

Response to Reviewer 2

We thank the reviewer for the appraisal of the manuscript. A point-by-point response to the reviewer's comments is given below.

Reviewer #2:

Major comments

1. "'3D issue': Already the title of the manuscript leaves the false impression that 3D radiative transfer (RT) is of central importance to the method and that the consideration of it is a core innovation of the method. Both is not true. A 3D model was used to solve RT for simple 1D homogeneous cloud/ sky situations only. Something that could have been accomplished with every 1D RT model. The use of the 3D model might have been for convenience only, but reasons are not given. Neither a detailed quantitative analysis of 3D RT influence specific to the suggested method nor any way to compensate or correct for it is presented. This false impression should be completely removed or the content with respect to 3D RT should be considerably strengthened. "

Response: While we agree that the surface **irradiance** under our homogenous cloud domain could be derived from a 1D model, the 3D radiative transfer is needed since the measurement of **radiance** (incoming power per steradian) is a 3D measurement. As far as the authors know, no 1D model provides meaningful radiance as they are assumed to only be a function of ϑ_z , which as demonstrated in Section 4, leaves the variations in ϑ_s unaccounted for ground based imaging. For example the radiance of a two-stream model would be constant across a hemisphere. The reason for using the 3D model is described at the beginning of section 3:

"Surface irradiance under the homogenous cloud field is homogeneous and could be obtained from a 1D model. The 3D dependency of the sky radiance field requires a 3D-RT model to simulate sky images. SHDOM is an explicit 3D-RT model that uses discrete ordinates to integrate the radiative transfer equation spatially, while spherical harmonics are used to save memory when solving the source function. This method allows for better computational efficiency compared to other methods such as the Monte Carlo (MC) method when solving the whole sky radiance field. SHDOM is also found to be within 2-3% (close to the noise level) of the MC models in the Intercomparison of 3D Radiation Codes (I3RC) (Marshak et al., 2005; Cahalan et al., 2005). Because of its computational efficiency and accuracy, SHDOM is selected for this analysis."

2. "Discussion of results/ requirements: Accuracy requirements for the method to be developed should be derived in the introduction section. These are missing completely until very late in the discussion section. Errors found for the new method seem large. Especially for the regime of small OT most relevant for solar energy production. Discussion of this with respect to the needed accuracy is weak. "

Response: A discussion of accuracy requirement was added to the introduction.

"Differences in τ_c can greatly affect the irradiance available for solar energy production. For this analysis we consider the of accuracy requirement of global horizontal irradiance (GHI) to be $\pm 5\%$ Fig. 1a. Fig. 1b

and 1c demonstrate the corresponding absolute and relative error in τ_c for a 5% error in GHI. Relative τ_c accuracy required for solar forecasting is large for thin cloud ($\tau_c \sim 1$) and thick clouds ($\tau_c > 30$). A minimum occurs at $\tau_c = 16$ where a 21% error in τ_c is permissible for avoiding an under-prediction of GHI by 5%.

