Interactive comment on "Effective resolution concepts for lidar observations"

By M. Iarlori et al.

Anonymous Referee #2

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Review of "Effective Resolution concepts for Lidar Observations" by Marco Iarlori, Fabio Madonna, Vincenzo Rizi, Thomas Trickl and Aldo Amodeo Paper ID: amt-2015-89.

The authors would like to thank the reviewer#2 for the detailed revision of the manuscript. The reviewer has provided several valuable comments and suggestions to improve the quality of the manuscript and to correct some inaccuracies. English is improved in the new version of the manuscript along with its general structure. The authors have got benefit from the notes reported by the reviewer in form of hand-written comments. The manuscript length is reduced, the Sect 2.2, 2.2.1, 2.2.2, 2.3 and 3.5 are now moved in the Appendix. The math notation is now more consistent.

Note: unless otherwise stated, page/line numbers refer to the Annotated Manuscript from Referee #2.

Answers to "**Major Comments**".

(1) Reviewer comment: "The first part of the manuscript (whole Sect. 2) and, more intensively until Eq. (7), is just a review – in summary "book" fashion – of well-known concepts on digital signal processing, which conveys no original material. Suggestions are given next to improve it in terms of notation, terminology and clearer structuration of ideas."

The Sect.2 is improved: it rewritten and the relevant references are cited. In details:

1. *Response of linear-time-invariant (LTI) systems to arbitrary inputs: convolution sum*

   Eq.(R1) is now the Eq.(1).

   "The general time-domain relationship between the system output value at time *n* and the input values for a linear-time invariant (LTI) system can be written as:

   $$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)\,. \qquad (1)$$

   In Eq. (1) the sequence $h(k)$ is the so-called impulse response and it represents the wanted LTI transformation."

2. *Causality*

   The causality characteristics of the FIR filters used in this work are briefly commented in the revised manuscript (see below the point 3., 4. and 5.):

   "In the case of real-time DSP the Eq. (1) cannot be used directly since it requires future discrete-time input samples to be available, i.e. those with positive (n–k) time-index (for this reason the LTI system is required to be *causal*) moreover it has infinite limits. However, for the current study only Finite Impulse Response (FIR) filter will be examined, i.e., only those filters for which $h(k)$ has a finite number of non-zero elements. Furthermore the analysis of lidar signals is typically carried out offline, and for this reason, "future" samples are always available (Orfanidis, 2010)."

3. *Finite Impulse Response (FIR)*

   The FIR definition is inserted as suggested by the reviewer (Eq.R3).

   "According to the above considerations, the Eq. (2) can be written as:

$$y(n) = \sum_{k=-N}^{N} h(k)x(n-k), \qquad n = N+1,\ldots,n_{\max} - N\,. \tag{2}$$

   The Eq. (2) is a representation of the so-called non-causal Linear Time Invariant (LTI) Finite Impulse Response (FIR) digital filter (Orfanidis, 2010)."

4. *Highlight the importance of linear-phase FIR filters, which is what you use all over the papers.*

   The role of linear-phase FIR filters is discussed in the manuscript:

   "The FIR frequency response can be written as:

$$H(\omega) = A(\omega)e^{j\theta(\omega)} \tag{4}$$

   where $A(\omega)$ is the amplitude and $\theta(\omega)$ is the phase. FIR filters can have $\theta(\omega)=-b\omega$, where b is a constant. This form of $\theta(\omega)$ simply shifts the different frequency components of a signal, avoiding phase distortion. A sufficient condition to have linear-phase is a real and symmetric impulse response (Oppenheim and Schafer, 2009). In this study, non-causal low-pass FIR filters are considered for which $\theta(\omega)=0$ (zero-phase) and the symmetry condition is $h(k)= h(-k)$. Then $H(\omega)$ is a real function (Kuc, 1988) which can assume positive and negative values."

5. *Fourier Transform (FT) of a discrete-time aperiodic signal (your Eq.(2)). Introduce the concept of "frequency for discrete-time signals".*

   The equivalent concept of *digital frequency*. The symbol ν is preserved (Kaiser,1977) because it is also used in other papers about the lidar effective resolution (Godin, 1999; Masci, 1999):

   "The impulse response of a non-causal FIR filter is a real-valued sequence (as this is true also for the signals considered in this study) and its Discrete Time Fourier Transform (DTFT) (Karam et al., 2009; Hamming, 1989; Smith, 2007):

$$H(\omega) = \sum_{k=-N}^{N} h(k)e^{-j\omega k}, \qquad \omega = \frac{\Omega[rad/\sec]}{f_S[sample/\sec]} = \frac{2\pi f}{f_S}[rad/sample]. \tag{3}$$

