Answer to reviewers

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We thank both reviewers for their constructive and relevant comments and suggestions. We have taken into account all the suggested points and we think that the revised version of the paper has gained considerably in clarity and accuracy. Our answers to the reviewers' comments (which are in italic) are shown in black regular font, the new additions or modifications in the revised paper are shown in blue font.

Anonymous Reviewer #1

Specific comments

1. *Page 14, line 11 ff: The exact definition of the scattering matrix elements s which relate the incident and scattered $\vec{E}$ field as function of direction (which angles?) remains somewhat unclear, which is not uncommon in the literature. However, I would find it useful to see the exact equation and a sketch defining the scattering angles. Also, which sign convention for the imaginary part of the refractive index of the scatterers is applied?*

   We have added some explanation and a sketch about the scattering matrix formalism at the very beginning of Section 3.5.

   The mathematical formulation of the radar observables involves the *scattering matrix* $\mathbf{S}$, which relates the scattered electric field $\mathbf{E^s}$ to the incident electric field $\mathbf{E^i}$ (Bringi and Chandrasekar, 2001) for a given scattering angle.

$$\begin{bmatrix} E_h^s \\ E_v^s \end{bmatrix} = \frac{e^{-ik_0 r}}{r} \mathbf{S}_{\text{FSA}} \begin{bmatrix} E_h^i \\ E_v^i \end{bmatrix} \quad (1)$$

   where k_0 is the wave number of free space ($k_0 = 2\pi/\lambda$).

   The scattering matrix $\mathbf{S}_{\text{FSA}}$ is a 2×2 matrix of complex numbers in units of m^{-1} (e.g., Bringi and Chandrasekar, 2001; Doviak and Zrni, 2006; Mishchenko et al., 2002).

$$\mathbf{S}_{\text{FSA}} = \begin{bmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{bmatrix}_{\text{FSA}} \quad (2)$$

   The FSA subscript indicates the forward scattering alignment convention, in which the positive z-axis is in the same direction as the travel of the wave (for both the incident and scattered wave). A sketch illustrating the reference unit vectors for the scattered wave in the FSA convention is given in Figure 5.

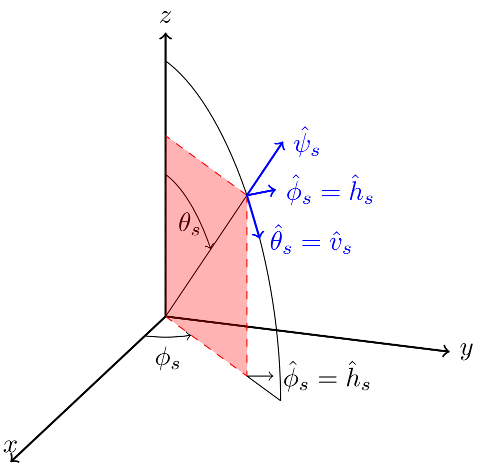

Figure 5: The direction of the far-field scattered wave is given by the spherical angles θ_s and ϕ_s, or by the unit vector $\hat{\psi}_s$. In the FSA convention, the horizontal and vertical unit vectors are defined as $\hat{h}_s = \hat{\phi}_s$ and $\hat{v}_s = \hat{\phi}_s$. The unit vectors for the spherical coordinate system form the triplet $(\hat{\psi}_s, \hat{\theta}_s, \hat{\phi}_s)$, which in the FSA convention becomes $(\hat{\psi}_s, \hat{v}_s, \hat{h}_s)$, with $\hat{\psi}_s = \hat{v}_s \times \hat{h}_s$. This figure was adapted from Bringi and Chandrasekar (2001).

We have also added an explanation on the permittivity at the beginning of Section 3.6.

In the following, the term (complex) *permittivity* will be used for the relative dielectric constant of a given material. It is defined by:

$$\epsilon = \epsilon' + i\epsilon'' \quad (3)$$

where ϵ' is the real part, related to the phase velocity of the propagated wave, and ϵ'' is the imaginary part, related to the absorption of the incident wave.

2. *Maxwell-Garnett is only one of many known Effective Medium Approximations, and Eq. (13) is the special case of a 2-component mixture (n-component mixture see Bohren and Huffman (1983)) where small ice spheres are suspended in air (matrix). You could mention some alternative formulations from the literature (see Blahak (2016) for a summary) but stating that, if none of the components is a strong dielectric, all these formulas approximately agree to first order (Bohren and Huffman (1983)). This will become more important later in your section 3.7.2, Permittivity*

   Thanks, we have added some discussion regarding this point in the paragraph following Eq. 13:

   Note that other EMAs exist, such as the Bruggemann (1935) and Oguchi (1983) approximations. If none of the components is a strong dielectric, all these EMAs approximately agree to first order (Bohren and Huffman, 1983). The interested reader is referred to Blahak (2016), for an intercomparison of these EMA in the context of simulated reflectivity fields.

