Reponse to Zeng et al's response to Comment by I D Culverwell

We thank Zeng et al for their detailed reponse to our comments on their paper "Ionospheric correction of GPS radio occultation data in the troposphere".

The accuracy of the simplest, 'delta function', approximation to the bending angle remains in question.

Here are the data that were plotted in Fig C1, as well as the fractional difference between the numerically calculated bending angle ('Z(l)', shown in green on Fig C1) and the analytically estimated bending angle obtained by assuming an infinitely thin ionosphere ('delta(l)', shown in red on Fig C1).

l=(r0-r)/H	Z(l)	delta(l)	delta(l)/Z(l)-1
0.0000000	-0.91588359	Infinity	-Infinity
0.10000000	-0.84389117	79.266546	-94.929821
0.20000000	-0.75919754	28.024956	-37.913919
0.30000000	-0.66169702	15.254854	-24.054137
0.40000000	-0.55164468	9.9083182	-18.961414
0.50000000	-0.42973138	7.0898154	-17.498249
0.60000000	-0.29715382	5.3934053	-19.150214
0.70000000	-0.15567202	4.2799912	-28.493645
0.80000000	-0.0076447467	3.5031195	-459.23879
0.90000000	0.14396798	2.9357980	19.392021
1.0000000	0.29564676	2.5066283	7.4784567
1.1000000	0.44340959	2.1727057	3.8999971
1.2000000	0.58297096	1.9068567	2.2709292
1.3000000	0.70996550	1.6911219	1.3819775
1.4000000	0.82022973	1.5132054	0.84485558
1.5000000	0.91012480	1.3644356	0.49917418
1.6000000	0.97687119	1.2385398	0.26786397
1.7000000	1.0188546	1.1308801	0.10995234
1.8000000	1.0358553	1.0379613	0.0020331162
1.9000000	1.0291522	0.95710488	-0.070006507
2.0000000	1.0014642	0.88622693	-0.11506880
2.1000000	0.95671196	0.82368469	-0.13904631
2.2000000	0.89961656	0.76816747	-0.14611679
2.3000000	0.83518598	0.71861815	-0.13957111
2.4000000	0.76817160	0.67417566	-0.12236320
2.5000000	0.70259097	0.63413237	-0.097437356
2.6000000	0.64140345	0.59790187	-0.067822485
2.7000000	0.58639025	0.56499459	-0.036487076
2.8000000	0.53824003	0.53499890	-0.0060217309
2.9000000	0.49678978	0.50756648	0.021692683
3.0000000	0.46133886	0.48240084	0.045654037
3.1000000	0.43095041	0.45924813	0.065663504
3.2000000	0.40467897	0.43788994	0.082067433
3.3000000	0.38170123	0.41813741	0.095457338
3.4000000	0.36136176	0.39982648	0.10644380
3.5000000	0.34316410	0.38281405	0.11554224
3.6000000	0.32673811	0.36697475	0.12314647
3.7000000	0.31180515	0.35219838	0.12954639
3.8000000	0.29815035	0.33838768	0.13495650
3.9000000	0.28560349	0.32545656	0.13953984
4.0000000	0.27402652	0.31332853	0.14342413
4.1000000	0.26330543	0.30193548	0.14671196
4.2000000	0.25334470	0.29121651	0.14948727
4.3000000	0.24406345	0.28111709	0.15181970
4.4000000	0.23539248	0.27158821	0.15376760
4.5000000	0.22727214	0.26258576	0.15538031
4.6000000	0.21965066	0.25406988	0.15669984
4.7000000	0.21248280	0.24600455	0.15776218