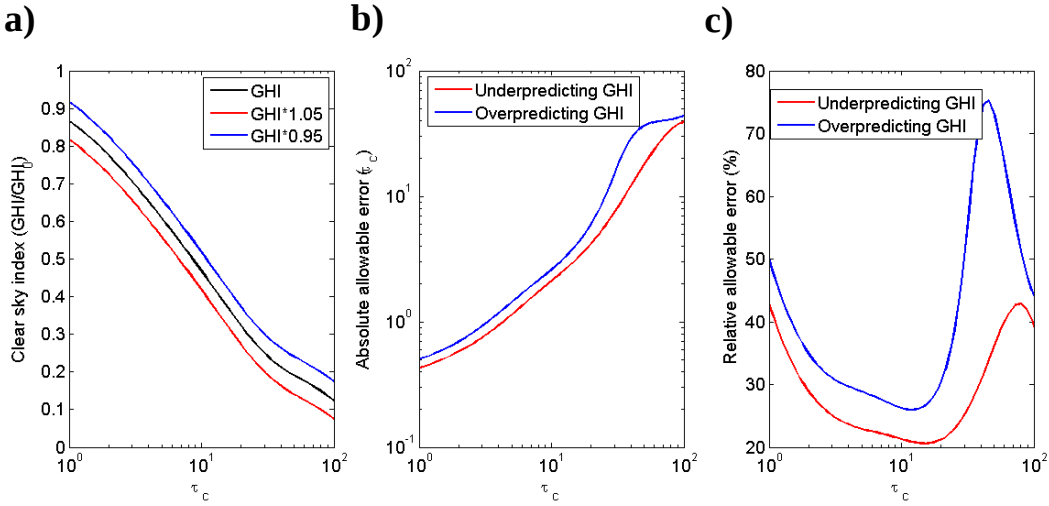


Fig. 1a) Irradiance divided by clear sky irradiance as a function of τ_c for homogenous clouds as derived from SHDOM. The black line represents the results while the blue and red are 5% offsets in GHI. 1b) Error bounds of $\pm 5\%$ on 1a converted to absolute intervals for τ_c . For example, for GHI to stay within 5% of its value at $\tau_c = 30$, τ_c cannot be more than 7.7 below 30 and not more than 15.6 above 30. 1c) Same as 1b but the y-axis is divided by τ_c .

3. “Calibration issue: The authors introduce the use of a radiance measurement as fundamental to the method. The needed absolute calibration of the proposed RGB cameras to provide such a radiance stays very unclear and seems to be insufficient. Related to the “discussion of requirements and errors” issue. “

Response: We are unsure of the intent of the reviewer. Was the description of methodology on how to calibrate RGB cameras insufficient or was the issue with the accuracy of the calibration method used for our camera? Additional analysis was added to section 3.2 to address the latter concern:

“Fig. 3 demonstrates the three signal calibrations with a relative root mean square error (RMSE) of 0.155, 0.148, and 0.144 for the red, green and blue channel respectively. This RMSE is within the range of the variability expected in overcast clouds Szczodrak et al. (2001).”

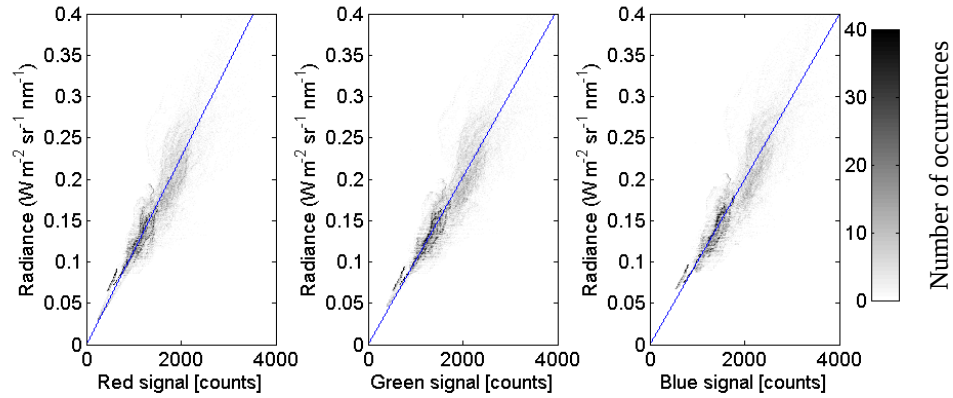


Fig. 3 SHDOM radiance ($I_{\Delta\lambda}(\vartheta_z, \vartheta_s)$) versus USI pixel signal value ($v(\vartheta_z, \vartheta_s)$). Dots with greyscale indicate density while the blue line is the best fit line.”

4. “Unprecise presentation: In some places throughout the manuscript acronyms are not introduced. Quantities are called “large” or “small” without orientation on values. Important information on requirements and assumptions is given too late.”

Response: The manuscript was corrected to address this concern.

Minor Comments

1. “11286, line 2: What is USI?”

Response: A ground-based sky imager. The sentence was revised for clarity:

“A method for retrieving cloud optical depth (τ_c) using a UCSD developed ground-based Sky Imager (USI) is presented.”

2. “11286, abstract and beginning of introduction: Already after the first mentioning of errors the reader wants to know which range of optical thickness values is of interest for you and which range of values were tested. What OT range and result accuracy is relevant for solar energy production? “

Response: The range of OT was added to the abstract as follows,

“ τ_c values ranged from 0-80 with values over 80 being capped and registered as 80.”

A discussion of accuracy requirement was added to the introduction.

“Differentiating τ_c becomes crucial since differences in τ_c can greatly affect the irradiance available for solar energy production. For this analysis we consider the of accuracy requirement of global horizontal irradiance (GHI) to be $\pm 5\%$ Fig. 1a. Fig. 1b and 1c demonstrate the corresponding absolute and relative error in τ_c for a 5% error in GHI. Relative τ_c accuracy required for solar forecasting is large for thin cloud ($\tau_c \sim 1$) and thick clouds ($\tau_c > 30$), while a minimum occurs at $\tau_c = 16$ with 21% error for under predicting GHI.