   Please see also the answer to point 5 for more general modifications to this section.

3. *Your Eq. (10) is wrong, because the argument of log should be dimensionless. Did you mean something like*

$$Z_H(r_g) < S(r_0) + G + SNR_{thr} + 20 \cdot log_{10}\left(\frac{r_g}{r_0}\right) \quad (4)$$

   where $S(r_0)$ is the sensitivity at a certain reference range r_0. Please review this Equation. Is this just a "typo" or are your results affected?

   We agree that the distance in the log should be normalized to yield a dimensionless argument. This is why we modified this Equation as recommended, by introducing a reference distance r_0.

The received power at the radar antenna decreases with the square of the range, which leads to a decrease of signal-to-noise ratio (SNR) with the distance. To take into account this effect, all simulated radar variables at range r_g are censored if:

$$Z_{\mathrm{H}}(r_g) < S + G + \mathrm{SNR}_{\mathrm{thr}} + 20 \cdot \log_{10}\left(\frac{r_g}{r_0}\right) \tag{5}$$

where G is the overall radar gain in dBm, S is the radar antenna sensitivity in dBm, Z_{H} is the horizontal reflectivity factor in dBZ, and $\mathrm{SNR}_{\mathrm{thr}}$ corresponds to the desired signal-to-noise threshold in dB (typically 8 dB in the following). r_0 is a distance used to normalize the argument of the logarithm. If all units are consistent and SI based then $r_0 = 1$ m.

Concerning S, it is related to the sensitivity of the receiver, so to a certain extent S is defined at a reference distance of 0, this is why we define it as a constant.

4. *Please define D: melted diameter or actual diameter of a melting particle?*

   It is the the maximum dimension of a melting particle. We have added this information in the text following the equation:

   The considered diameter D is the actual maximum dimension of a melting particle, and not the melted diameter.

5. *The description of your applied EMA is too short and omits necessary detail: Please give the exact formula of ϵ_{eff} that you applied for partially melted particles. Describe the role of the air inclusions. Note that there is an ncomponent version of Maxwell-Garnett given in Bohren and Huffman (1983), as well as a variant that assumes spheroidal inclusions instead of spherical inclusions in the matrix medium. Note also that there are other EMAs on the market, derived under different assumptions on the internal melting morphology and it is not clear which one is best. This might also depend on the specific radar observable under consideration and is really hard to determine. Definition of ρ_{total} ? Also, please illustrate in a new figure the typical dependence of the mass fraction of water $f_{water} = m_{water}/m_{ice}$ for single particles as function of D and f^m_{wet}, as derived from your Eq. (24) together with (21). This will shed more light on your implicit assumptions about the distribution of melt water among the particle sizes for given average degree of melting.*

   We thank Reviewer 1 for this comment. It is quite true that this part had to be improved. We have rewritten the beginning of Section 3.6.2 (paragraph: Snow, graupel, hail and ice crystals) by starting from the general Maxwell-Garnett EMA before introducing the simpler form used for dry solid hydrometeors:

   The permittivity of composite materials, such as snow, which consists of a mixture of air and ice, can be estimated with a so-called Effective Medium Approximation (EMA). A well known EMA is the Maxwell-Garnett approximation (Bohren and Huffman, 1983), in which the effective medium consists of a matrix medium with permittivity ϵ^{mat} and inclusions with permittivity ϵ^{inc}:

$$\epsilon_{\mathrm{eff}} = \epsilon^{\mathrm{mat}}\left(\frac{1 + 2 f^{\mathrm{inc}}_{\mathrm{vol}} \frac{\epsilon^{\mathrm{inc}} - \epsilon^{\mathrm{mat}}}{\epsilon^{\mathrm{inc}} + 2\epsilon^{\mathrm{mat}}}}{1 - f^{\mathrm{inc}}_{\mathrm{vol}} \frac{\epsilon^{\mathrm{inc}} - \epsilon^{\mathrm{mat}}}{\epsilon^{\mathrm{inc}} + 2\epsilon^{\mathrm{mat}}}}\right) \tag{21}$$

   where ϵ_{eff} is the effective permittivity of the composite material, and $f^{\mathrm{inc}}_{\mathrm{vol}}$ is the volume fraction of the inclusions.