4.8000000	0.20572884	0.23835709	0.15859834
4.9000000	0.19935369	0.23109780	0.15923515
5.0000000	0.19332624	0.22419965	0.15969592
5.1000000	0.18761878	0.21763797	0.16000101
5.2000000	0.18220654	0.21139023	0.16016823
5.3000000	0.17706728	0.20543580	0.16021322
5.4000000	0.17218101	0.19975575	0.16014975
5.5000000	0.16752964	0.19433271	0.15998999
5.6000000	0.16309682	0.18915067	0.15974470
5.7000000	0.15886769	0.18419492	0.15942345
5.8000000	0.15482871	0.17945185	0.15903471
5.9000000	0.15096755	0.17490890	0.15858605
6.0000000	0.14727292	0.17055445	0.15808423
6.1000000	0.14373448	0.16637773	0.15753526
6.2000000	0.14034271	0.16236873	0.15694454
6.3000000	0.13708888	0.15851819	0.15631691
6.4000000	0.13396493	0.15481747	0.15565671
6.5000000	0.13096342	0.15125854	0.15496784
6.6000000	0.12807746	0.14783390	0.15425380
6.7000000	0.12530069	0.14453657	0.15351775
6.8000000	0.12262717	0.14136001	0.15276253
6.9000000	0.12005142	0.13829813	0.15199072
7.0000000	0.11756833	0.13534520	0.15120461
7.1000000	0.11517313	0.13249589	0.15040631
7.2000000	0.11286137	0.12974517	0.14959770
7.3000000	0.11062890	0.12708832	0.14878049
7.4000000	0.10847185	0.12452093	0.14795621
7.5000000	0.10638657	0.12203883	0.14712628
7.6000000	0.10436967	0.11963811	0.14629194
7.7000000	0.10241795	0.11731508	0.14545434
7.8000000	0.10052840	0.11506627	0.14461451
7.9000000	0.098698217	0.11288839	0.14377339
8.0000000	0.096924736	0.11077837	0.14293182
8.1000000	0.095205461	0.10873326	0.14209057
8.2000000	0.093538035	0.10675031	0.14125033
8.3000000	0.091920235	0.10482691	0.14041171
8.4000000	0.090349964	0.10296059	0.13957529
8.5000000	0.088825237	0.10114899	0.13874157
8.6000000	0.087344178	0.099389901	0.13791100
8.7000000	0.085905013	0.097681216	0.13708400
8.8000000	0.084506060	0.096020934	0.13626093
8.9000000	0.083145724	0.094407158	0.13544213
9.0000000	0.081822494	0.092838084	0.13462790
9.1000000	0.080534935	0.091311998	0.13381849
9.2000000	0.079281683	0.089827268	0.13301414
9.3000000	0.078061444	0.088382343	0.13221507
9.4000000	0.076872985	0.086975744	0.13142145
9.5000000	0.075715135	0.085606063	0.13063344
9.6000000	0.074586776	0.084271958	0.12985120
9.7000000	0.073486845	0.082972148	0.12907484
9.8000000	0.072414329	0.081705411	0.12830447
9.9000000	0.071368261	0.080470582	0.12754018
10.000000	0.070347718	0.079266546	0.12678204
10.100000	0.069351820	0.078092239	0.12603013
10.200000	0.068379723	0.076946643	0.12528450
10.300000	0.067430625	0.075828785	0.12454518
10.400000	0.066503757	0.074737734	0.12381221
10.500000	0.065598381	0.073672598	0.12308562
10.600000	0.064713794	0.072632524	0.12236541
10.700000	0.063849322	0.071616694	0.12165159
10.800000	0.063004319	0.070624324	0.12094416
10.900000	0.062178165	0.069654662	0.12024313