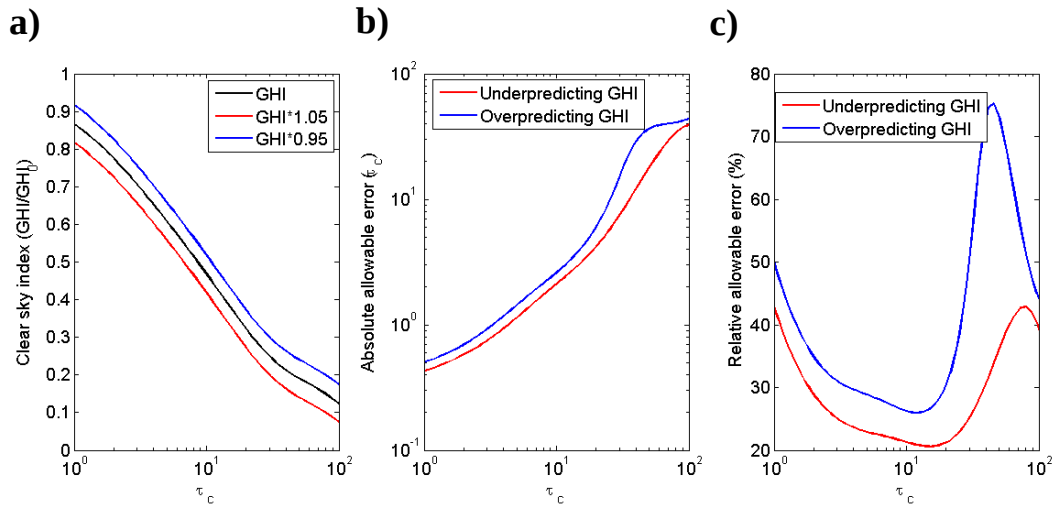


Fig. 1a) Irradiance divided by clear sky irradiance as a function of τ_c for homogenous clouds as derived from SHDOM. The black line represents the results while the blue and red are 5% offsets in GHI. 1b) Error bounds of $\pm 5\%$ on 1a converted to absolute intervals for τ_c . For example, for GHI to stay within 5% of its value at $\tau_c = 30$, τ_c cannot be more than 7.7 below 30 and not more than 15.6 above 30. 1c) Same as 1b but the y-axis divided by τ_c .”

3. 11287, line 19: What is the threshold OT to differentiate between clouds and clear sky?

Response: For existing methods the connection between OT and what is a cloud is typically not made. Rather the categories of clear sky and cloud are defined subjectively.

4. 11288, l 2: some text missing here

Response: The sentence has been corrected:

“To analyze this relation, the Spherical Harmonic Discrete Ordinate Method (SHDOM) (Evans et al., 1998; Pincus et al., 2009) is used to produce synthetic overcast sky images (Section 3) and analyze the determinants of sky imager radiances (Section 4).”

5. “11288, l 20: Unclear sentence. Where do the contrails come from? You mean, if one knows the given clear sky values, one can detect thin clouds? What dependencies have been considered to do so? Please clarify.”

Response: Manuscript was modified as follows to the address concern:

“However, in a study of contrail clouds Koehler et al. (1991) observed that the ratio of RBR to the clear sky RBR was similar between contrail cases and permitted a method for identifying thin clouds. In other words, knowing the clear sky value, one can detect thin clouds. For clear sky images the main factors affecting the RBR were found to be the solar zenith angle (θ_0 , Fig. 1), solar pixel angle/scattering angle (ϑ_s) and pixel zenith angle/view angle (ϑ_z) and changes in aerosol properties. This lead to the development of clear sky libraries (CSL) (Shields et al., 1993; Chow et al. 2011) to express clear sky RBR value under any condition.”

6. “11289, sentence line 19 ff: What is a large optical thickness typical for clouds? Larger than 0.3? Please clarify. I do not understand the end of this sentence. “

Response: Manuscript was modified as follows to address the concern.

“Although a direct relationship with aerosol optical depth (τ_a) and RBR is observed for small τ_a , ($\tau_a < 0.3$) (Ghoniya et al., 2012) no direct relationship has been found between RBR, or other variables determined from sky imagers, and larger optical depths ($\tau > 0.3$) such as those found typically in clouds. This has limited sky imager cloud detection to a binary classification in which the image is segmented into cloud or clear sky.”

