

## Response to Referee #1

### A variational regularization of Abel transform for GPS radio occultation

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Overall, the paper is clear and concise and presents a case where using a variational technique can be used to improve the retrieval of a refractivity profile from GNSS-RO. The math and application of the technique all appear sound. However, the improved refractivity profile is acquired by the assimilation of ECMWF forecasted atmospheric profiles. This then poses a fundamental question as to the goal of GNSS-RO, is it to solely obtain the best refractivity profile, or to gain atmospheric state information such as temperature and moisture from the profile? If it's the latter there may then be an incestuous relationship where if the ECMWF model were to ingest these refractivity profiles, they would be ingesting forecast data from their own model. Does this technique then need to be applied for each NWP model independently, and the background error covariance calculated for the NWP model it is going to be applied to?

**Response)** I appreciate this observation. I admit that the incestuous relationship is unpleasant in theoretic perspective. However, it is difficult, if not impossible, to fully incorporate VR into data assimilation because of the EIV issue. Unfortunately, breaking the problem into two sequential steps (i.e., VR and then assimilating the retrieved refractivity) seems the only feasible option at the moment in order to avoid the EIV issue. The practical question is how significant the adverse effect could be compared to the benefit of VR. We have compared AI and VR by assimilating the refractivity data produced by them into WRF (Weather Research and Forecasting) DA. The DA experiments show that VR yields clear positive impacts on analysis and forecast over AI (manuscript is in preparation for separate publication). Based on the results, we strongly believe that the benefit of VR outweighs the incest effect. For instance, the 1D-Var+4D-Var approach for precipitation-affected microwave radiance at ECMWF (Bauer, P., Lopez, P., Salmond, D., Benedetti, A. and Moreau, E.: Implementation of 1D+4D-Var Assimilation of Microwave Radiances in Precipitation at ECMWF. I: 1D-Var, Q. J. R. Meteorol. Soc., 132, 2277–2306, 2006) is shown to be beneficial. Yes. It might be the best practice for individual NWP centers to apply VR independently using their own forecast and by constructing and using the background error covariance that is consistent with the forecast. This was stated in the last paragraph of section 5 (conclusions) of the original manuscript. By doing so, VR and data assimilation are consistent in the background and the potential contamination arising from external a priori can be avoided.

Further, in the technical description, I could use a little more detail on the computation particularly of the error covariances. The vertical coordinate is never discussed for these matrices and should be described. And though a reference is stated for the control-variable transform its application seems to merit a sentence or two. Lastly, the abstract of the paper itself never mentions that the regularization method will ingest ECMWF model forecast data, or more generically, NWP model forecast data, for input to the method. This is a key point, and obviously impactful on the final result and should be mentioned plainly in the abstract. Considering these points, revisions are required before publication can be considered, though minor revisions, they are fundamental and need to be addressed.

**Response)** Thank you so much for the valuable comments. All these are well taken in the revised manuscript.

One last philosophical point, the paper should try to address the question as to what is the benefit of the final result of such a technique. The antagonist would say that in a full data assimilation system, would you acquire the same result assimilating bending angle or refractivity profile which utilizes the traditional AI approach, with appropriate observation error, and then also assimilating the ECMWF forecast model profiles? The benefit to the VR refractivity profiles is coming from the ECMWF model data, so if they are available why not just assimilate the ECMWF model data directly as proxy radiosondes? To address this

concern, you could start by clearly stating that the goal or focus of this study is on creating the highest quality refractivity profile and what the benefits of such a dataset may be. Then follow up in the final summary and conclusions with a discussion about what may be the next steps in advancing this technique. It would seem that the logical extension would be to formulate a way to create a new forward operator for the bending angle profiles in the observation height coordinate which uses the NWP systems background (forecasts) to create an adjusted bending angle and PHD, and then subsequently transforms this back into innovations and Jacobians in the model space which can be used in the full solver minimization. It could be thought of as something similar to a 1D-Var step which would be embedded before passing information onto the main DA solver.

**Response)** The manuscript has been significantly revised to follow the reviewer's suggestions. With all due respect, I believe that many of these points are already explained in the original manuscript, but not clearly enough. The introduction is completely rewritten to articulate these points. In case of AI-produced refractivity, it is impossible for data assimilation to undo the vertical propagation of bending angle error. For bending angle data assimilation, the EIV problem makes it impossible to acquire the same result with AI. For instance, the data assimilation is unable to "retrieve" perfect refractivity out of error-free bending angle unless the provided background refractivity is initially perfect. Therefore, it is impossible to yield the same result with VR by assimilating bending angle or refractivity profile even with assigning proper observation error. The practical constraints of NWP models (limited top height and vertical resolution) are an additional issue. Note that RO data processing typically uses a significantly higher data resolution (number of layers in the order of thousands or more) and top height (2,000 km as described in the manuscript) for AI. Numerous Observing System Experiments have shown the positive impact of RO data (even for AI-produced RO refractivity), which cannot be attained by assimilating forecast profiles. In addition, the error estimation described in section 2.3 of the manuscript shows that AI is superior to ECMWF forecast in the tropospheric refractivity. It means that the information in VR mainly comes from RO bending angle rather than the forecast. Although it is not possible to eliminate the forecast influence completely, VR-produced refractivity is certainly better than the forecast in the quality.

