Response to Referee#1's comments

Dear Referee#1,

Thank you for spending your time reviewing this manuscript and providing detailed and insightful comments. Your comments have been addressed and the manuscript will be revised accordingly at the end of the interactive discussion. We are not permitted to conduct manuscript revision until the interactive discussion finishes.

A point-by-point response to your comments are given in the following. Your comments are given in bold and followed by our responses which are in normal text and indented.

2 Specific Comments

1. P 10438 L 13: How do you know the magnitude of A at this point? Please elaborate.

> The magnitude of **A** here is an approximation by its order of magnitude. Based on Eq. (6), it can be shown that the radial velocity variance (i.e. the diagonal term of **A**) is approximately $\cos^2\phi\,\sigma_u^2$ where ϕ is the elevation angle and σ_u is the standard deviation of wind speed. It should be a good assumption that $TI = \sigma_u/V_0$ is about 10% and wind speed V_0 > 4 m s^{-1}. Then the radial velocity variance > $0.16\cos^2\phi$. Because the elevation angle is commonly kept low, say, $\phi < 20°$, the radial velocity variance > 0.14 m^2 s^{-2}.
>
> The following explanation will be added to the manuscript for clarification: "> 0.1 m^2 s^{-2} given that the turbulence intensity is about 10% and wind speed > 4 m s^{-1}".

2. Further model investigation: In Sect. 4 you investigate hypothetical output of the model. You observed e.g. an approx.. linear dependence (for bigger TI) on TI.

- **Could you also plot of the error versus the mean velocity?**

> The relationship between the error and the mean velocity is implicitly included in Figure 4 in which the lines with different symbols represent different wind velocity. The error increases with increasing wind speed mainly for the following

reason: the turbulence intensity here is invariant with wind speed based on the definition $TI = c_n\kappa / \ln(z/z_0)$. Increasing wind speed will increase the variance of wind speed and consequently increase the error in the estimated wind speed. The error dependence on wind speed is also the result of the sample locations determined by the wind speed and direction.

The following sentence will be added to the manuscript to state the dependence of the error on the wind velocity: 'The error increases with increasing wind speed through its relationship with the variance'.

- **Can you comment on the robustness of the pure model results? For example, how strongly do the results change with another lidar window $w(r-s)$? How sensitive are they to small changes of the integral length scale which can only be estimated approximately.**

  In the model, the integral length scale is calculated as a function of the turbulence intensity (TI), the height and the Coriolis force (see Appendix A). Errors can be calculated without information of the integral length scale. The sensitivity of the estimated error to the integral length scale has been implicitly demonstrated by the error-TI relationship.

  The lidar window acts as a low pass filter and removes variation associated with turbulent eddies of scales that are smaller than $\Delta R/2$ where ΔR is the size of the lidar window. When $\Delta R/2$ is much smaller than the turbulence integral length scale, the error derived from the model should not be sensitive to the type of lidar window used. However, the sensitivity will increase when the turbulence length scale approaches $\Delta R/2$ (30 m for the lidar used in the manuscript).

3. Is these definitely no way to estimate the standard deviation σ_V directly from the data without the comparison to cup measurements? For example, the inverse method you use to obtain the lidar estimate can be seen as a linear regression also yielding estimated uncertainties for the estimated regression coefficients u_0 and v_0. Maybe you can use that or even estimate σ_V more directly? Can you comment on that?

It is possible to estimate the standard deviation σ_V from the data. As shown in Wang et al. (2015), the estimated radial velocity variance can be used to approximate the diagonal terms of the matrix **A** and the uncertainty can be quantified using the weighted least squares. We have added a paragraph at the end of Sect. 3 to state that it is possible to estimate the standard deviation from the data.

The observational data have been already processed and were provided in the format of 10-minute mean wind speed and direction. Raw data were not available; therefore, we cannot apply the weighted least squares method to estimate the uncertainty from the data.