7. “11290, line 15 and 11291, line 3: Your RT simulations are on 161 homogeneous single layer cloud cases only? No need for 3D model.”

Response: As discussed in the major concern segment, the 3D radiative transfer is needed since the measurement of radiance is a 3D measurement. It is true that a homogenous cloud domain such as the one used can be created and input into a 1D model but the radiance from the 1D model do not provide any real meaning as they have an assumed shape to create the 1D model. For example the radiance of a two-stream model would be constant across a hemisphere.

8. “11290, l 24: You do this for liquid water clouds only? This would be an important information which should be given much earlier (abstract, title).”

Response: Abstract was adjusted to include water cloud description:

The RRBR method is applied to images of liquid water clouds taken by a USI at the Oklahoma Atmospheric Radiation Measurement program (ARM) site over the course of 220 days

9. “11292, line 11: These are strong assumptions which are most likely only valid to some extent. How big are the likely errors due to the non-perfect lens? “

Response: A_{in} and $\Delta\lambda$ are not dependent on the lens but on the camera. Errors due to a non-perfect lens affect $\Delta\Omega$ and result in vignetting. A discussion of vignetting was added to address this concern.

“To adjust for errors due to an imperfect lens the effects of vignetting were adjusted by using a labsphere integrating sphere (LIS). The LIS is an integrating sphere that provides uniform light inside of the sphere. The USI was placed inside the LIS and images were taken. Fig. 4 demonstrates the vignetting effects of one of our instruments. Vignetting was corrected as,

$$v_c = \frac{v_0}{v_x} v, \quad (7)$$

where v_c is the corrected signal and v_0 is the average signal value at the center of the uniform image and v_x is the signal value of the uniform image at the pixel location being corrected.

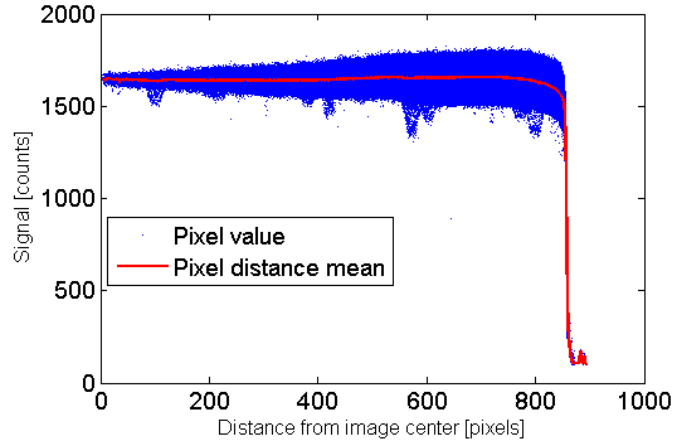


Fig. 4 Uniform signal values versus pixel distance from center, taken from 12 images.

USI 1.7 was corrected using the LIS. Unfortunately the USI 1.8 was not available to be corrected for vignetting effects but instead was corrected by comparing to USI 1.7 under an overcast sky. USI 1.8 was corrected as follows,

$$v_c = \frac{v_7}{v_8} v \quad (8)$$

v_7 is the signal value of USI 1.7 under the overcast sky and v_8 is the signal of USI 1.8 value under the overcast sky.

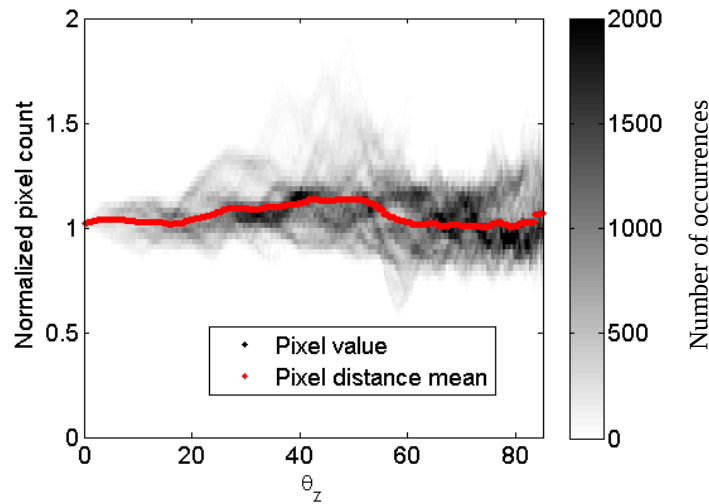


Fig. 5 USI 1.7 signal for red, green and blue divided by USI 1.8 signal versus θ_z for an overcast sky. The colorbar shows the number of occurrences. Since laboratory tests to eliminate vignetting for USI 1.8 were not available, field data from a lab-calibrated adjacent imager (USI 1.7) and USI 1.8 were used for calibration.”