11.000000	0.061370266	0.068706987	0.11954847
11.100000	0.060580053	0.067780609	0.11886017
11.200000	0.059806980	0.066874862	0.11817821
11.300000	0.059050522	0.065989110	0.11750256
11.400000	0.058310174	0.065122739	0.11683320
11.500000	0.057585454	0.064275161	0.11617009
11.600000	0.056875894	0.063445811	0.11551319
11.700000	0.056181049	0.062634143	0.11486248
11.800000	0.055500487	0.061839636	0.11421789
11.900000	0.054833795	0.061061784	0.11357940
12.000000	0.054180574	0.060300105	0.11294696
12.100000	0.053540440	0.059554129	0.11232051
12.200000	0.052913023	0.058823409	0.11170001
12.300000	0.052297968	0.058107510	0.11108542
12.400000	0.051694932	0.057406016	0.11047667
12.500000	0.051103582	0.056718523	0.10987372
12.600000	0.050523601	0.056044644	0.10927651
12.700000	0.049954680	0.055384004	0.10868499
12.800000	0.049396523	0.054736242	0.10809910
12.900000	0.048848841	0.054101009	0.10751879
13.000000	0.048311359	0.053477969	0.10694400
13.100000	0.047783809	0.052866796	0.10637467
13.200000	0.047265932	0.052267176	0.10581076
13.300000	0.046757479	0.051678806	0.10525219
13.400000	0.046258208	0.051101393	0.10469892
13.500000	0.045767886	0.050534652	0.10415088
13.600000	0.045286288	0.049978311	0.10360803
13.700000	0.044813193	0.049432103	0.10307030
13.800000	0.044348392	0.048895772	0.10253764
13.900000	0.043891680	0.048369070	0.10200998
14.000000	0.043442859	0.047851756	0.10148728
14.100000	0.043001736	0.047343599	0.10096948
14.200000	0.042568126	0.046844372	0.10045652
14.300000	0.042141849	0.046353857	0.099948343
14.400000	0.041722731	0.045871844	0.099444899
14.500000	0.041310602	0.045398127	0.098946132
14.600000	0.040905299	0.044932507	0.098451986
14.700000	0.040506662	0.044474793	0.097962409
14.800000	0.040114538	0.044024797	0.097477346
14.900000	0.039728777	0.043582339	0.096996745
15.000000	0.039349233	0.043147243	0.096520553
15.100000	0.038975766	0.042719338	0.096048718
15.200000	0.038608238	0.042298460	0.095581189
15.300000	0.038246518	0.041884447	0.095117914
15.400000	0.037890476	0.041477145	0.094658843
15.500000	0.037539987	0.041076401	0.094203927
15.600000	0.037194929	0.040682069	0.093753115
15.700000	0.036855184	0.040294007	0.093306358
15.800000	0.036520637	0.039912075	0.092863610
15.900000	0.036191175	0.039536138	0.092424820
16.000000	0.035866692	0.039166067	0.091989944
16.100000	0.035547080	0.038801733	0.091558933
16.200000	0.035232237	0.038443012	0.091131742
16.300000	0.034922064	0.038089786	0.090708325
16.400000	0.034616462	0.037741935	0.090288637
16.500000	0.034315337	0.037399347	0.089872634
16.600000	0.034018597	0.037061910	0.089460272
16.700000	0.033726153	0.036729518	0.089051507
16.800000	0.033437917	0.036402064	0.088646296
16.900000	0.033153803	0.036079447	0.088244599
17.000000	0.032873730	0.035761568	0.087846371
17.100000	0.032597616	0.035448329	0.087451574

17.200000	0.032325383	0.035139636	0.087060165
17.300000	0.032056954	0.034835398	0.086672105
17.400000	0.031792255	0.034535525	0.086287355
17.500000	0.031531213	0.034239929	0.085905874
17.600000	0.031273757	0.033948527	0.085527626
17.700000	0.031019817	0.033661234	0.085152571
17.800000	0.030769327	0.033377971	0.084780672
17.900000	0.030522220	0.033098658	0.084411892
18.000000	0.030278432	0.032823219	0.084046195
18.100000	0.030037902	0.032551580	0.083683545
18.200000	0.029800567	0.032283666	0.083323905
18.300000	0.029566368	0.032019408	0.082967242
18.400000	0.029335247	0.031758735	0.082613520
18.500000	0.029107148	0.031501580	0.082262706
18.600000	0.028882014	0.031247877	0.081914765
18.700000	0.028659791	0.030997561	0.081569665
18.800000	0.028440428	0.030750569	0.081227373
18.900000	0.028223872	0.030506840	0.080887856
19.000000	0.028010072	0.030266314	0.080551083
19.100000	0.027798980	0.030028932	0.080217022
19.200000	0.027590548	0.029794636	0.079885643
19.300000	0.027384728	0.029563372	0.079556915
19.400000	0.027181474	0.029335084	0.079230807
19.500000	0.026980742	0.029109719	0.078907290
19.600000	0.026782488	0.028887225	0.078586335
19.700000	0.026586668	0.028667551	0.078267913
19.800000	0.026393241	0.028450647	0.077951995
19.900000	0.026202166	0.028236465	0.077638554
20.000000	0.026013403	0.028024956	0.077327561