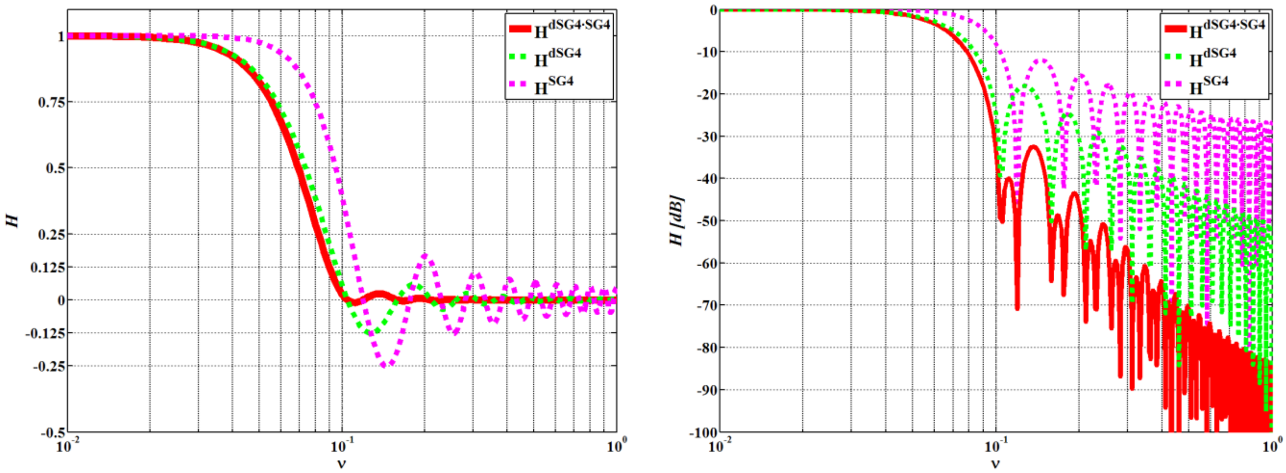
   gives the frequency response, with ω (the *digital frequency*) that is the ratio between the analog angular frequency Ω, and f_S the sampling frequency, "it represents a convenient normalization of the physical frequency f "(Orfanidis, 2010). In general $H(\omega)$ is a complex, continuous and periodic function of ω with period 2π. For real $h(k)$, $H^*(\omega)=H(-\omega)$, and all the information carried by the frequency response are confined in the range $0\leq \omega \leq\pi$ (Kuc, 1988). If f is normalized to the Nyquist frequency f_N=$fs/2$ (Orfanidis, 2010), i.e. reported in ν (=f/f_N=ω/π) units, then 0≤ ν ≤1. The variable ν is usually called reduced or normalized frequency (Kaiser, 1977; Godin, 1999, Masci, 1999; Orfanidis, 2010); it will be used as an independent variable in all the frequency response plots presented in this work."

6. *Fig. 1 Why the negative part of the spectrum is not represented? Explain that only positive frequencies need be represented.*

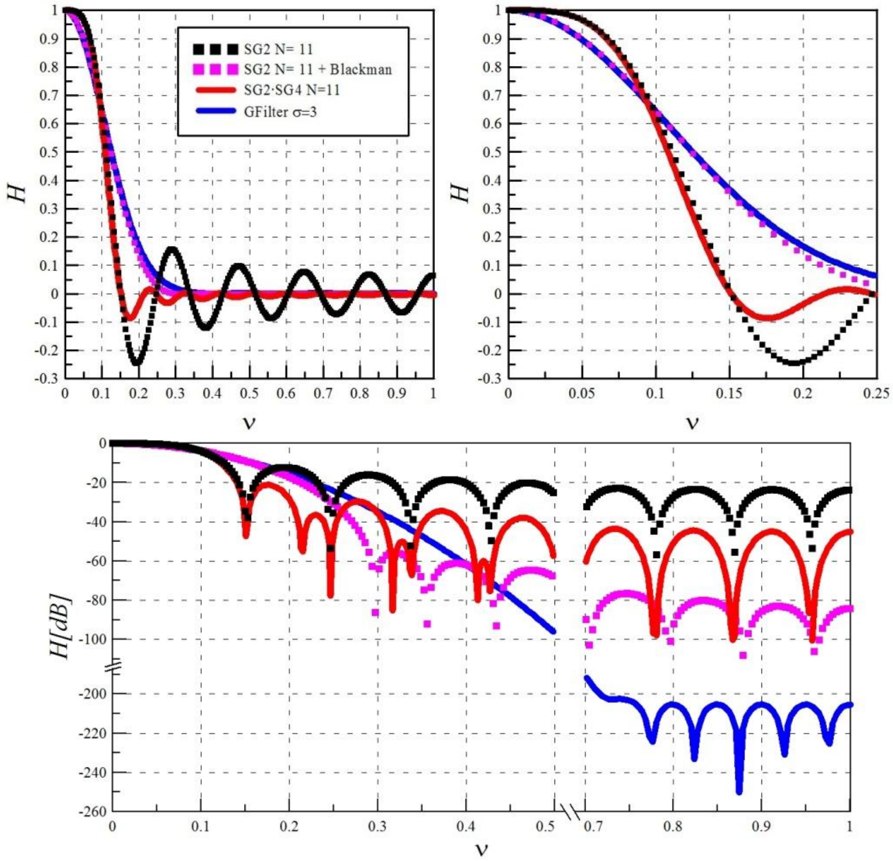
   See the above answer.

(2) Reviewer comment: FIGURES. Please consider plotting 20log$_{10}$|H(ω)| all over most of (not all) the spectral figures of the paper instead of plain H(ω). The standard representation in signal processing is 20log$_{10}$|H(ω)| and arg[H(ω)] [deg]. This will help identifying sidelobes in the STOPBAND. The correct notation for decibel is [dB] (not db).