10. “11292, line 16: You calibrate with a remote sensing retrieved OT? No lab calibration? I guess accuracy of such a “calibration” of absolute radiance can only be in the range of 10-25% or

whatever is the accuracy of the retrieval used for calibration. What accuracy do you need for your radiance measurement? You have to discuss that. “

Response: Regarding the required accuracy of radiance measurements, see the response to comment 2. Manuscript was modified as follows:

“The calibration constant $C_{2\lambda}$ is obtained as the average (denoted as overbar in Eq. 6) of 131 all overcast (cloud fraction (CF) is greater than 0.9) images on 98 different days. Overcast skies are preferred because the radiance is more homogeneous and since the method by Min et al. (2003) could be applied to obtain the τ_c that is input to SHDOM. $C_{2\lambda}$ values are 1.16×10^{-4} , 1.11×10^{-4} , and $9.69 \times 10^{-5} \text{ W}\cdot\text{m}^{-2}\cdot\text{st}^{-1}\cdot\text{nm}^{-1}$ for the red, green and blue channels, respectively. Fig. 3 demonstrates the three signal calibrations with a relative root mean square error (RMSE) of 0.155, 0.148, and 0.144 for the red, green and blue channel respectively. This RMSE is within the range of the variability expected in overcast clouds Szczodrak et al. (2001). These calibration values are assumed to be correct as they are calibrated to the data used for validation, in this way calibration errors are removed. Further development will require lab calibration to validate this calibration method.”

11. “11293, line 10: These have to be calibrated somehow. I guess most are not. Please discuss.”

Response: The method described in section 3.2 is how you calibrate these instruments.

12. “11293, line 22: What is “near 0”? Stray light probably affects the measurement up to optical thickness of about 5, don’t you think? As long as you do see a clear solar disc through the cloud. Again this is related to the question in which OT values PV solar energy production is interested in? “

Response: “ τ_c near 0” was changed to $\tau_c=0$. Stray light is believed to be dependent on DNI, which follows $\text{DNI}=\exp(-0.57\cdot\tau_c/\cos(\theta_0))$. So for example at a $\theta_0 = 60$ the DNI for $\tau_c=2$ is 10% of clear sky DNI and would greatly reduce Stray light.

13. “11294, line 4: It is hard to understand terminology only introduced in the following sections (“interpolant described in Sect. 5”). “

Response: Manuscript was modified as follows:

“To mitigate some of the stray light effects the SHDOM results for clear sky ($\tau_c = 0$) are replaced by the measurements from the CSL for the rest of this analysis.”

14. “11299, line 6: Shouldn’t the references read “Min and Harrison, 1996b, 2003”?”

Response: The references were changed:

Min and Harrison. 1996b; Min et al. 2003

15. “11299, line 16: GHI not have the ambiguity problem as radiance, right? That means, growing OT is directly related to optical thickness always. Please mention.”

Response: Ideally yes, but in practice there is still ambiguity, as thin clouds can create GHI enhancement under apparent homogeneous clouds.

16. “11299, line 17: What is the impact of your 8 μ assumption on accuracy? How do you decide that you have liquid water clouds? What is the assumed accuracy of the Min et al method? Which version do you use in the following, with or without MWR? Please discuss.”

Response: Min et al.’s sensitivity study demonstrated that the 13% uncertainty in LWP leads to a 12.7% uncertainty in effective radius but only 1.5% uncertainty in optical depth. The weak sensitivity of optical depth to effective radius occurs because the phase function is only weakly dependent on effective radius. Min et al. concluded that the uncertainty in the inferred cloud properties was less than 5%.

Liquid clouds were assumed if a cloud height less than 9 km was observed.

