Responses to the Comments from Referee #2

Dear Referee,

Thank you for spending your time reviewing this manuscript and providing detailed and insightful comments. A point-by-point response to your comments are given in the following. Your comments are colored with blue and followed by our responses starting with **Response**.

This paper concerns estimation of the uncertainty of the mean wind speed when estimated from a Doppler lidar arc scan, i.e. a certain fraction of a normal conical scan. The subject is timely, as more and more scanning Doppler lidars are used, especially in the wind energy industry. The authors has got accepted a similar paper in Journal of Oceanic and Atmospheric Technology (Wind Measurements from Arc Scans With Doppler Wind Lidar by Wang, Barthelmie, Clifton and Pryor), and although there is some overlap, the present paper contains a series of new experiments and an emphasis on the implication for annual energy production (AEP) estimation of wind turbines. The papers shows how wind direction relative to the direction of the arc is quite important for the uncertainty, and also that, in general, a wider arc gives lower uncertainty.

Some general improvement is needed at several sections:

Comment 1 Much of the theory is very similar to the JTECH paper already published. I suggested that most of section 3 and 4 should be deleted and replaced with reference to the appendices in the JTECH paper. The same isotropic turbulence model is used, the same exponential correlation function is used, so what is new?

Response: The same theory is used in both paper to predict uncertainty. This paper added a method (see appendix A) to calculate the integral length scale L_u from the turbulence intensity and height and uses the standard error of 10-minute horizontal wind speed from lidar measurements instead of the mean square error of the least squares fit to quantify uncertainty. We kept Eq. (13) and Eq. (14) in Section 4 because having these two equations is beneficial for reproducing the results in the paper. Section 3 is reduced as suggested by the referee. The reduced part of Section 3 is given below (see Page 5 Line 144):

The uncertainty in the wind velocity estimated from arc scans can be derived from the covariance matrix A of the measured radial velocities. Assuming a horizontally homogenous wind field with zero mean vertical wind speed (i.e. w_0 = 0), the solution of the ordinary least squares ($\boldsymbol{V}_l$) based on Eq. (1) is the estimate of horizontal wind velocity (Wang et al., 2015):

$$\boldsymbol{V}_l = \mathbf{G}\mathbf{v}_R \tag{6}$$

where $\mathbf{G} = (\mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T$ and $\mathbf{v}_R$ is a vector including N radial velocities measured in, for example, 10 minutes. The matrix $\mathbf{D}$ is a $N \times 2$ matrix with its ith row given by $[\cos\phi \sin\theta_i, \cos\phi \cos\theta_i]$. The estimated wind velocity is characterized by its covariance matrix ($\mathbf{C}_l$) given by:

$$\mathbf{C}_l = \mathbf{G}\mathbf{A}\mathbf{G}^T \tag{7}$$

assuming zero random error for radial velocity (Wang et al., 2015). The variance of the random error σ_e^2 is ~ 0.01 m^2 s^{-2} (see Sect. 2) which is much smaller than the diagonal term $\mathbf{A}$ (> 0.1 m^2 s^{-2} given wind speed > 4 m s^{-1} and turbulence intensity ≈ 10%); therefore, it can be neglected.

Comment 2 also figure 1 seems superfluous. Isn't it concluded that random errors due to the instrument itself are swamped by the random errors due to turbulence?

Response: The points/errors in Figure 1 are results of random errors from both the instrument and atmospheric turbulence. However, the random error due to turbulence will vanish when the autocorrelation at lag one approaches one. The autocorrelation value cannot be one and this is why the points are scattered. However, it is clear that there is lower bound for the error values. The errors close to the lower points also have large autocorrelation values. Therefore, it is reasonable to use the bottom 5% of errors to approximate the error-SNR relation, which is consistent with the theoretical prediction.

Comment 3 Figure 2 is about systematic errors, which is not the subject of the paper. I think it should be removed, since it is not used in the paper.

Response: Figure 2 has been removed as suggested by the referee. The related text has been changed to (see Page 4 Line 98):

For a well-secured ground-based lidar pitch (displacement from the horizontal) and roll (i.e. tilt) angles can be measured and usually much lower than 1°, causing negligible errors.

