functions converge even if they have only one solution. There are criteria for convergence,

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and the iterations can alternate around the solution, or they can monotonically converge toward the solution. We can include a line here to show the path the solution takes, for instance starting at  $(-1,-1)$ , down to  $(-1,f(-1))$  over to  $(f(-1),f(-1))$  down to  $(f(-1),f(f(-1)))$  etc. like a zig-zag line going from top right to bottom left.

Fig 6: Yes, we could also plot the solution for  $\zeta > 0$  in this figure.

Fig. 9: We could get rid of this figure, but we'd have to include the mathematical functions  $z(u)$  in the text. We think that this would obstruct the flow of the paper more than a figure. We'll definitely adjust the legend and y axis though.

P. 50 lines 3-5. We can shorten this.

P. 50 line 9: We meant local maxima. We can change "concentrations" to "local maxima".

P. 50 line 18-19: We can get rid of this sentence.

P. 50 lines 15-21: We can get rid of this explanation of what  $S_{H,z}(u)$  isn't, and instead explain what it is. We can include a discussion about the magnitude of  $S_{H,z}(u)$  compared to the sensitivity functions of other variables. A local value of 4 is quite high.

Response to Reviewer #3:

We thank Reviewer #3 for their comments. We agree that guidance for beam setup is unchanged with our results, and that it was previously suspected that scintillometers were most sensitive to areas of topographic protrusion. However, this sensitivity was not quantified and there was no method to produce error bars on plots, so our goal for this paper was to take the first step in that direction. There will always be a certain precision and accuracy with which we can determine the topography, and quantifying the uncertainty on the heat fluxes that results from this is the matter of our paper. We agree that the assumption of independent  $u^*$  measurements may introduce significant error, however this was done as a first step in order to compare with the sensitivity functions for flat terrain in Andreas (1989). We have produced a similar variable ter-

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rain study with path averaged  $u^*$  measurements for displaced-beam scintillometers, and the results can be found at: <http://arxiv.org/abs/1405.2309> Differences in stability function choices could be an interesting matter for future research. In the discussion we can include a section comparing the resulting values of  $S_{H,z}(u)$  to the sensitivity functions for heat from other variables such as  $u^*$  itself (assuming it was measured independently). The error introduced by assuming that  $u^*$  is independently measured and from not knowing which stability function to use is systematic error (error bar offset), whereas the error propagated from  $S_{H,z}(u)$  is random error (error bar width) so we'll have to compare keeping this in mind. It will be most important to compare to sources of random error from the sensitivity functions of other variables.

1. Title: We can change “measurements of” to “derived”.
2. Scintillometers measure intensity fluctuations, not  $C_n^2$  directly, so this is what we were trying to accommodate.
3. We can get rid of “and independent friction velocity measurements”.
4. We'll specify that we are considering height above ground, since we are not considering the displacement height for the purposes of our study.
5. We'll change “normalized path length” to “relative path position”.
6. We can delete “source”.
7. We are citing this source as an example of the use of LIDAR topographic data, perhaps we can re-arrange and cite this earlier. We will compare the values of  $S_{H,z}(u)$  to the values of the sensitivity functions for other variables to support our statement, and we can re-write this sentence to make a weaker claim. It is true that assuming an independent  $u^*$  measurement may introduce more uncertainty, but in our experience with displaced-beam scintillometers and path averaged  $u^*$  measurements, the sensitivity to  $z(u)$  is even higher. (see: <http://arxiv.org/abs/1405.2309> ). It is true that we are not taking into account uncertainty in which stability function to use (systematic error

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or error bar offset) in this statement. To address both these issues, we can make it clearer to reflect that we are considering the magnitude of random error propagation (error bar width).

8. Since our terrain features very short, sparse, and even vegetation, we are not considering the displacement height. Our analysis is the first step towards taking into account variable terrain, and displacement height would be an interesting topic for future study. We will include a discussion on this.

9. Lines 19-21, P. 50: The amount of error that is reduced by increasing the precision on the topography measurements will vary from site to site, and even with atmospheric conditions, but we can supply some examples on relative error in H reduction for some sample data. The high sensitivity regions do have an extreme dependence on  $z(u)$ , we explain this physically on P. 50 line 2.

10. In unstable conditions, the mean sensitivity approaches 100% as seen in Fig. 8, so the error propagates quite strongly. We will provide comparison of  $S_{H,z(u)}$  to the sensitivity functions for other variables. We will provide examples of the effect of the error propagation on “end stream” relative error in H. We will emphasize the practicality of the paper in now being able to solve for H with compact computer code, and being able to produce error bars for H with a closed form analytic sensitivity function instead of overly complicated and rarely used numerical methods.

Response to Dr. Lysy:

We thank Dr. Lysy for his comments.

1: We agree that we should quantify the effect of the sensitivity function on “end stream” relative error on H. We can include the effect of reducing error in  $z(u)$  by using LIDAR measurements over DEM data by tracking a reduction in relative error in H for some sample data, as requested by Reviewer #3.

2.1: In Gruber & Fochesatto (2013), we included a description of this equation stating

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that the first and last terms represent an offset from the true value, and that the square root term represents the width of the error bars. We recognize that we can't really use a mathematical expression like that, however, so we will use your modified expression.

2.2: We will modify the notation in Eq. 11 to include  $x_i$ . We agree that the Dirac-Leibniz derivative turns out to actually be a functional derivative, although a different one from the one we are used to using in the calculus of variations. We found what you are referring to in "Field Quantization" by Greiner & Reinhardt (1996), chapter 2, and we understand that this is equivalent to what we are doing in this paper, although not for an error analysis application. We are familiar with the calculus of variations, but we are not able to understand how Fernholz (1983) and Beutner & Zahle (2010) relates to our paper. Perhaps you would be willing to aid us in explaining this better, and in finding a proper source using these functional derivatives in error propagation over functionals. Perhaps this is Beutner & Zahle but we are not able to understand it.

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Interactive comment on Atmos. Meas. Tech. Discuss., 7, 33, 2014.

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