“By default a cloud effective radius (r_e) of 8 μ m is assumed in the Min method, but when liquid water path (LWP) values are available from a microwave radiometer (MWR), then r_e is iteratively solved. r_e is first solved for with Eq. 8 and used as an input in the discrete ordinate model, which provides a different τ_c , which leads to a different r_e , and this process is repeated until the changes in τ_c are within a threshold value. Min et al. concluded that the uncertainty in the inferred cloud properties was less than 5%. Since the Min method uses *GHI* measurements to estimate τ_c , the τ_c is representative of the sky hemisphere. At the ARM site the Min τ_c is sampled and reported every 20 sec. Since the Min method only works for liquid clouds, liquid clouds were assumed if a cloud height less than 9 km was observed.”

17. “11300, line 9: Does that mean that the accuracy of the MWR is about 50% in the OT range 6-12? This is a large uncertainty? “

Response: Yes. That is a large uncertainty and the method will need further development to narrow the uncertainty.

18. “11300, line 15: It’s not only the non-linearity. There are real 3d effects also. One relevant here is named “tunnelling” or “channelling”, which works in the same direction. Then there is the apparent cloud cover problem at oblique solar zenith angles, working the opposite direction ...”

Response: These are addressed in the discussion section.

“Characterizing the cloud heterogeneity effects may improve the RRBR method. As the RRBR method is based on interpolants developed from simulations of homogeneous overcast skies, cloud heterogeneity violates the assumptions and is the leading source of errors. Errors due to cloud heterogeneity have been analyzed mainly in the context of satellite remote sensing. Varnai et al. (1998) and Chambers et al. (1997) observed that the spatial reflectance variation is smoother than variations in τ_c . They hypothesized that optically thicker clouds would scatter more light to their thinner neighboring clouds causing the thinner clouds to appear brighter and thicker (looking from space), while the thinner clouds would scatter less light to the thicker clouds making them appear darker and thinner than expected for a homogeneous cloud scene. A similar but opposite effect is observed in ground based imagery, where thicker clouds shade their neighboring thinner clouds making them appear darker and thicker but this effect is moderated by the location of the sun relative to the clouds.”

19. “11300, line 16: How does that adjust for heterogeneity? Please give more detail? Equation 9 does not make sense without the sum over all logs.”

Response: The brackets represent average not sum over all logs.

This modification does not adjust for heterogeneity, but for the nonlinear characteristics of averaging optical depth. The RRBR method provides OT for all pixels in a sky image which need to be averaged to compare with one measurement provided by the Min method.

20. “11300, line 20ff: Please explain. You used the Min OT data to calibrate your sensor in the first place. Instead of finding perfect match of results, you do find 20-50% RMSE. What does that mean? Does your method worsen the results from Min algorithm by using RBR? Why should one use your method if Min is the more reliable for overcast skies as you state. You do not have any solution for broken skies either? You have to explain and discuss these points.”

Response: Calibration factors were updated to correct some errors and the RMSE is now 6-31%. The manuscript was modified as follows to address this concern.

“Fig. 9 compares results from both methods. An R^2 of 0.99 reflects the high correlation between the two methods. The relative RMSE decreases as τ_c increases as demonstrated in Table 2, with thin clouds ($\tau_c < 10$) having an RMSE of 31.3% and thick clouds ($\tau_c > 30$) having an RMSE of 5.8% with the overall RMSE being 9.1%. The low RMSE at $\tau_c > 10$ validates the RRBR method for overcast clouds with $\tau_c > 10$, but for $\tau_c < 10$ the Min method is no longer valid (Turner et al., 2004) and the RMSE increases drastically. In the case of overcast clouds, the main benefit of the RRBR over the Min method is that it is able to produce pixel-by-pixel τ_c . “

21. “11301, line 6: You think 48% RMSE is good agreement? You have to explain your requirements.”

Response: A section discussing requirements was added to the introduction. Specific sentence was modified as follows:

“Overcast conditions again result in good agreement with root mean square error (RMSE) of 5.0 or 17.7% and R^2 of 0.96 again well within the minimum error requirement of 21%.”

22. “11301, line 8: What is this in percent? Must be much higher than above mentioned?”

Response: Manuscript was changed to address this concern:

“The RMSE is 2.33 for the heterogeneous cases, which is well within the uncertainty of the MWR measurements of ± 5.6 but is a relative RMSE of 84.2% that exceeds the objectives set at the beginning.”

23. “11301, line 11: Who measures “entire hemisphere”? The MWR? Rather not? Please explain.”