Comment 4 The theory is about the relative uncertainty of the arc scan, while the data analysis is the relative uncertainty of the difference between the arc scan and the cup measurement. One could argue that if the uncertainty of V_{cup} is very small, then the two quantities are the same. But the uncertainty of v_{cup} over a 10-minute period is several percent, which is the same order of the uncertainty as that of the arc scan. Therefore, you should expect (17) to be larger than (16) (depending on how uncorrelated V_0 and V_{cup} are, which also depends on the distance between the measurements). This difference is certainly something that should be investigated when comparing with data, and before it is concluded that theory predicts data well. The uncertainty given by (20) is inadequate because it relates to the variation of systematic errors a certain type of cup anemometers would have if they were subjected to a constant, laminar flow. The random error that also occur in the experiments is much larger and can be estimated by Lenschow et al (1994), which is very much along the same line of reasoning than the present paper in sections 3 and 4. So (20) is about what systematic errors you can expect on a cup, while the relevant ε_c is the one that has to do with the random error due to turbulence.

Response: A definition is given of the difference between the lidar and cup measurements in the revised manuscript (see Eq. (16) on Page 9). The cup uncertainty due to turbulence has been added to the analysis using the formula from Lenschow et al. (1994). The same input parameters used to estimate the lidar relative standard error (RSE) are used to estimate the cup RSE due to turbulence. The RSE defined in Eq. (20) is derived from cup anemometer errors in simulated turbulent wind field (Pedersen et al., 2006); therefore it represents cup RSE due to instrument uncertainty.

The following change has been made in the manuscript to address the referee's comment (from Page 9 to Page 10):

In all cases the analysis is based on the estimated 10 minute horizontal mean wind speed (V_l) from the Galion measurements as derived using the ordinary least squares method. The RSE of V_l will be evaluated through the relative difference between V_l and the measurement (V_c) from cup anemometers installed on nearby meteorological masts (in compliance with the standard (IEC, 2005)):

$$e_d = \frac{V_l - V_c}{V_c} \tag{14}$$

Periods with V_c < 4m s^{-1} or lidar SNR < 20 dB are excluded from the analysis. To quantify the measurement uncertainty, the observed RSE ($\hat{\varepsilon}_d$) is defined as the standard deviation of e_d binned by wind direction or turbulence intensity, and the 95% confidence interval, CI95, of $\hat{\varepsilon}_d$ is estimated by (Ahn and Fessler, 2003):

$$\mathrm{CI}_{95} = \hat{\varepsilon}_d + 1.96\hat{\varepsilon}_d / \sqrt{2(n-1)} \tag{15}$$

where *n* is the number of samples in a bin. Note that this definition means that only the spread of values is evaluated and bias is not considered.

The value of $\hat{\varepsilon}_d$ has contributions from random errors related to both instrument and turbulence. The lidar instrument errors are not considered; hence, the expected RSE (ε_d) based on the relative difference between lidar and cup anemometer measurements has the following definition:

$$\varepsilon_d^2 = \varepsilon_l^2 + \varepsilon_c^2 - 2\rho_{lc}\varepsilon_l\varepsilon_c + \varepsilon_{cup}^2 \tag{16}$$

Terms on the RHS of Eq. (16) represent sources of errors and will be estimated as follows in order to differentiate the lidar RSE and the difference between lidar and cup anemometer measurements:

- ε_l is the lidar RSE due to turbulence defined in Eq. (13) that can be estimated with the isotropic turbulence model.
- ε_c is the cup RSE due to turbulence which is a function of the integral time scale and the sampling duration (Lenschow et al., 1994). Eq. (13) in Lenschow et al., (1994) will be used to estimate ε_c by assuming that the streamwise velocity autocorrelation decays exponentially. The integral time scale is derived from the integral length scale (L_u) in Eq. (A12) and the observed mean wind speed. The sampling duration is 10 minutes.
- ρ_{lc} is the correlation between the turbulence-related errors of lidar and cup anemometers that depends on the spatial structure of turbulence and the distance between cup and lidar measurement locations. Estimating ρ_{lc} is difficult because lidar measures a volume and cup measures a point (or a line assuming

frozen turbulence). A simple approximation is used here to estimate ρ_{lc}. The separation distance is the distance between the center of an arc and the cup location which are 150 m for both Site A and Site B and 120 m for Site C. The correlation decays exponentially with the same integral length scale that is used to estimate ε_l and ε_c at each site