4. In Sect. 4 you estimate the standard deviation of the difference between the lidar estimate and the cup estimate of the ten minute mean. For the cup you use an error which is independent of turbulence intensity and is probably just an estimation of the pure error? However, I am not completely sure if this is the way to go here. For example, if you had two perfect point measurement devices with a distance d there would still be a non-zero standard deviation for their difference depending on the turbulence intensity and the spatial correlation between the points. This is probably also relevant in the lidar and cup scenario which are not exactly at the same point. Can you comment on that? Do you think this effect is negligible?

The cup error (in measuring the 10-minute mean) used in the manuscript is derived from "simulations of systematic deviations of the cup anemometers under the given ranges of operational conditions for a given class" (Friis Pedersen et al., 2006). One of the operational conditions is the turbulence intensity which is defined as $TI = 0.12 + 0.48/V$ for Class A and $TI = 0.12 + 0.96/V$ for Class B where V is the mean wind speed.

We agree with the referee that, in addition to the measurement errors of the lidar and the cup anemometer, there are other factors causing the difference between the measurements from the two instruments. The separation distance is definitely a factor. Measurements from two locations can be considered as two realizations of the underlying stochastic process. The instantaneous values of the two realization are different, but they should have the same statistics (mean and variance) and are correlated. The magnitude of the correlation depends on the separation distance and the spatial correlation of the turbulence. The averages of the two realizations are the unbiased estimates of the mean of the stochastic process and should converge to the same value as the averaging time increases. Because of the turbulent fluctuation, the random error will inevitably exist for the mean estimate (10 minutes) and contribute to the difference between the two instruments. The random error for the lidar has been addressed in the manuscript in terms of the relative standard error (RSE). The random error for the cup has been included in the cup error defined in Eq. (20) because the cup anemometer class number in Eq. (20) is derived from a turbulent wind field simulated with a wind-speed-dependent turbulence intensity and a turbulence spectrum model.

In conclusion, for the inter-comparison between the lidar and the cup, the separation distance between the two instruments matters when the measurements are instantaneous, but the effect should be negligible for the comparison of mean wind speed (with a long averaging period). The turbulent fluctuation is the dominant source of error for both instruments, and its effect on error has been considered for the lidar (through the isotropic model) and the cup (through the error equation in Eq. (20)) in the manuscript. The following sentence has been added to state that the cup error has already taken into account the turbulence intensity: 'Note the expression in Eq. (20) is derived from the simulated wind field with its turbulence intensity defined as a function of wind speed; therefore, errors from Eq. (20) has already taken into account the effect of turbulence intensity on the cup anemometer error.'

5. The rescaling procedure in Eq. 21 is obviously not exact. Please remark this clearly. As far as I can see the effect of $\boldsymbol{\beta}$ is not included to investigate the dependence on $\boldsymbol{\Delta\theta}$. Please comment on that.

As the referee suggested, it will be more appropriate to rescale by binning the data by the relative direction β. However, there is no overlapping β at the three sites with a sufficient sample number for the rescaling. The high error at site B (6.3%) might be related to the large extent of β associated with the data.

We will comment on the manuscript that the rescaling is not exact and the effect of β is not considered in rescaling: 'Note the effect of β has not been removed in the rescaling because an overlapping bin of β with a sufficient sample number cannot be found at the three sites. Therefore, the rescaling may change if beta is considered.'

3. Minor Comments

1. P10432 L25: please check English in "propagates through into uncertainty"

This will be changed to "is propagated to the uncerttainty".

2. The word "uncertainty" is used a bit too much in my opinion, and sometimes it is not a 100% clear to me if you just mean the standard deviation. Please check if you define uncertainty clearly or use other terms like standard deviation, where possible.

The uncertainty will be defined, when it appears the first time in the manuscript, as the standard error of the estimated wind velocity.

3. In Eq. (2): I do not really like $\langle v_R \rangle$ as true and v_R as measured radial velocity. The brackets are often used to indicate averaging but here $v_R \neq \langle v_R \rangle$ since generally $\langle \delta \rangle \neq 0$.

We will replace $\langle v_R \rangle$ with v_{R0} as the notation for the true radial velocity.