Response: We apologize for this error which has been removed from the manuscript and now reads as follows:

“The lower correlation of 0.66 between the two methods is probably related (i) the uncertainty of the MWR, and (ii) the fact that τ_c retrievals under heterogeneous cloud conditions introduce random errors

due to incomplete overlap of the field-of-view of the USI and MWR, (iii) 3-D cloud effects and (iv) uncertainty in the MWR τ_c related to the assumption of $r_e = 8 \text{ um}$.”

24. “11301, line 20: What is MBE? Some bias? Please introduce terms.”

Response: Definition was added to the manuscript as follows:

“A mean bias error (MBE) of -6.6% is observed demonstrating a tendency for the RRBR method to under predict τ_c .”

25. “11302, line 2: All uncertainties for thin clouds seem to emphasize that no method produces trustworthy values for OT.”

Response: Yes, we are currently working on a better way of validating thin clouds but it continues to be a prominent and important issue.

“Although these methods are able highlight some errors in the RRBR method no method was accurate enough to provide information about thin clouds ($\tau_c < 10$) and future development requires a new validation method for thin clouds.”

26. “11302, line 13: You obtained this A value by comparison to RRBR measurements and then you validate RRBR against the DNI measurements? This sounds awkward. This way any bias is excluded?! Please clarify or discuss.”

Response: A new method of retrieving A with SHDOM simulations was used and is described below:

“Another option for validation is to calculate τ_c directly from Beer’s law using the DNI measurement of the MFRSR, which validates τ_c in the circumsolar region (Fig. 11). Rearranging Beer’s law and solving for τ_c we obtain,

$$\tau_{MFRSR} = -\log\left(\frac{DNI}{DNI_0}\right) * \frac{\cos(\theta_0)}{A}, \quad (10)$$

where DNI is the measured DNI from the MFRSR, DNI_0 is clear sky DNI computed with the Ineichen model (Ineichen et al., 2002). To solve for A the DNI results from the homogeneous SHDOM simulations were compared to τ_c , from these results the A value was solved for and found to be 0.57. A is a constant factor to account for the strong forward peak in the phase function of liquid and ice water clouds. Fig. 11 demonstrates the SHDOM DNI and the exponential fit.

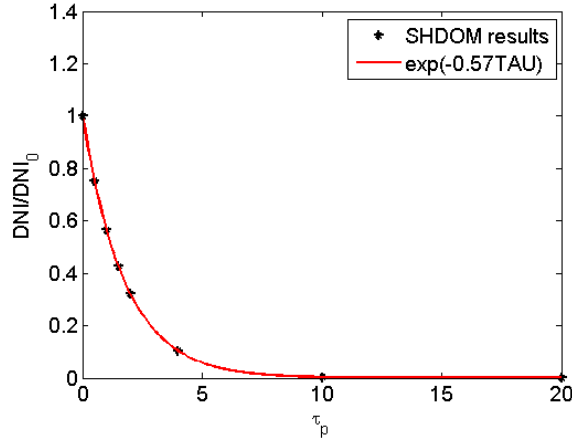


Fig. 11 DNI results from overcast homogeneous clouds and an exponential fit.

The τ_c of the RRBR method that is compared with τ_{MFRSR} are based on the average measurement for pixels in the solar region ($\vartheta_s < 5^\circ$)."

27. "11304, line 10: What do you mean with "consistent"? RMSE 20-80% and bias up to 30% in some regimes is pretty questionable, isn't it?"

Response: Results have been improved so that RMSE is in the range of 6-30%.

28. "11304, line 18: Finally a requirement is given. OT 0-10 is relevant for PV. You need to discuss all errors with respect to this range. Unfortunately this means that no method, neither old nor your new one is good enough."

Response: A discussion of the accuracy requirements was added to the manuscript.

"Summary statistics of the different validations are presented in Table 2. For overcast skies the RRBR yields τ_c that are consistent with the Min method. For heterogeneous cloud fields (cloud fraction < 0.7), comparisons with microwave radiometer (MWR) measurements of LWC at zenith demonstrated that the RRBR method provides τ_c estimates with typical r^2 of 0.66 and RMSE of 2.3 which is well within the uncertainty of the MWR instrument ($^{+/-} 5.6$) but more work needs to be done to validate heterogeneous clouds to be within the 21% uncertainty required. A RMSE of 0.71 between the USI and DNI retrieved τ_c is observed in the range of $0 < \tau_c < 10$. As demonstrated by the relative RMSE in Table 2, the current method provides accurate τ_c estimates for overcast clouds, while the relative RMSE is larger for $\tau_c < 10$. These results validate the RRBR method for overcast clouds but consistent under prediction of heterogeneous clouds require improvement for this method to be valid under heterogeneous clouds."