- ε_{cup} is the instrument error that can be found from the following equation:

$$\varepsilon_{cup} = \left(\frac{k}{3}\right) \cdot \left(\frac{0.05\ m\ s^{-1}}{V_0} + 0.005\right) \quad (17)$$

where k is the cup anemometer class number that represents the maximum relative error of a cup anemometer in turbulent wind fields (IEC, 2005; Pedersen et al., 2006). The k values for cup anemometers used at the three sites are listed in Table2.

Comment 5 It is simply not correct that a small azimuthal angle range should let a inhomogeneity in the horizontal wind have a smaller effect on the mean wind determination. Rereading Schwiesow 1985, I cannot find that statement anywhere

Response: We agree with the reviewer that arc scan is more error-prone than full conical scans when inhomogeneity appears. This is stated in the text "VAD scans are commonly used for wind resource assessment because in homogeneous terrain or under a constant wind gradient the function used to derive the wind velocity should have the smallest errors while arc scans can potentially have large errors if the fit is distorted by a small number of erroneous points". However, we also want to emphasize that when homogeneity exists over a small area where full-conical scans cannot fit, arc scans will have a role. One example is given in the manuscript:

> "For example, a VAD scan centered at the hub of an operating wind turbine will be affected by inhomogeneity because of the wind turbine wake. If the purpose of measurement is the freestream wind speed, a smaller sector scan or arc scan upwind of the wind turbine can be more suitable than a full conical scan."

The quote "arc scans are less affected by inhomogeneities in the wind field on scales of the scan diameter than are the full circle scans" can be found on page 13 at the beginning of the 2nd paragraph on the right in Schwiesow et al (1985).

Comment 6 Section 7 should be shortened or removed. The point of random error is not that is gives uncertainty on the yearly average wind or AEP, which it actually doesn't at all as shown in much detail in the paper (a small fraction of a percent).

Response: There was an error in the old manuscript. Section 7 has been shortened and a revised method has been used to calculate the uncertainty in AEP prediction based on IEC (2015). The shortened text is given below (see Page 12 Line 395):

> If wind speed measurements deriving from arc scans of a lidar are used to predict annual energy production (AEP) at a given site, naturally, the uncertainty in the wind speeds will propagate into AEP prediction and contribute to the uncertainty in wind resource assessment. The annual AEP is predicted as follows:
>
> $$E_y = \sum_{j=1}^{J} \sum_{i=1}^{I} \left(T_y F_{V,i} F_{D,j}\right) P_i \qquad (19)$$
>
> where $F_{V,i}$ and $F_{D,j}$ are the probabilities of the ith wind speed bin and jth wind direction bin, respectively, P_i is the power production of a wind turbine at wind speed V_i, and T_y is the total hours in a year. Assuming statistical independence between lidar measurements, the contribution of the arc scan measurement uncertainty to the uncertainty of E_y is quantified by the standard error (σ_y) defined as follows (IEC 2005):
>
> $$\sigma_y = \sum_{j=1}^{J} \sum_{i=1}^{I} \left(T_y F_{V,i} F_{D,j}\right) c_i^2 \sigma_{l,ij}^2 \qquad (20)$$
>
> where $\sigma_{l,ij}$ is the lidar measurement standard error (see Eq. (13)) for the ith wind speed bin and jth direction bin, and c_i is the sensitivity factor determined by:
>
> $$c_i = \left|\frac{P_i - P_{i-1}}{V_i - V_{i-1}}\right| \qquad (21)$$

References

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Pedersen, F. T., Dahlberg, J.-Å., and Busche, P.: ACCUWIND-Classification of Five Cup Anemometers According to IEC 61400-12-1, Forskningscenter Risøe, Denmark, Risøe-R, No. 1556(EN), 2006.

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