The linear scale helps to explain the effects of the negative values of H. In the revised manuscript Fig.6 and Fig.8 are shown also using dB scale. The correct decibel notation is included.



New Fig.6



New fig.8

(3) Reviewer comment: Notation (not exhaustive): (…)
Notation is now consistent and homogeneous.

(4) Reviewer Comment: <u>Equivalent SG filter</u>. The author propose cascaded SG filter combinations of the type SG2(L1) & SG4(L2). How does a single filter, for example SG2(L1+L2), compare with the cascaded solution? More generally, the authors should discuss on an equivalent SGx(y), i.e., a suitable pair of order "x" and length "y" based on nice parametric studies carried out. This is better oriented to the focus of the paper.

The authors think that a complete parametric study on this topic is beyond the scope of the manuscript. We also think that our examples are sufficient to describe the cascade methodology and its performance.
The cascade SG filter is proposed as one of the possible solutions to reduce the side-lobe effects of the SG filters. The cascade filter $SGx(y)SGx_1(y_1)$ "equivalent" to SGx(y) (i.e. with nearly the same pass-band and less pronounced side-lobes of the SGx(y)), is found with the simple rule: $\Delta x=x_1-x=2$, $y_1=y$.
A single SG2 filter of any length could not overcome the side-lobe effect, see Fig.A1. In Fig. A1 (right plot) it is shown that the simple rule for the construction of the "equivalent"

cascade SG filter works beyond the validity limit of Eq.(10), and this is included in the revised manuscript.

In addition, in Fig. A1 (left plot) the cascade SG2(N)·SG2(N) is plotted to show that it remove efficiently the side-lobe effects, but it is not “equivalent” to SG2(N) from to pass-band point of view.

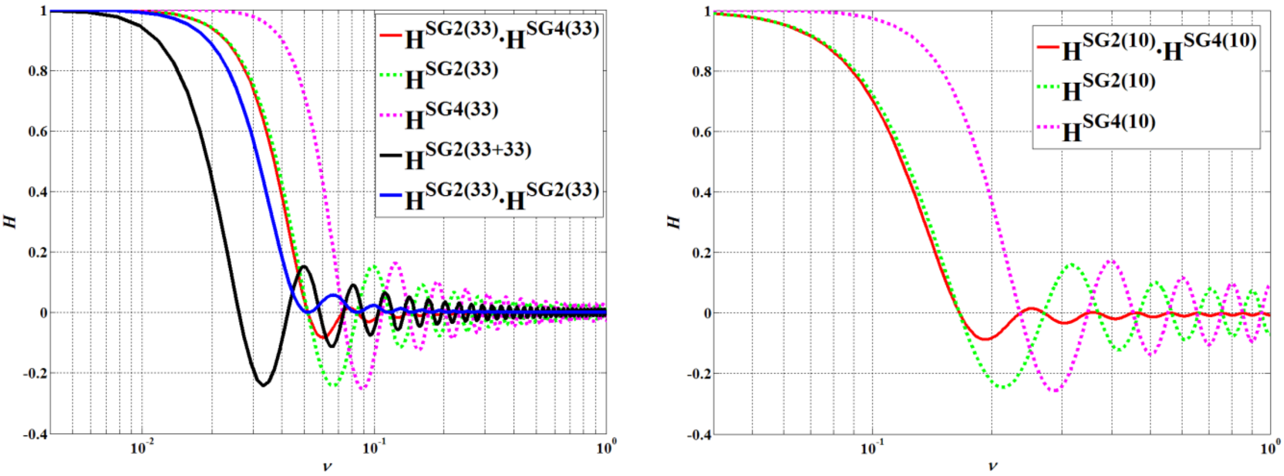


Fig. A 1. Left plot: the cascade filter “equivalent” to the SG2(33) should have nearly the same pass-band of SG2(33) (green-dot line) and less pronounced side-lobes. The best results (red line) is obtained adopting the cascade construction with the simple rule: Δx=2 and same y. Right plot: it is shown our simple rule works well (red line) beyond the validity limits of Eq.(10).

(5) Reviewer comment: <u>Variable spatial resolution</u>. How lidar signals are processed under a variable spatial-resolution approach? Using the same SG filter over the full range of the lidar sensor (or other remote-sensing sensor) means processing the lidar signal with the same spatial resolution (i.e., constant all over the inversion range). However, when faced with data records several km long, the rhythm of decrease of SNR forces to use much longer spatial resolutions at further ranges, where the SNR is getting lower and lower. It is difficult to believe that the same SG filter can solve the problem for the full inversion range (say 0-8 km). I advise the authors to expand/discuss this part in detriment of others.

The reviewer is right, it is a common practice to change the smoothing parameters to overcome the decrease of the SNR of the lidar signal as the range increases. We briefly describe the application of low-pass filters with variable parameters. Further analysis is outside the scope of this paper:

“Due to the large dynamic range of a lidar signals, digital filters with a different frequency response could be applied at different ranges, in order to deal properly with local values of the signal-to-noise ratio (SNR). This means that the filter’s smoothing parameters are range dependent, therefore the smoothed lidar returns will be characterized by a range dependent effective resolution (see Sect.3).”