Typos and grammatical changes: Multiple times in the paper, data assimilation(s) is used. The final "s" is not needed as it can already be considered plural. One could use data assimilation methods/systems is you wanted to add another word, but it is perfectly adequate to leave this out. For example: Page 1, line 13: In contrast to variational data assimilation, VR holds . . .

**Response)** Done. Thank you very much for pointing this out.

Page 1, line 17: . . . purposely corrupted synthetic sounding with a known true solution.

**Response)** Corrected.

Page 7, line 1: This differs from meteorological data assimilation of in-situ observations, where state variables are usually the same as those of the prediction model. — The original statement was not correct as currently the majority of observational data in meteorological data assimilation originates from satellite radiances which are not in state space, but need a forward operator similar to GNSS-RO. Please note my addition of "in-situ" but revise as you deem appropriate.

**Response)** The above-mentioned sentence does not state that the observed variables are of the same type with model (state) variables. The sentence explains that the state/control variable of VR (refractivity), instead of observed variable, is different from model variables. The control (state) variables in the data assimilation of satellite radiances are still a subset of model prognostic variables (temperature, moisture, surface pressure, winds, ...) and the forward operator (radiative transfer model) maps the state variables into the observation space. Assimilation of indirect variables does not necessitate any change of the state vector (i.e., constituent elements or structure).

Page 7, line 3: . . . the location of the state-vector elements is represented in relation . . .

**Response)** Changed. Thanks for this excellent suggestion.

Page 7, line 4: For this reason, . . .

**Response)** Done.

Page 7, line 33: The method attempts to separate forecast and observation errors from the variance of  $y - H(x)$ , using the assumption that . . .

**Response)** The method is able to separate the errors.

Page 9, line 4: Question, does the sampling rate of 1 second correspond to 4 meters throughout the entire occultation or just in the lower troposphere. Please clarify, “4 meters though the depth of the occultation” or “4 meters in the lower troposphere” as appropriate.

**Response)** The sampling rate is meant for radiosonde data. The balloon ascent rate is quite variable and depends on many factors (e.g., atmospheric stratification and vertical air motion). The ascent rate shows large sounding-to-sounding variability but does seem to have strong height dependency. After rechecking the radiosonde dataset, the ascent rate is now changed to “about 5 meters” and “on average” is added to indicate the high variability.

Page 10, line 8: On the other hand, regularization methods include the penalty term, which acts like a filtering and invokes a reverse . . .

**Response)** Changed. Thanks for this good suggestion.

Page 11, line 10: Note, the total cost function in complex systems often does not always monotonically decrease. It can occur but often requires aggressive pre-conditioning to be applied and appropriate conjugate gradient descent methods chosen. You may want to preface that you have seen this behavior which can be attributed to the small order and general simplicity of the problem.

**Response)** I agree that it is more difficult for minimization algorithms to find the steepest descent direction in large-scale problems. Subsequently, the cost function may not decrease as fast as shown in Fig. 4. However, the total cost function must decrease monotonically with iteration. Otherwise, the iteration simply terminates (although the cost function may increase in trial steps between iterations, the trial steps are not considered as a successful iteration unless a reduced cost function can be found). The sentence is rephrased to imply that the convergence speed can be dependent on the problem size. Your suggestion is greatly appreciated.

Page 12, line 1: . . . Monte Carlo approach is larger than . . .

**Response)** Corrected. Thank you.

Page 12, line 4: Data assimilation methods/systems are ORD.

**Response)** Changed.

Page 16, line 20: . . . RO and FCST is crucial to allow for VR to reduce the bias.

**Response)** As suggested, “though” is now deleted.

Page 18, line 10: ... observations on average at the tropical HVRRD stations than the tropical ORD stations.

**Response)** Thank you; “tropical” is added.

Page 18, line 12: At the tropical HVRRD stations, RO observations are assumed to be more accurate, while the (simulated refractivities?) from the model background are less reliable. – I did not follow the use of “background sounding” so please be explicit here.

**Response)** Your comment is well taken. The sentence has been rephrased to make it easy to understand. Thanks for pointing this out.

Figure 2: It would maybe help to have the “key” for the lines shown explicitly for figure 2a (true and measured), and figure 2b (smoothed and measured).

**Response)** Figure 2a-b are modified as your kind suggestion.

Figure 3: I believe the x-axis in Figure 3a should be “Climatological B [N,%]”

**Response)** Correct. Decided to use “statistical”. Thank you.

Figure 4: The purple and black lines are very hard to distinguish particularly in figure 4b.

**Response)** The blue (looks like purple somehow) line is now dashed. Thank you for this great suggestion.

Figure 8: Similar to figure 4, the black and purple lines in figure 8b are very hard to distinguish.

**Response)** The Figure is revised to improve the clarity.

Page 12, line 32: The slope is indeed the critical refractivity gradient, GC, . . . (This abbreviation was not defined, but subsequently used).

**Response)** It is now defined explicitly.

Page 13, line 24: . . . actual RO events and compare these results with . . .

**Response)** The sentence is reworded. Thank you.

Page 14, line 16: ... above the ECMWF model top up to a height of 2,000 km.

**Response)** Changed as your suggestion.

Page 15, line 20: . . . does not always deviate discernibly from

**Response)** Done. Thank you.