29. "11305, line 13: You have to introduce these points much earlier (introduction). Throughout the manuscript it stayed unclear what you try to reach."

Response: Agreed. See response to comment #4.

30. “Table 1: What is an effective radius of 3.9 μ m? Is this a cloud value? I thought you use 8 μ m? Please explain in text.”

Response: It is the aerosol effective radius. The table was modified:

“Table 1. Atmospheric radiative properties for the ARM site used as input to SHDOM. τ_a and Rayleigh optical depth are averages for the year 2013 from AERONET data.”

	Red (620 nm)	Green (520 nm)	Blue(450 nm)
τ_a [-]	0.0784	0.1010	0.1212
Rayleigh optical depth [-]	0.0875	0.1627	0.2296
Aerosol Effective Radius(R_e) [μ m]	3.9	3.9	3.9
Aerosol R_e distribution	Lognormal	Lognormal	Lognormal
Refractive index [-]	1.42 -0.002i	1.41 -0.002i	1.40 -0.002i

31. “Table 2: What is MAE? How can it be that R2 values for “All” cases of Min method are not reached in any sub-category? Same for MWR. Does the line “MWR OT<5” mean that bias of 5.1 is reached for values smaller than 5?”

Response: R2 of ‘All’ cases is larger than any of the subcategories because the RRBR estimates the OT with a smaller MAE than the differences between the subcategories. In other words, a scatter plot with ‘All’ data would provide a ‘zoomed out’ version of the results which reduces the randomness of the data that is apparent when zooming in to a particular subgroup.

The table legend now reads:

“Table 2. Statistics of RRBR validation against the Min method in overcast skies, microwave radiometer measurements, and DNI measurements from the MFRSR. RMSE[-] is the absolute root mean square error, RMSE[%] is the relative root mean square error, MAE[%] is the relative mean average error, and MBE[%] is the mean bias error.”

32. “Figure 4: For tau_c=10 please use a different line style. It is hard to differentiate from tau_c=0. Caption: “red and blue channel”.”

Response: Markers were changed on graphs”

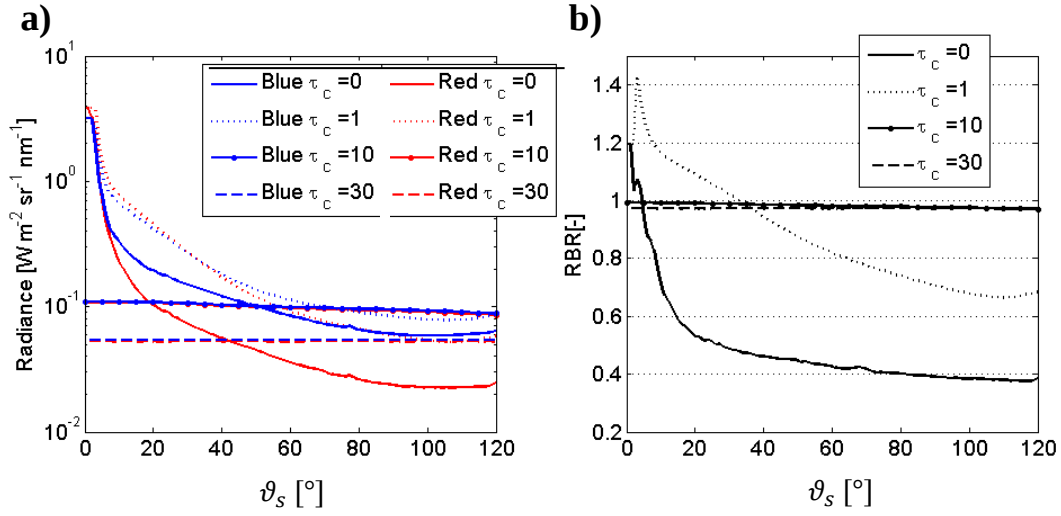


Fig. 4a) SHDOM red channel radiance over various sun pixel angles (ϑ_s) at $\vartheta_z = 60^\circ$, and $\theta_0 = 60^\circ$ (Pixels used for Fig. 4 are highlighted as a red line in Fig. 2c). Results are shown for different cloud optical depths from clear ($\tau_c = 0$) to thick clouds. b) RBR as a function of ϑ_s at constant $\vartheta_z = 60^\circ$ and $\theta_0 = 60^\circ$.