(6) *Reviewer comment: <u>Conclusion</u>. Just a preliminar review: Important is that –from what you have in hands- the scope of the paper at least covers: 1) an in-depth parametric study on SG and Gaussian filters and 2) Effective resolutions studies/methodologies. Both are equally important and nice to contribute, not only the second! The paper title could also be better adapter to reflect this.*

This study was carried out focusing on the effective resolution of lidar measurements. We agree with the reviewer and for this reason in the revised manuscript the corresponding part (p. 32 lines 1-13 in the old version of manuscript) have been changed as in the following:

"Concerning the latter point, a parametric study of several smoothing filters has been performed to provide recommendations about the type to be used. The estimation of the ERes does not provide any definitive indications on the best filter to be applied in the data analysis: because the ERes does not contain any information about the spectral behavior of the different filters. Among the investigated filters, for example the cascade Savitzky – Golay filters are well suited for the lidar data analysis. This class of filters is characterized by:

- the preservation of the spectral features of the signal in the pass-band;
- steep transition band;
- reduction of the side-lobe effects;
- larger transient zone.

The Gaussian filter characterized by :

- an efficient suppression of the noise;
- less accurate preservation of the signal in the pass-band;
- more extended transition-band."

Answers to the "Detailed comments". Note: unless otherwise stated, page/line numbers refer to the Annotated Manuscript from Referee #2.

1. *Reviewer comment: p.4, lin. 8. Clarify that according to SG (1964) paper there are two big families of SG filters: "smoothers" and "n-th order differentiators" and that derivation always implies a certain level of smoothing because of the inherent construction of the SG filter.*

   The details of the filters properties (SG and Gaussian) are now in Appendix:

   "These kinds of filters are widely known as Savitzky – Golay (SG) (Savitzky and Golay, 1964), a family of filters which comprise both pure smoothers and n^{th} order differentiators: this is discussed in the Appendix A1."

   In Appendix it is stated that for the SG filters, the differentiator has a low-pass inherently present by construction:

   "The impulse response, for a given P and N, of the SG n^{th} order derivative filter is directly calculated (Savitzky and Golay, 1964): this is another useful property of the Savitzy – Golay recipe. The way the SG filters are constructed implies that derivation has a low-pass filter inherently embedded."

2. *Reviewer comment: p.4, lin. 13. Notation. Unclear indexes. In the convolution equation, it is standard to say that the impulse response has "M" samples, the signal (x(n)), N samples, and the output (y(n)) N+M-1 samples. Please consider to include a figure sketch to help the reader (otherwise lins 21-23 becomes cumbersome…).*

   In the new version of the manuscript, the authors have corrected the index and rephrased the implication of the convolution (the transient effect) in a more clear way and for this reason the authors think that the figure sketch is not needed. This is done following the reviewer corrections/suggestions in the annotated manuscript:

   "Then, the Eq. (1) can be written as:

$$y(n) = \sum_{k=-N}^{N} h(k)x(n-k), \qquad n = N+1,\ldots,n_{\max} - N \,. \tag{2}$$

   The Eq. (2) is a representation of the so-called non-causal Linear Time Invariant (LTI) Finite Impulse Response (FIR) digital filter (Orfanidis, 2010). If the impulse response is composed of $2N+1=M$ elements and if the length of the input signal $x(n)$ is n_{max} (n=1,2,...,n_{max}), than the length of the output signal $y(n)$ will have normally $M+n_{max}-1$ elements, if we do not consider the limits for n in Eq. (2). When $y(n)$ is calculated for n outside those limits, i.e. for $n<N+1$($n>n_{max}-N$), there is always a lack of one or more of the needed input samples. Those missing inputs will be treated as zeroes in the convolution sum, and this will cause an *transient effect*, and hence, a distortion on the smoothed signal. To remove this transient effect, the limits in n have been inserted in Eq.(2) and of course this implies an overall loss of information in the output signal because the necessary removal of N points at the beginning and at the end of the output signal. Anyhow, it should be noted that there are techniques (Gorry, 1990; Khan, 1987; Leach et al. 1984; Orfanidis, 2009) that are able to deal with this problem."

3. *Reviewer comment: p.5, lin. 12. Remove "aliasing". "Sampling of a continuous-time aperiodic signal (e.g., the lidar signal) causes the spectrum of the discrete-time signal (i.e., H(omega)) to be a continuous and periodic function of variable omega (with period 2*pi (-pi<=omega<=pi))." "Aliasing" usually refers to the unwanted effect of multiple spectrum folding on the frequency axis due to a too low sampling rate.*

   The term "aliasing" is removed. See the answer [point (1), sub-point 5] in "Major Comments" answer section.

4. *Reviewer comment: p.5, lin. 15. Normalized frequency. Please see major comments.*

   See the answer [point (1), sub-point 5 and point (3)] in "Major Comments" answer section.

