

Review comments for “Smoothing data series by means of cubic splines: quality of approximation and introduction of an iterative spline approach” by Sabine Wüst et al.

The current paper introduces an alternative way of applying cubic splines for smoothing data series. Based on simple test data, the authors first showed the dependence of the approximation error on the sampling distance for the traditional way of applying cubic splines (Fig. 3). It was shown that when the sampling distance is equal or close to half of the wavelength, the approximation error does not decrease smoothly with decrease of the sampling distance, but oscillates strongly. In this case, the approximation error also depends very strongly on the phase of the wave/ exact positions of the sampling points (Fig. 4). In addition, artificial oscillations can appear in this case (Fig. 2b, c).

The authors suggested an alternative method (iterative approach) of applying cubic splines, which can reduce these phenomena. Indeed, using this iterative approach, the approximation error decreases quite smoothly with decrease of the sampling distance (Fig. 6a). Moreover, the dependence of the approximation error on exact positions of sampling points also reduces significantly (Fig. 6b). However, it should be noted that this improvement is only helpful for the sampling distances, which are equal or close to half of the wavelength. Another important note is that the amplitude of the smoothed series (and therefore of the residuals) changes significantly.

The iterative approach was also applied to another simple test dataset, which was created by superimposing 3 oscillations with different wavelengths on a realistic temperature background profile. Again, the approximation error decreases smoothly with decrease of the sampling distance. Finally, the iterative approach was applied to vertical temperature profiles measured by the SABER instrument. The residuals are then calculated and averaged over one year for the years 2010-2014. Results and their implication for gravity wave (GW) studying was discussed.

The paper is generally well written and has a clear structure. Introduction, Method description and Discussion are of appropriate length. My major and minor comments for this paper are given below:

Major comments

1. The interactive approach has advantages of stabilizing the traditional approach: (1) the approximation error decreases smoothly with decrease of the sampling distance, and (2) the approximation error depends very slightly on the exact locations of sampling points. However, these advantages are only helpful if the sampling distance is equal or close to half of the wavelength.

In practice, a constant sampling distance is chosen for all measurement profiles (for example, the authors chose 10 km for all SABER profiles in this study). It means the interactive approach may be advanced for only some profiles, which contain waves with wavelength of ~ 20 km. For other waves in the same profile or for other waves in other profiles, which have wavelengths different from ~ 20 km, I would not expect advantages of the interactive approach. It is well known that GWs have a very broad spectrum. Therefore, wavelengths, which are equal or close to a certain wavelength, can only be a very small part of that broad spectrum.

It is therefore interesting to see how much this stabilizing contributes to the total approximation error:

- (a) Starting with the test data, where the 3 oscillations of different wavelengths were superimposed on a realistic CIRA temperature background, can you please compare the approximation error of the traditional approach and the approximation error of the interactive approach? (Using a constant sampling distance of 10 km, for example)
 - (b) Similarly for SABER profiles per year: For each profile, please calculate the approximation error δ_{1i} for traditional approach and δ_{2i} for interactive approach, using a constant sampling distance of 10 km. For one year, the total approximation error is $\Delta_1 = \sum_i \delta_{1i}$ for traditional approach and $\Delta_2 = \sum_i \delta_{2i}$ for interactive approach (\sum_i means sum over all profiles of one year). Can you please compare Δ_1 and Δ_2 for the years 2010-2014?
2. The term called “approximation error” by the authors has the meaning of error in the case study, where the test data are described by an artificial sine. If an oscillation is superimposed on a realistic CIRA temperature background, the error of GW activity should be the difference between the residual (after the spline fit) and the original oscillation you used for superimposing. How does this error of GW activity vary with sampling distance for non-interactive and interactive approaches, for different superimposed wavelengths?
 3. When we average the temperature residuals over the whole year for enough number of profiles, due to the arbitrary distribution of phases and amplitudes of GWs, the non-squared mean residual should be approximately zero. However, as shown by Fig. 8d, the interactive method still produces non-zero amplitude oscillations in the temperature residuals. In page 8, lines 20-22, the authors suggested that changing in temperature gradients could be one of the reasons for this problem. However, if it is the main reason, we should see non-zero amplitude oscillations only in limited altitude regions near the stratopause or mesopause, but not in the entire altitude range in Fig. 8d. There seem to be systematic errors that have not been removed. Can you please comment on this?
 4. The authors recommended the interactive approach for estimating squared residuals for studying GW activity. Perhaps, the most convincing way to demonstrate if the interactive approach is suitable for GW studies, is comparing the GW squared temperature derived by this approach to the one derived by another method, using the same original data. For example, Ern et al. (2011) showed zonal averages of GW squared temperature for SABER measurements. For your method, taking the sampling distance of 10 km allows for vertical wavelengths up to 20 km. This can cover the main part of the GW spectrum in the stratosphere and mesosphere. It would be very interesting to see if the zonal averages provided by this interactive approach is similar to the ones in Ern et al. (2011).