As is clear from the above , and Fig C1, the Z(l) and delta(l) curves cross each other at l = (r0-r)/H ~ 2.8 (and also l ~ 1.8, in fact). The fractional difference between the two is therefore very small near l = 2.8. But it grows steadily as l increases, reaching a maximum of about 16% at l ~ 5.3, before slowly decreasing. 10% accuracy is not met again until l ~ 14.3. This is why I said that "... one needs to be about 15 ionospheric scale heights below the peak height before the delta function solution is even within 10% of the true bending angle ...". I was concentrating on the large l behaviour, because you said "... at heights lower than the maximum by two height scales the response is well approximated by (z0-h)^{-3/2} ..." – by implication, at *all* such heights.

This does not, however, explain your Fig C2. For example, the solid green curve (r0 = 100 km, H = 7.5 km) has l = 2.7 at r = 80 km. According to our model, the red and green curves should be very close here, but they are far apart on your figure. Indeed, your two curves cross at about 70 km, which implies l = 4.0, where the difference between our curves is about 14%. Similar discrepancies appear in the other curves. There is therefore a difference in the results of your analysis, based on ray tracing, and our analysis, based on numerical evaluation of the Abel integral for the bending angle. This is not so surprising, and it's not hugely important, because you scale your delta function curves. I don't suggest that the difference needs resolving now[*]. But I would ask you to please amend the statement on p 7788 that C&H 2012 found that, for l > 2, the response is "well approximated" by the delta function limit. It's not really what we saw, and certainly not what we said. If you wanted to be precise, you could perhaps say that we showed that |Z(l)-delta(l)| < 10% if 2.5 < l < 3.3[**] or if l > 14.3. You could of course say that your ray-tracing experiments suggest otherwise. Thank you.

Ian Culverwell.

[*] If you wanted to, a good place to start would be calculate your equivalent of the table of numbers above.

[**] Fortunately, 2.5 < l < 3.3 turns out to cover much of the region likely to be of use in practice, as your Fig C2 suggests.

Response to Dr. Culverwell: (comments are in blue; replies are in black)

The authors are grateful for discussion and commenting on the paper.

"There is therefore a difference in the results of your analysis, based on ray tracing, and our analysis, based on numerical evaluation of the Abel integral for the bending angle."

We don't believe that this difference is related to the difference between our ray-tracing and numerical calculation of the integral in (Culverwell and Healy, 2012). Validation of both methods by inversion is simple and straightforward, and we are confident that errors are extremely unlikely.

"This is not so surprising, and it's not hugely important, because you scale your delta function curves."

Yes, we believe the difference is related to the way the true bending angle is modeled by delta-function response. We apply least squares fitting (by solving for the scaling coefficient) between the transition height (20 km in Figure C2) and 80 km. For our used models of F and E layers, we found that this fitting results in <5% fractional error between 0 and 20 km (the error may be greater for negative heights but we did not check).

The figure on slide 6 (Culverwell and Healy, 2012) does not seem to use fitting, as this can be understood from slide 5. However, fitting is discussed on slides 9 and 10 and the figure from slide 6 is repeated on slide 10. Thus we believe that including this reference in support of our modeling approach without a detailed discussion may be misleading and it was our mistake. Instead, in the revised paper, we explain our bending angle modeling (which includes fitting) in more details, with the figure and error estimate.

Again, we are grateful for pointing to this.