5. *Reviewer comment: p.6, lins 3-5. A negative value of H… results in artifacts. This assertion is FALSE. What causes "artifacts" of unwanted effects is the frequency content in the transient+stop-band of the spectrum, be it positive, negative, or with a given phase (in the case of a complex-valued spectrum, not your case). This part of the spectrum conveys "unwanted" leakage frequencies. The fact that an unwanted frequency has arg (H(omega1)) = 0 or 180 deg (i.e., H(omega_sub_1)=+1 or -1, respectively) means that this high-frequency will show up in the grey zone of Fig.2 without or with a sign reversal in the oscillatory behaviour of the Chirp signal. KEY is to have low sidelobes and no discontinuities in the time domain.*

   Thanks for pointing out this problem. We have revised this paragraph. A more complete discussion about artifacts is done in the new version of the manuscript:

   "Ideal low-pass filter cannot be practically realized and the departure from the ideal behavior will cause artifacts in the output signal. In fact, real low-pass filters are characterized by a finite transition region between the pass-band and the stop-band and at higher frequencies H could be significantly different from zero and with alternating positive/negative values (ringing or side-lobe effect). As an example, a negative value of H in the stop-band region could results in a particularly evident high frequency artifacts in the output signal. For this reason the filter's design tries to minimize the transition region and the side-lobe effects as discussed in Appendix."

6. *Reviewer comment: p.6, lin. 8, Eq. (3). Revise/wrong. See Eq. (R8b) next.*

Eq. (3). Corrected (now is Eq. (5)):

$$x_n = \cos\left[2\pi\left(\frac{f_0}{f_s}\right)n + \pi k\left(\frac{n}{f_s}\right)^2 + \phi\right];\ f_0 = 0; k = \frac{2}{\pi}; \phi = 0$$
$$\Rightarrow x_n = \cos\left[2\left(\frac{n}{f_s}\right)^2\right] = \cos(2t_n^{\ 2});\ t_n = \left(\frac{n}{f_s}\right); n = 0,1,2,\ldots \quad (5)$$

7. *Reviewer comment: p.8, lin. 4, Eq. (5). See annotated ms.*

Eq. (5). Corrected (now is Eq. (7)):

$$H^{(1)}(\omega) = j\omega = \omega\, e^{j\pi/2}$$
$$\left|H^{(1)}(\omega)\right| = \omega \quad (7)$$

8. *Reviewer comment: p.8, lins. 8 and 17. Clearly expose that the goal is to implement a low-pass-limited differentiator with the"j*omega" slope reset to zero between* ω_c *and* $\omega=\pi$ *,* ω_c *the low-pass cut-off frequency.*

We have further clarified this concept:

"(…) in Fig. 2, the portion of the differentiated signal we want to preserve is the low frequency one (for example the part corresponding to the 0÷1 interval of the time axis), but the high frequencies (the noisy portion) are strongly amplified respect those originally included in Eq. (5) and the higher are the frequency the higher is the amplification, as described by Eq. (7). Therefore a proper tradeoff has to be considered between perform the ideal derivative procedure for the whole signal and the necessary cut of high frequencies. For this reason, the goal is to design a band-limited differentiator that for frequencies greater than a certain cut-off value (i.e. for $\omega>\omega_c$) will ideally remove the high frequency component in Eq. (7). In other words a low-pass differentiator is wanted, i.e. one whose overall frequency response can be written as $H^{(1)L}$ and that can be thought as a cascade of a low-pass filter H^L and the ideal derivative $H^{(1)}$ (Luo et al., 2005; Zuo et al., 2013)"

9. *Reviewer comment: p.8 lins 8-12. Slang, magazine oriented (in my opinion).*

This part has been removed.

10. *Reviewer comment: p.9. Eq. (7). Revise/wrong.*

The suggested equation and our expression are equivalent: this can be seen if the different summation limits are considered. We keep [-N,N] interval to be consistent with the previous and subsequent equations. In the new math notation:

$$H^{(1)L}(\omega) = -j\sum_{k=-N}^{N} h^{(1)L}(k)\sin(\omega k)$$

11. *Reviewer comment: p.9, lins 18-20. Somehow slang and difficult to follow.*
    Suggestion: The SG-derivative filter can be understood as a cascaded system of two filters, the first one being a low-pass filter (LPF) acting on the raw input signal and the second one being the ideal differentiator (H(ω)=jω) and acting on the LPF signal. Prefix "d" stands for the low-pass filter prototype associated to this first stage of the SG differentiator (inherently, low pass).

    The suggestion is considered in rewriting the paragraph.

    "The SG low-pass derivative filter can be understood as a cascaded system of two filters, the first one being a low-pass filter acting on the raw input signal and the second one being the ideal differentiator ($j\omega$) and acting on the low-pass filtered signal. Prefix "d" stands for the low-pass filter prototype associated to this first stage of the SG differentiator (inherently low-pass), i.e. dSG2 for those in Fig. 4."

12. *Reviewer comment:* p.13 lin 3. *This is just a valid approximation. In fact there are hundred of alternative low-pass filter design approaches.*

    The authors agree that it is just one of the possibilities. For this reason, some comments have been inserted in the text (note: the cascade section is in Appendix now):

    "Operatively, for an efficient "equivalent" cascade of two smoothing filters (*L1*,*L2*) one possible solution is to get the frequency response of filter *L2* to have a ν_c large enough to cover the frequency response of *L1* up to the beginning of its stop-band."

13. *Reviewer comment: p.13, lins. 13-14. How does a SG2(L1+L2) compares with the cascaded SG2(L1)-SG4(L2)? What about SGx(y), i.e., a suitable pair of order "x" and length "y"?*

    See the answers reported in the "Major Comments" section, point *(4)*.