   Note that other EMAs exist, such as the Bruggemann (1935) and Oguchi (1983) approximations. If none of the components is a strong dielectric, all these EMAs approximately agree to first order (Bohren and Huffman, 1983). The interested reader is referred to Blahak (2016), for an intercomparison of these EMA in the context of simulated reflectivity fields.

   Dry solid hydrometeors consist of inclusions of ice in a matrix of air. In this case $\epsilon_{\mathrm{mat}} \approx 1$, which leads to a simplified form of the mixing formula (e.g., Ryzhkov et al. 2011).

$$\epsilon^{(j)} = \frac{1 + 2 f^{\mathrm{ice}}_{\mathrm{vol}} \frac{\epsilon^{\mathrm{ice}} - 1}{\epsilon^{\mathrm{ice}} + 2}}{1 - f^{\mathrm{ice}}_{\mathrm{vol}} \frac{\epsilon^{\mathrm{ice}} - 1}{\epsilon^{\mathrm{ice}} + 2}} \tag{22}$$

The rest of the section is unmodified. We have also added some information about how we compute the permittivity of melting hydrometeors in Section 3.7.2. Note that ρ^{total}, was somewhat of a typo, as it should have been ρ^{water} instead. We have fixed this in Equation 30 and we have added the mathematical expression for ρ^m, the density of the melting hydrometeor.

In Equation 18, we have previously introduced the general two-component Maxwell-Garnett EMA. However, melting hydrometeors are a mixture of three components: water, ice, and air. To compute their permittivity, the general two-component formulation is used recursively, first to derive the permittivity of dry snow (as was done previously for dry snow, graupel, hail and ice crystals), and then the permittivity of the dry snow and water mixture.

The necessary volume fractions of all components f_{vol} can again be estimated with the mass-diameter model:

$$f_{\text{vol}}^{\text{water}} = f_{\text{wet}}^m \frac{\rho^m}{\rho^{\text{water}}} \tag{34}$$

$$f_{\text{vol}}^{\text{ice}} = \frac{\rho^m - f_{\text{vol}}^{\text{water}} \rho^{\text{water}}}{\rho^{\text{ice}}} \tag{35}$$

$$f_{\text{vol}}^{\text{air}} = 1 - f_{\text{vol}}^{\text{water}} - f_{\text{vol}}^{\text{ice}} \tag{36}$$

$$\tag{37}$$

where $\rho^m = \frac{m^m(D)}{\pi/6D^3}$ is the density of the melting hydrometeor.

In a first step, Equation 21 is used with $f_{\text{vol}}^{\text{inc}} = \frac{f_{\text{vol}}^{\text{ice}}}{f_{\text{vol}}^{\text{ice}}+f_{\text{vol}}^{\text{air}}}$, $\epsilon^{\text{mat}} \approx 1$, $\epsilon^{\text{inc}} = \epsilon^{\text{ice}}$, to yield ϵ^{d}, the permittivity of the dry part of the melting hydrometeor. For the second step, however, the estimated permittivity of the melting hydrometeor will depend on whether water is treated as the matrix and snow as the inclusions or the opposite, giving two different possible outcomes. To overcome this issue, a formulation proposed by Meneghini and Liao (1996) is used, where the final permittivity is a weighted sum of both permittivities and where the weights are function of the wet fraction. This method is also used by Ryzhkov et al. (2011). Precisely, Equation 18 is used first with $f_{\text{vol}}^{\text{inc}} = f_{\text{vol}}^{\text{water}}$ and $\epsilon^{\text{mat}} = \epsilon^{\text{d}}$, $\epsilon^{\text{inc}} = \epsilon^{\text{water}}$, to yield $\epsilon^{\text{m},(1)}$, and at second with $f_{\text{vol}}^{\text{inc}} = f_{\text{vol}}^{\text{air}} + f_{\text{vol}}^{\text{ice}}$ and $\epsilon^{\text{mat}} = \epsilon^{\text{water}}$, $\epsilon^{\text{inc}} = \epsilon^d$, to yield $\epsilon^{\text{m},(2)}$. The final ϵ^{m} is a weighted sum of $\epsilon^{\text{m},(1)}$ and $\epsilon^{\text{m},(2)}$:

$$\epsilon^{\text{m}} = \frac{1}{2}\left[(1+\tau)\epsilon^{\text{m},(1)} + (1-\tau)\epsilon^{\text{m},(2)}\right] \tag{38}$$

where parameter τ is a function of f_{wet}^m:

$$\tau = \text{Erf}\left(2\frac{1-f_{\text{wet}}^m}{f_{\text{wet}}^m} - 1\right) \quad \text{if } f_{\text{wet}}^m > 0.01, \tag{39}$$

