Date: October 4, 2017

Manuscript #:   amt-2017-228
Manuscript title: *A variational regularization of Abel transform for GPS radio occultation*

Brief Summary of the Manuscript
This manuscript proposes the variational regularization technique to estimate the atmospheric refractivity, given a-priori information from ECMWF. This proposed method is compared against radiosonde data and is applied in numerous GPS-RO retrievals. The advantage of this approach is that information about the error characteristics of the bending angle are also considered when estimating the refractivity, leading to better agreement with high resolution radiosondes at low altitudes. Despite the effort made, the manuscript in its current form is densely written, lacks motivation and objectivity, the methodology is not well placed within the context of the new approach, and the proposed technique is the 1-DVar that is routinely used in the retrieval of thermodynamic variables (except in this case is used to estimate the atmospheric refractivity). Sections of the paper are unnecessary and must be removed, because they provide textbook details that are beyond the scope of this study. The presentation of the results needs major re-writing with clear and concise explanation placed within the objective and motivation of this investigation. For examples, Section 2.1 can be reduced, Section 2.2 can be removed, Section 3.2 I do not understand its purpose, Section 3.3 the EIV issue is too detailed, Section 3.5 again too detailed and what purposed does it serve in this investigation, and so forth for the rest of the section. There are no references in the manuscript to back up the claims either. The proposed approach in the estimation of the atmospheric refractive index appears to introduce more effort and more error characterization than the usual AI approach and it also makes the refractivity more dependent on a-priori information. Given the above, I am afraid I cannot recommend this manuscript for publication in its present form.

Major Comments:

1) ***The introduction lacks motivation and objectivity and needs revising.*** The way the introduction is written does not establish the need for a new retrieval technique of the atmospheric refractivity and does not highlight the advantage(s) of the VR technique over the traditionally used Abel transform. Additionally, there are many unnecessary details on the VR that overwhelm the reader and overshadow any potential motivation and goals of this research. Proper referencing to the Abel transform limitations, other refractivity retrieval techniques, and the new VR method are also needed. What new information this VR could reveal over the Abel transform? Are the improvements within the Abel transform retrieval uncertainty and statistically significant?

2) ***The methodology lacks detailed explanation and is weak.*** The VR technique proposed in this study is practically a description of the 1-DVar assimilation approach, with the only difference being the assignment of the state variable (refractivity) onto the impact parameter space (unlike what is traditionally done in assimilation systems). Despite the fact that this may reduce the EIV problem, it is hardly a "breakthrough" as noted on Page 3 in Line 16 and the methodology lacks the following:
    a) Description of the error covariance matrix estimation
    b) How is the error covariance matrix for the forward model, **H(x)**, is defined in $\mathbf{R^{-1}}$?
    c) Page 9; Line 9: How is the "typical (climatic) size of the measurement" is defined?
    d) Page 9; Line 15: How sensitive is the bending angle error to the value of the parameter *L*?
    e) The forward modeled bending angle (**H(x)**) is estimated via Eq. (4), which is subject to the spherical symmetry approximation the author introduces as a limitation. Does this introduce additional retrieval errors to constraint through the VR?
    f) Page 7; Line 21: Since the initial guess matters, under what conditions can the author claim that the final solution does not depend on every detail on it? And what is meant by the "every detail"?
    g) How are the bending angle errors are defined in real GPS-RO measurements?
    h) What is the sensitivity of the VR to the initial guess from ECMWF?
    i) Page 16; Line 20: How does the "weighting" between observations and FCST is decided and

how sensitive this "weighting" is to the final solution?

j) Page 18; Lines 9–11: Again, how is the "weighting" being defined in the VR technique?

k) Page 19; Lines 6–7: Since the error carried forward into the refractivity, does not this imply that the forward model, **H(x)**, will also carry forward errors in the bending angle via the errors inherited through Eq. (4)?

l) Page 20; Line 28: The estimation of the atmospheric refractivity appears to depend highly on the information provided by the initial guess, in this case ECMWF.

m) Page 11; Line 10: How is the threshold of the cost function defined and how sensitive the final solution is to that threshold?

3) ***The results provide a marginal contribution to the state-of-the-art AI method of retrieving the atmospheric refractivity.*** Looking at Figure 4b, the refractivity error of the AI and of the VR methods falls within the retrieval uncertainty of the refractivity and is < 1.0% between the two. Is the difference between the black and the red lines statistically significant? The final solution also seems to depend heavily on the initial conditions (Page 20; Line 28), yet on Page 7 Line 21 the opposite is claimed, and the advantages of using the VR technique to estimate the refractivity are not clear through this presentation.

Minor Comments:

a) **Line 5:** It should read: "In Radio Occultations (ROs), the refractivity is obtained from..."

b) **Line 6:** It should read: "...AI is primarily susceptible to..." The only reason I want to see the word "primarily" in this sentence is because there are secondary mathematical issues that also limit the accuracy of the Abel transform.

c) **Line 15:** How do you define "... known true solution... "

d) **Lines 23-24:** It should read: "Knowledge of the refractive index vertical structure in the..."

e) **Line 26:** Clarify what you mean by "flaws as well as signal components" and put a comma after the word "flaws"

f) **Page 2; Line 1:** It should read: "... the atmosphere is one of the sources of..."

g) **Page 2; Line 1:** I disagree that the non-spherical symmetry of the Earth's atmosphere qualifies as a bending angle measurement error. Bending angle is retrieved by the phase measurement; so I consider the non-spherical symmetry as a retrieval error, because the phase and amplitude of the GNSS signals measured at the LEO antenna are not affected by the spherical symmetry. Please, revise accordingly.

h) **Page 2; Line 6:** Perhaps, change the word "measurement" with the word "retrieval"?

i) **Page 2; Line 33-34:** It should read: "... to realistically estimate the uncertainties of RO measurement and *a-priori* and properly take them into account."

j) **Page 3; Line 1:** Clarify what you mean by "reliable method"

k) **Page 3; Line 1:** Remove the word "they"

l) **Page 3; Line 2:** Add a recent reference regarding this statement.

m) **Page 3; Line 16:** I would remove the word "breakthrough" and re-write the sentence as: "This necessitates the implementation of more rigorous techniques that can potentially improve the quality of the refractive index..."

n) **Page 3; Lines 17-18:** It should read: "This study explores this possibility through the VR technique."

o) **Page 3; Line 20:** It should read: "In Section 4, a real data validation..."

p) **Page 3; Line 21:** It should read: "Section 5."

q) **Page 3; Line 24:** It should read: "... and a receiver..."

r) **Page 4; Line 5:** It should read: "... to be unstable."

s) **Page 4; Line 17:** It should read: "... operator in the RHS, in addition..."
t) **Page 4; Line 20:** Add a reference.
u) **Page 4; Line 27:** It should read: "... of the transmitted radiowave signal."
v) **Page 9; Line 18:** It should read: "Noise in the measured bending angle negatively affects the quality of the refractive index, unless properly mitigated."
w) **Page 15; Line 10:** Explain what you mean by "soundings that are dubious in quality are discarded".
x) **Page 16; Line 15:** Add a reference to back up this statement.
y) **Page 16; Line 28**: It should read: "... (especially in moisture)..."
z) **Page 17; Line 29**: It should read: "noise"
aa) **Page 18**; **Line 27**: Place a period instead of a comma after the word HVRRD.