14. *Reviewer comment: p.14, lins 13-25. This part can be moved to a discussion sub-section in Sect. 3 or simply removed. Your paper is too long.*

    Paragraph removed.

15. *Reviewer comment: p.15. Unclear writing/structuration for Sect. 2.2.2*
    Please distinguish between time windowing and spectral windowing.

Windowing in the time domain means multiplying x(n) by a window w(n) as indicated by Eq. (11) + handwritten block diagram. According to FT properties, multiplication in the time domain equal convolution in the frequency domain. However in Fig. 7 frequency response it is not clear if the SGBlackman window is the result of multiplication in the time domain (correct) or the result of multiplication of a Blackman window in the frequency domain by the SG spectral response (incorrect). Please clarify.Most readers will not understand what is "leakage": Better introduce the problem by saying that "truncating" x(n) in the time domain is equivalent to multiplying by a rectangular window, w(n), which causes sinc=sin(x)/x (the FT of the rectangular window) to convolve in the frequency domain, i.e., many unwanted secondary lobes in the frequency response!

The reviewer is right, this section needs a revision. For the SG2+Blackman, the multiplication is done in the time domain. This section is in the Appendix, and it is rewritten as:

"The frequency response and the impulse response of an ideal low-pass filter are (Mitra, 2001):

$$H(\omega) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < \omega \le \pi \end{cases}$$
$$h(k) = \frac{\sin(\omega_c k)}{\pi k}, \quad -\infty \le k \le \infty \qquad \text{(C1)}$$

The previously mentioned necessity to deal with finite length impulse response, i.e. with FIR filter of Eq.(2), requires that the impulse response in Eq. (C1) must be truncated within $-N \le k \le N$. This operation can be seen like the multiplication of the impulse response by a rectangular *window* function $w(k)$:

$$w(k) = \begin{cases} 1, & |k| \le N \\ 0, & \text{elsewhere} \end{cases}$$
$$h_w(k) = h(k)w(k) \qquad \text{(C2)}$$

However, the FIR filter frequency response calculated with Eq. (3) and Eq. (C2), however is not identical to the ideal one. In particular it will exhibit undesired oscillations in both pass-band and stop-band (i.e. the side-lobes) which is the so-called *Gibbs phenomenon*. Moreover the transition-band will be of finite extent (Mitra, 2001). The main reason of the insurgence of the Gibbs phenomenon with the rectangular window consist in its sudden switch to zero at both ends. In order to reduce the Gibbs phenomenon more efficient window functions which taper-off more smoothly to zero at each end, are generally applied. However the more efficient the Gibbs phenomenon elimination is the wider the transition-band results.
For this reason, each window function is a kind of the usual compromise between the requirements of higher spectral selectivity, i.e. the narrowest the transition-band and the highest suppression of unwanted spectral features (Mitra, 2001). Several window functions are reported in literature (Harris, 1978). Eisele has introduced an efficient window function of the Blackman–type to lidar work:

$$w^{BL}(k)=\begin{cases}0.42+0.5\cos(\pi\frac{k}{N})+0.08\cos(2\pi\frac{k}{N}), & -N\le k\le N\\ 0, \quad \text{elsewhere}\end{cases} \tag{C3}$$

The filter constructed with this window (Eisele, 1998; Trickl, 2010) does not exhibit significant side-lobes.

As sn example of the application of the window method, a Blackman–type window of Eq. (C3) has been applied to an SG2 low-pass filter impulse response. The result, reported in Fig.7, shows a frequency response with very small side-lobes (maximum magnitude ~0.001); see also Fig.8 (lower panel). It can be also seen that using the same *N*, the pass-band extension of the windowed SG2 is nearly the same of the SG2.

As can be seen from the right plot in Fig. 7, the transition band in the SG2 filter with the application of the Blackman–type window, is quite larger (of about a factor 2) than the SG2 filter one (see the left plot in Fig. 7). This illustrates the previously mentioned compromise between better spectral selectivity and side-lobe suppression. From the comparison between the SG2 filter and SG2-windowed filter in Fig.7, for the latter it can be observed a less efficient damping in the stop-band of those frequencies over the pass-band and before the first side lobe of the (not windowed) SG2 (i.e. for $0.2<\nu<0.3$) and also a less flat behavior in pass-band i.e. worst signal preservation for $\nu<0.2$."

16. Reviewer comment: p.15, Eq. (12). Eq. (12) Revise/wrong.

The equation should be correct, if the limits are [-N,N]. This could be checked with the Matlab function *window(@blackman,2*N+1)*, see Trickl, 2010. The reviewer correctly noted that information on the limits is needed, and these have been added :

$$w^{BL}(k)=\begin{cases}0.42+0.5\cos(\pi\frac{k}{N})+0.08\cos(2\pi\frac{k}{N}), & -N\le k\le N\\ 0, \quad \text{elsewhere}\end{cases}$$

Other Comments found in the Annotated Manuscript. Note: unless otherwise stated, page/line numbers refer to the Annotated Manuscript from Referee #2.

The other relevant points of the reviewer #2 and reported as hand-written notes on the manuscripts are considered below.