We think that this revised version better describes our approach. The dependency of $f_{\text{vol}}^{\text{water}}$ on the diameter D and the wet fraction f_{wet}^m is illustrated in Figure 6. It basically shows that there is a roughly polynomial increase in the volume fraction with the wet fraction for a fixed diameter, which can be expected. In terms of diameter dependence, the relation is more complex. It can seem surprising, that, for a fixed wet fraction, $f_{\text{vol}}^{\text{water}}$ increases with the diameter for graupel, but decreases for snow. It is easy though to figure out though that this is caused by the mass-diameter relations of dry snow and graupel: dry snow has a power of 2 in its power-law, whereas graupel has a power of 3.1. This implies that dry graupel becomes denser with the size, whereas dry snow becomes less dense (because 3.1 > 3 > 2). Since melting hydrometeors depend on the density of water (which is constant) and their dry counterparts, the same trend can be found for melting hydrometeors.

6. *In contrast to your Eq. (28), in the original literature Szyrmer and Zawadzki (1999) the equation reads*

$$N^r(D_r)v_t(D_r) = N^m(D)v_t^m(D)$$

[...]. I see two possible ways forward: (a) change your computation of $N_m(D)$ using the correct transformation for the one-to-one-correspondence, or (b) keep your parameterization but discuss the implicitly contained shedding/aggregation parameterization somehow. [..]. Because $N_m(D)$ through the melting layer is extrapolated from the rain DSD at the bottom, the transition to the dry snow PSD just above the melting layer is not continuous. How large is the jump?

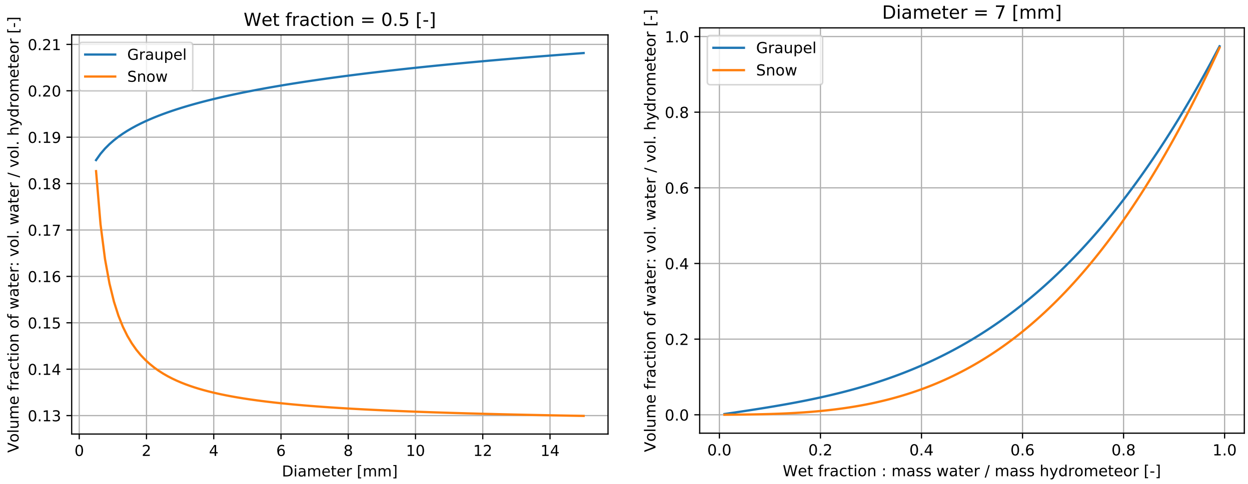

Figure 6: Dependency of melting graupel and snow $f_{\text{vol}}^{\text{water}}$ on the diameter (top) and wet fraction (bottom).

Thanks to Reviewer 1 for pointing this out. Indeed there has been some misinterpretation of Zawadzki's paper. Hence, we have decided to improve this part in the following way: we have first corrected the approach based on Szyrmer and Zawadzki (1999), by talking into account Reviewer 1's corrections. By closer visual inspection, we realized that, indeed, there is a jump between the dry snow PSD and the PSD of melting snow when $f_{\text{wet}} = 0$ and it is quite large. It can lead to an unrealistic sharp drop of Z_{H} of several dBZ over one or two radar gates, above the melting layer. We have then added another approach based on a simple empirical weighting between the DSD of raindrops and the PSD of dry solid hydrometeors (snow/graupel). We have then compared these two approaches, and it seems the second ones performs better in comparison with radar data, and it also allows for a seamless transition between the PSD of melting hydrometeors and the PSD of dry hydrometeors. In the end, the second approach is favored.

