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Atmospheric Measurement Techniques Discussions

Interactive comment on "Novel approaches to estimating turbulent kinetic energy dissipation rate from low and moderate resolution velocity fluctuation time series" *by* Marta Wacławczyk et al.

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We thank the Referee for the comments and suggestions. We will improve the manuscript accordingly. Below, we reply and discuss the issues raised up by the Referee.

1 Reply to general comments

1. *Section 2 ("State of the art") is a collection of equations that relate various turbulence characteristics with each other. These equations have been taken from*

a large number of different sources, and it is not clear what the underlying physical assumptions are and to what extent they are consistent across the various sources. For example, it is not explained whether the one-dimensional spectra $E_{11}(k)$ and $E_{22}(k_1)$ and the frequency spectrum $S(f)$ are meant to be one-sided spectra or two-sided spectra.

$E_{11}(k)$, $E_{22}(k_1)$ and $S(f)$ are the one-sided spectra which, after integration over argument from 0 to ∞ equal the variance of the signal. $E_{ij}(k_1)$ are defined as twice the one-dimensional Fourier transform of $R_{ij}(r_1\mathbf{e}_1) = \langle u_i(\mathbf{x} + r_1\mathbf{e}_1, t)u_j(\mathbf{x}, t)\rangle$ (Pope, 2000). Here, we assume that the flow is homogeneous and statistically stationary and statistics do not depend on point **x** or time. Equations [11] in the manuscript will be corrected to

$$R_{11}(r_1\mathbf{e}_1) = \frac{1}{2}\int_{-\infty}^{\infty} E_{11}(k_1)\mathrm{e}^{ik_1r_1}\mathrm{d}k_1 = \int_0^{\infty} E_{11}(k_1)\cos(k_1r_1)\mathrm{d}k_1, \tag{1}$$

$$R''_{11}(r_1\mathbf{e}_1) = -\int_0^{\infty} E_{11}(k_1)k_1^2\cos(k_1r_1)\mathrm{d}k_1. \tag{2}$$

We will amend Section 2, correct Eqs. [11] and improve the manuscript in order to make the considerations more consistent. (In this reply we refer to equations from the manuscript using square brackets)

2. *Moreover, it is not mentioned which of the relationships follow from Kolmogorov's theory of fully developed, locally homogeneous and isotropic turbulence Kolmogorov (1941a,b) and which are valid for any statistically homogeneous vector fields, regardless of whether or not they are isotropic (Monin and Yaglom, 1975, pp. 16-22)*

Eqs. [11] in the manuscript are valid under the assumption of homogeneity alone, however, for further relations (relationship between λ_g and λ_f and Eqs. [15] and [16]) the assumption of local isotropy (Kolmogorov, 1941) is needed, to finally find the value of the dissipation rate.

As the airborne measurements provide signals of velocity along the $1D$ aircraft flight path, the local isotropy assumption is needed to estimate the dissipation rate ϵ of a 3D turbulent field and the assumption of homogeneity alone (Monin and Yaglom, 1975) is not sufficient.

3. *Additionally, I find it worrisome that the authors sometimes confuse k_1 and k and that some of their equations contain transcendental functions with dimensional arguments.*

   Integration in equations [25], [26] and [31] is in fact performed over non-dimensional variables. In the manuscript we denoted them by k and k_1 which was confusing. In "Reply to specific comments" below we present the derivation in detail and denote the variables by ξ and ξ_1.

4. *I find it difficult to follow the flow of the authors' reasoning in detail. I am not surprised that the zero-crossing methods can provide ϵ estimates with a quality comparable to the ϵ estimates obtained with traditional spectral retrieval methods. The relative advantages and disadvantages, however, are less clear, and the authors do not discuss and explain them in sufficient depth from a physical point of view.*

   In the manuscript we proposed two extensions of zero-crossing method to estimate TKE dissipation rate for low-pass filtered signals, in particular from airborne turbulence measurements with spatial resolution of meters or tens of meters along $1D$ aircraft tracks. The first of them, described in Sections 3.1 and 4.1 applies additional filtering of the signal and, similarly as the structure function or power spectra methods, is based on the inertial-range arguments.

   In spite of the same underlying physical arguments the structure function and power spectra methods are often used simultaneously, for better ϵ estimates (Chamecki and Dias, 2004). Here, the proposed method offers yet another option.

   The possible advantages are:

   - simplicity (e.g. it is not necessary to choose averaging windows),
   - robustness to measurement errors in recorded amplitude of velocity fluctuations (see discussion in conclusion section).

   In all the three, listed above, methods based on the inertial range arguments it is necessary to use a constant C_1 or C_2, which has to be determined by independent measurements. In order to avoid this limitation we propose the second extension of zero crossings, described in Sections 3.2 and 4.2. This extension assumes a model spectrum for the inertial and the dissipation range. We apply the particular, exponential model, see Pope (2000). The advantage of this method is that the inertial-range constant C_1 cancels in Eq. [25] and the resulting value of ϵ is unaffected by the uncertainty in C_1 estimates. Clearly, the approach is as good as the model itself. The only parameter present in the model equations [25] and [26], β, is fixed by theoretical constrains, as the dissipation spectrum $2\nu k^2 E(k)$ should integrate to ϵ. The second approach is based on different physical arguments than the methods based on the inertial-range scaling only, it additionally makes use of the first similarity hypothesis of Kolmogorov (1941) and a model for the dissipation range spectrum. Still, it can be used for signals with spectral cut-offs, hence it offers an alternative to the spectral retrieval methods.