Introduction

1. *Reviewer comments p.2, lins 9-11: The reviewer said that the statement "Indeed, the smoothing is a low pass filter, while the numerical derivative has a low pass filter inherently associated (see Sect. 2.1)." is false.*

   The text modified as it follows:

   "The smoothing is a low-pass filter, instead the ideal numerical derivative enhances high frequencies (Mollova, 1999). This means that in order to effectively perform a numerical derivative of a signal, a smoothing filter has to be coupled in cascade with the ideal derivative (see, Sect. 2.1)."

Sect.2.2 (now in Appendix)

We accomplish to the reviewer request to organize this section (now Appendix A1) in bullets.

2. *Reviewer comments p.11, lin 3: The reviewer corrects "P+1" with "P".*
   The authors think that "P+1" is correct. In Bromba and Ziegler (1981): "[...] a Savitzky-Golay smoothing filter of degree 2M (=P) exactly conserves every existing moment (from the zeroth) up to m = 2M + 1 (=P+1)." (in parenthesis authors' notes.)

Sect. 2.3 (now in Appendix)

3. *Reviewer comments on Sect 2.3: Correction on Eq.(13).*

The Eq.(13) has been corrected as suggested by the reviewer.

4. *Reviewer comment, p.16 lin.9. Why low-pass?*

Here, low-pass first derivative indicates that also the numerical differentiation filter originating from a Gaussian filter has a low pass filter embedded:

"This filter is characterized by a single parameter (σ, the standard deviation), and its impulse response (a zero mean Gaussian) has the advantage that can be written analytically as:

$$\begin{aligned} h_\sigma(k) &= (2\pi\sigma^2)^{-1/2} e^{-\frac{k^2}{2\sigma^2}}; \qquad -\infty \le k \le \infty \\ h_\sigma^{(1)L}(k) &= \frac{dh_\sigma(k)}{dk} = -\frac{k}{\sigma^2} h_\sigma(k); \qquad -\infty \le k \le \infty \end{aligned} \tag{D1}$$

In Eq. (D1) $h_\sigma(k)$ is the impulse response of the Gaussian smoothing filter and $h_\sigma^{(1)L}(k)$ is the one for the Gaussian first derivative filter. The convolution of a signal with a Gaussian first derivative is equivalent to differentiating that signal before (or after) the low-pass filtering operation. This means that, as seen for SG derivative filters, also the Gaussian derivative filter has a low-pass filter inherently embedded. Moreover, the frequency response of the low-pass embedded in the Gaussian low-pass first derivative filter (dG, in analogy with the denomination adopted for SG filters) is a Gaussian low-pass filter with the same parameter σ (Hale, 2011).

5. *Reviewer comment, p.17 lin.1-8. Simplify your reasoning!*

The authors have followed the reviewer suggestion and all the section is revised (see also the previous answer):

"To be used in FIR filters the impulse responses in Eq. (D1) must be truncated. Luckily, the Gaussian curve has a quick approach to zero and for this reason it can be truncated without a strong approximation. In fact, Eq. (D1) provides a value less than 0.0004 for $|k|\geq 4\sigma$. This latter condition implies that, to proper truncate the impulse response, it is sufficient to employ a value of N equal to 4σ (actually the nearest integer to 4σ) in Eq. (2) with no needs to go beyond this value. When σ increases, the pass-band reduces its extension and provides a stronger smoothing effect, although a Gaussian filter has a transition band quite wider than a SG filter with a similar pass-band. A Gaussian filter is also less flat in the pass-band (van Vliet et al., 1998) than a SG filter (for $P\geq 2$), but it has also the advantage of being almost without side lobes.

6. *Reviewer comment, p.17 lin.15. In fact is similar to the Blackman window not to the SG2+Blackman*:

In Fig.8 the function H is related to the use of a SG2 combined with a Blackman window not to the use of a Blackman window only.

Sect. 3

7. *Reviewer comments on Sect. 3: Paragraph organization. Title Correction. Too discursive/ambiguous: key is to begin with resolution<-> fcut-off*.

This section is reorganized according to the reviewer's comment. The authors will keep the title as is, because in the section the two main procedures to estimate quantitatively the Effective Resolution (ERes) are described also by giving the rationale of these approaches.

The cutoff frequency can be also related to the reduction of the vertical resolution: in the cited papers, the cutoff frequency is the main parameter needed to retrieve the effective vertical resolution. See also answer 16. below.

In this manuscript however, we choose to give more emphasis to the Rayleigh criterion and to the NRR criterion and to the link between the cut-off frequency and the ERes estimated with the previous criterions.

8. *Reviewer comment, p.18 lin.8. Which type of filter?*:

The used filter, i.e. the low pass embedded in the 1^{st} order differentiation filter based on the SG2 (or dSG2 according to our definition), is included.

9. *Reviewer comment. p.18, lin8: Define notation(about the aerosol backscatter)*

Defined in the Introduction.