4. Can you give a citation for Eq. (3)?

A citation has been added for Eq. (3).

5. Please check if σ_V should not be σ_{V_0} throughout the paper (e.g. Eq. (11))?

The standard error of the estimated wind speed is denoted as $\sigma_{\hat{V}}$ and this is consistent throughout the manuscript.

6. Could you simply reread the paragraph starting at P10433 L23. I find it bit difficult to follow. May you can rewrite it slightly.

We will rewrite this paragraph in the revised manuscript as follows:

The uncertainty in v_R scales with the turbulence intensity because turbulent fluctuations of both wind speed and backscattering particle locations in the sensing volume can broaden the signal spectrum and thus increase the uncertainty in v_R (Banakh et al., 1995; Frehlich, 1997). When the turbulence is sufficiently strong (i.e. $\sigma_{v_r}^2 > 0.5 w_R$ where w_R is the spectrum width of the lidar signal in velocity space and it is equal 0.877 m s-1 for the Galion lidar), the random error variance σ_e^2 becomes proportional to $\sigma_{v_r}^2$. If σ_{v_r} is extremely large, the spectrum width will be so wide such that the peak is indistinguishable from the noise and the radial velocity selected by the estimator can be any value within the velocity search space (±39 m s-1 for the Galion lidar).

7. I am sometimes confused by the usage of the hat notation. Please check where the hat should be and where not. In stochastics a hat is sometimes used to distinguish between stochastic variables and their estimates. This is not always case here (e.g. $\mathbf{C}$ and $\hat{\mathbf{C}}$). I suggest to change that. In Eq. (10) a hat on the left hand side of the Eq. would make sense in my opinion.

Our intention was to use the hat notation to denote estimates. Obviously $\hat{\mathbf{C}}$ is not an estimate of the covariance matrix. We have changed this notation to $\mathbf{C}_e$ where the subscript e denotes errors (see new Eq. 9 and Eq. 12).

The notation in Eq. (10) was a typographic error. The left hand side of the equation should be $\hat{V}_0$. We have corrected this.

8. I am not content with the notation for AEP estimation. In Eq. (11) for example it should be clear that σ_V depends on speed and direction by you just used V_i so far. Or did I misunderstand?

The uncertainty in AEP is defined in Eq. (25). The value of $\sigma_{\hat{V}}$ should be a function of wind speed and direction. We will change the notation $\sigma_{\hat{V}_i}$ to $\sigma_{\hat{V}_{i,j}}$ to denote it is a function of wind speed and direction.

9. P10434 L17: I would not use "Thus" since you have not shown the other results yet. Maybe you can write: "As shown in the next section", in the beginning of the sentence.

We have changed the text as suggested.

10. P10435 L13: $\boldsymbol{r_i}$ and $\boldsymbol{r_j}$ should be vectors here

The notations have been corrected.

11. P10437 L14: than not "that"

This has been corrected.

12. P10443 L24 following: I do not completely agree. Without cup error it seems to be partially much better $\boldsymbol{\beta} < -20°$.

The sentences will be added to the manuscript to address the reviewer's comment:

'When $\beta \leq -30°$, the predictions and observations are the almost the same. Adding the cup anemometer standard error causes overestimation of the relative error. Overall, in terms of the relationship between the relative error and the relative wind direction, the prediction is consistent with the observation, indicating that turbulent wind fluctuations are the main source of uncertainty and the assumptions made in applying the isotropic turbulence model are largely realized.'

References

Friis Pedersen, T., Dahlberg, J. Å., and Busche, P.: ACCUWIND - Classification of five cup anemometers according to IEC 61400-12-1, Denmark. Forskningscenter Risoe.Risoe-R; No. 1556(EN), 2006.

Wang, H., Barthelmie, R. J., Clifton, A., and Pryor, S. C.: Wind Measurements from Arc Scans With Doppler Wind Lidar, Journal of Atmospheric and Oceanic Technology, 10.1175/JTECH-D-14-00059.1, 2015.