2 Reply to specific comments

1. We will change the title of the section, according to the Referee's suggestion.

2. *Eq. [2] fails at wave numbers small compared to $1/L$, where the turbulence is usually anisotropic and is no longer universal.*

In principle, we agree here with the Referee. However, one can assume that the lowest wavenumbers of the spectrum E_{11} available from the measurements are within the validity of the local isotropy assumption and that the largest scales of the flow do not influence the value of dissipation rate.

3. We agree with the Referee, we will amend the manuscript, accordingly.

4. *The integrals in Eqs. [25] and [26] contain the term* e^{-k}*, the exponential function, however, is a transcendental function and its argument must be dimensionless, such as the argument* $\beta k \eta$ *in Eq. [24]. Because* k *is not dimensionless (its dimension is 1/Length), Eqs. [25] and [26] cannot be correct.*

   Integration in Eqs. [25] and [26] is performed over non-dimensional variables. Below we present the derivation in detail and change the notation to ξ and ξ_1 instead of k and k_1.

   In order to derive Eqs. [25] and [26] we considered a relation [22] between N_{cut} and N_L

$$u'^2 N_L^2 = u'^2_{cut} N^2_{cut} \left(1 + \frac{\int_{k_{cut}}^{\infty} k_1^2 E_{11} \mathrm{d}k_1}{\int_0^{k_{cut}} k_1^2 E_{11} \mathrm{d}k_1} \right), \tag{3}$$

   and in Eq. [23] assumed a certain form of the energy spectrum applicable in the inertial and the dissipation range

$$E(k) = C\epsilon^{2/3} k^{-5/3} \mathrm{e}^{-\beta k \eta}, \tag{4}$$

   the corresponding one-dimensional spectrum E_{11}, for the range of scales where the local isotropy assumption holds, was calculated using Eq. [2] form the manuscript

$$E_{11}(k_1) = C\epsilon^{2/3} \int_{k_1}^{\infty} k^{-8/3} \mathrm{e}^{-\beta k \eta} \left(1 - \frac{k_1^2}{k^2} \right) \mathrm{d}k. \tag{5}$$

   With the following change of variables $\xi = \beta k \eta$ in this integral we obtain

$$E_{11}(k_1) = C\epsilon^{2/3} (\beta\eta)^{5/3} \int_{k_1\beta\eta}^{\infty} \xi^{-8/3} \mathrm{e}^{-\xi} \left(1 - \frac{(k_1 \beta \eta)^2}{\xi^2} \right) \mathrm{d}\xi. \tag{6}$$

   We next introduce (6) into (3) and once again change the variables to $\xi_1 = k_1 \beta \eta$. We obtain

$$u'^2 N_L^2 = u'^2_{cut} N^2_{cut} \left[1 + \frac{\int_{k_{cut}\beta\eta}^{\infty} \xi_1^2 \int_{\xi_1}^{\infty} \xi^{-8/3} \mathrm{e}^{-\xi} \left(1 - \frac{\xi_1^2}{\xi^2} \right) \mathrm{d}\xi \mathrm{d}\xi_1}{\int_0^{k_{cut}\beta\eta} \xi_1^2 \int_{\xi_1}^{\infty} \xi^{-8/3} \mathrm{e}^{-\xi} \left(1 - \frac{\xi_1^2}{\xi^2} \right) \mathrm{d}\xi \mathrm{d}\xi_1} \right] = u'^2_{cut} N^2_{cut} \mathcal{C}_{\mathcal{F}}, \tag{7}$$

   where the correcting factor $\mathcal{C}_{\mathcal{F}}$ equals

$$\mathcal{C}_{\mathcal{F}} = 1 + \frac{\int_{k_{cut}\beta\eta}^{\infty} \xi_1^2 \int_{\xi_1}^{\infty} \xi^{-8/3} \mathrm{e}^{-\xi} \left(1 - \frac{\xi_1^2}{\xi^2} \right) \mathrm{d}\xi \mathrm{d}\xi_1}{\int_0^{k_{cut}\beta\eta} \xi_1^2 \int_{\xi_1}^{\infty} \xi^{-8/3} \mathrm{e}^{-\xi} \left(1 - \frac{\xi_1^2}{\xi^2} \right) \mathrm{d}\xi \mathrm{d}\xi_1}. \tag{8}$$

   The form of both equations is identical as [25] and [26], but the integration variables are denoted ξ and ξ_1, instead of k and k_1. We will amend the manuscript, accordingly. At the same time we note that the results of analysis performed in Sections 4 and 5 remain unchanged.

References

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