Sect. 3.2

10. *Reviewer comment, p.21 lin 14. Which parameter? Whose parameters? Plural? With 2 test parameters changed*:

All the parameters are changed, (N,P) for SG filters and sigma for the Gaussian filter. Operatively for SG, P is fixed and N is changed for any pulse couple, then the procedure is repeated for a different P:

"Operatively, two unitary pulses at a fixed distance are smoothed by a low-pass filter whose parameters are changed to achieve an increasing signal distortion. Increasing N for SG filters with fixed P, or σ for Gaussian filters, it is possible to find the maximum value of the filter parameter that allows to still resolve the two smoothed pulses according to the Rayleigh criterion. For SG filters, afterwards P will be changed and the whole procedure is repeated to have results of the application of the criterion also for different polynomial degrees."

11. *Reviewer comment, p.24 lins 1-11. Unclear (about how the linear fits are constraint)*

An example is included in which, to clarify this point, is explicitly stated how the constraints are used to find the linear fit parameters:

"As an example on how the constraints are applied, for the SG4, the condition $N=P/2$ implies that with N=2 the filter will not operate any smoothing and $\Delta R_{eff}=\Delta R_{raw}$(=1 in our plots), then to obtain the fit parameters, the linear fit have been constrained through to the point (N=2, ΔR_{eff} =1)."

Sect. 3.3

12. *Reviewer comments p.24, lin. 22. Noise Variance:*

We use the term "noise variance" and we comment that this refers to zero-mean white noise.

13. *Reviewer comments on Eq.(23) Say clearly that is just your "Departing Hypothesis":*

Done:

"From the noise reduction point of view, the Eq. (22) makes possible a more general hypothesis on how to estimate the effective resolution by means of the NRR. In fact it can be inferred that the ERes associated to the application of a generic low-pass filter *L* on a signal could be written by means of the general equation:

$$\Delta R_{eff}^{L}\Big|_{NRR} = \frac{\Delta R_{raw}}{NRR^{L}} \,. \qquad (23)$$

Adopting a slight different point of view, a proof or at least a solid hint of the validity of the hypothesis represented by the Eq. (23) could be provided."

14. *Reviewer comments p.26. "TIP":*

We think that the SNR matching criterion included as proposed, along with the Fig. 16/17, well supports our hypothesis for the NRR criterion.

15. *Reviewer comments p.27: Reverse Order*:

We reorganized this part of the manuscript according to the reviewer's suggestions.

Sect. 3.4

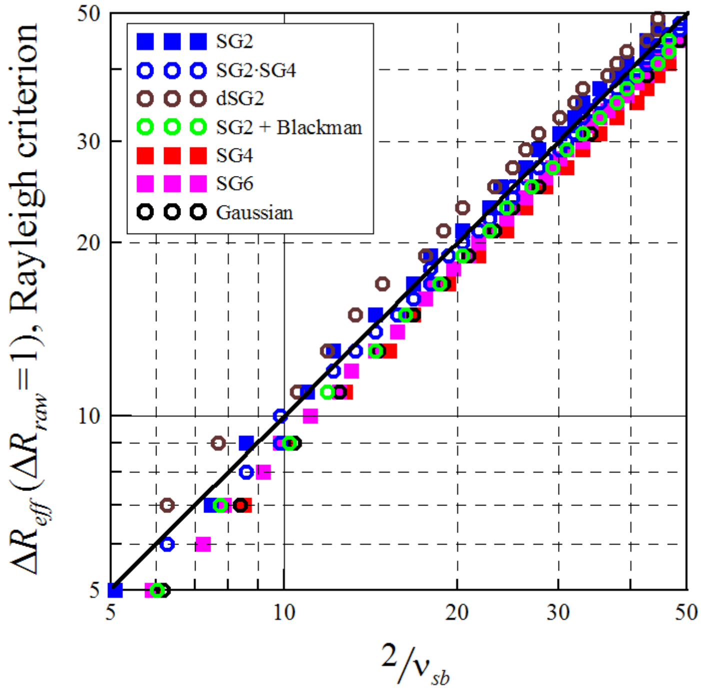
16. *Reviewer comment p.28 Eq(27).*

The main point in this Section is not to assume a particular cut-off definition but to find the definition that is more compatible with the ERes estimations found in the previous Sections. This is done because the results of Fig.18 suggest that a different criterion for the ERes requires a different cutoff definition in Eq.(27).

17. *Reviewer comment p.29 lin 8. I would suggest to repeat Fig.20 using the 1-st zero criterion, instead.*

This comment is not completely clear. Anyway Fig.20 (see below) is redone and the corresponding sentence in the main text is rewritten. In this way the data reported in Fig.20 have a clear connection with Eq.(29):

"For the Rayleigh approach, the ERes calculated via Eq. (27) are similar to those estimated using Eq. (19) and Eq. (20), if the cut-off frequency is $\nu_c\big|_{Ray} = \nu_{sb}/2$ (Orfanidis, 2009); this is shown in Fig. 20."



Sect. 3.5 (now in Appendix)

18. *Reviewer comment on Sect. 3.5. I would delete or reduce this section to a minimum(…).*

This section provides a more comprehensive review of the existing methods used throughout the remote sensing community. The authors agree with the reviewer that this Section is more speculative than the previous ones and that a deeper investigation is required. For this reason, this section is moved in Appendix.

Conclusions

19. *Reviewer comments on Conclusions*:

See response for the "Major Comments", point (5). All the remaining minor comments/correction and suggestions reported by the reviewer are considered in the new version of the manuscript.