Minor comments

1. Page 2, Lines 5-7: “Conclusions about ... sampling points used”. The Shannon’s sampling theorem described in Appendix 3 of Gubbins, 2004 is about reconstruction of the original time series from its samples. Since you use a constant sampling distance through each entire data series, it is rather the sampling distance between two samples, which decides which shortest wavelength can be resolved. It is rather about the Nyquist theorem than the Shannon’s sampling theorem. I would suggest to write straightforward that: “The shortest wavelength/period which can be resolved by the spline is twice of the sampling distance according to the Nyquist theorem.”
2. Page 2, Line 15: “between 48°N ...and 15°O”. Please shortly explain here why did you choose this region?

3. Page 2, Lines 19-22: “The distance of 10 km ... up to 20 km”. At this point, it is not straightforward for all readers to understand why choosing 10 km distance will lead to maximum wavelength of 20 km. Later, in Sect. 2, paragraph 2, you explained this very nicely. I suggest to move the explanation here and shortly refer to it again in Sect. 2 later.
4. Page 2 Line 27 and Page 7, Line 33: For the motivation of this paper, the authors used Fig. 1, where squared temperature residuals were averaged for the whole year. An oscillation with a wavelength of about 10 km is found and the authors suggested that this oscillation is an artefact of the non-interactive spline approach. However, due to seasonal variations of GW sources and of the background wind, GW activity over the same location at a certain altitude can be different for different seasons. Therefore, the oscillation with a wavelength of about 10 km could also be an artefact of averaging. Can you please comment on this?
5. Page 3, Line 12: “Depending on ... approximated”. This sentence is not clear to me. I guess the authors mean: “If we want to approximate the entire data series, the length of the data series must be an integer number of the distance between 2 spline sample points. That is why only certain distances between two consecutive spline sampling points can be chosen if the whole data series is approximated”. If this is what the authors want to say, I suggest to rewrite this sentence. This also makes the next paragraph become more understandable.
6. Page 5, Line 7: “gravity waves ... ”. Please cite the paper of Ern et al. (2011), which provides a comprehensive GW data set derived from SABER measurements.
7. Page 8, Line 20: Please clarify the “height regions”

Technical corrections

Page 2, Line 15 and Page 13, Lines 6, 7: $^{\circ}\text{O}$ and 15°O \rightarrow 5°E and 15°E

Page 7, Line 15: might be surprising \rightarrow is not straightforward

Page 8, Line 21: one reason \rightarrow one of the reasons for

Page 19, Line 5: there are no dashed-dotted lines in Fig. 5e, f.

References

Ern, M., Preusse, P., Gille, J. C., Hepplewhite, C. L., Mlynchak, M. G., Russell III, J. M., and Riese, M. (2011). Implications for atmospheric dynamics derived from global observations of gravity wave momentum flux in stratosphere and mesosphere. *J. Geophys. Res.*, 116.