Since this new part is quite long, we do not copy it here, but we would like to refer the reviewers to the new Section 3.7.3 in the revised paper.

7. *The numeric representation of the convolution with a Gaussian kernel in Eq. (39) is wrong. To correct, do either: [...]. Also, you have to divide by the sum of the Gaussian weights!*

We thank Reviewer 1 for pointing this error out. Fortunately, this is just an error in the text, as in the code, we used the *convolve* function from the well-known Python *scipy* library. We fixed the equation according to your second proposition:

$$S^{\text{corr}}[i] = \sum_{j=0}^{N_{\text{FFT}}} S[i-j] \frac{1}{\sigma_{t+\alpha}\sqrt{2\pi}} \exp\left[-\frac{(v_{\text{rad,bins}}[j])^2}{2\sigma_{t+\alpha}^2}\right] \tag{52}$$

where $\sigma_{t+\alpha} = \sigma_t + \sigma_\alpha$

However, we think that there is no need to divide by the sum of the Gaussian kernel because this sum should be one, indeed, the term $\frac{1}{\sigma_{t+\alpha}\sqrt{2\pi}}$ yields a normalized Gaussian kernel.

8. *This section should perhaps be better named attenuation computation instead of correction, because the latter is usually used to denote the inverse procedure applied to observations. Also, the attenuation computation given in Eqs. (40) to (42) is wrong. According to Lambert-Beers law, attenuation in the space of linear reflectivities (such as your reflectivity S) is given by [...]. Please correct also the text of this section accordingly and recompute the data of your figure 14.*

Reviewer 1 is right and we thank him for this correction. We somehow got quite confused, and ended up solving a uselessly difficult problem...This derivation as well as the title have been changed in the text. Note that k_h (one-way attenuation in linear units) is used now (as can be seen in the new appendix C2):

In reality, attenuation will cause a decrease in observed radar reflectivities at all velocity bins within the spectrum. To take into account this effect, the path integrated attenuation in linear units at a given radar gate (k_h in Equations C2) is distributed uniformly throughout the spectrum.

$$S(r_g, \phi_g, \theta_g)^{\text{att}}[i] = S(r_g, \phi_g, \theta_g)[i] \cdot \exp\left(-2 \int_{r=0}^{r_g} k_h(r, \theta_g, \phi_g)\, dr\right) \tag{53}$$

Please note that the appendix C has changed, and a single-way linear attenuation k_h is used instead, which explains why there is still a factor of 2 in the equation above. This modification leads to a slightly different Figure 14 (please see the revised paper), which does not change the conclusions that are being drawn. Note that Figure 14 also shows some differences in SNR censoring of the radar data. This is because, while recomputing the plot, we realized that we were not being consistent with the SNR threshold of 8 dB, that was mentioned in the paper (a smaller value was used). Now this is fixed as well.

9. *Page 34, line 1: While I agree with the findings of the DSD-comparison in this special case, the well-known general difficulties of such comparisons (vastly different sampling volumes, shapes of normalized spectra strongly depend on rain rate) should be discussed a bit more and why their influence is presumably small in this case. Also, whether or not to use this improved shape parameter value in the forward operator instead of the microphysics-consistent value depends on the application (model verification vs. data assimilation). Applying it in the model microphysics may be a good idea, but without re-tuning other parameters in the model, one might end up with a degradation of the surface precipitation, because one of the compensating errors has been taken away.*

   Thanks to Reviewer 1 for providing this complement of information. We have added the recommended remarks in text, at the end of the paragraph:

   However, one must keep in mind the numerous difficulties in the comparison of these DSDs. First of all, the sampling volumes are vastly different (around 80 millions of cubic meters for the COSMO grid cell, around 10000 cubic meters for the three Parsivels integrated over a time interval of 5 minutes and averaged over 520 of these time intervals. Secondly, the shape of the DSDs depend strongly on the simulated precipitation intensity which is not always agreeing with observations (rain gauges). Regarding the first point, giving the large homogeneity of the studied precipitation events (widespread stratiform rain), the representativity issue comparison still has some relevance. Concerning the second point, since precipitation intensity is a moment of the DSD, one can expect a better agreement with Parsivel observations with more realistic COSMO microphysics, especially for larger particles.

   As conclusion, changing the shape parameter in the COSMO microphysics is a delicate task, as without re-tuning other parameters in the model, it might lead, *in fine*, to a degradation of the surface precipitation. Using it solely off-line in the context of the forward radar operator might be a better choice, as it can help to reduce the bias in simulated polarimetric variables.

Technical corrections

1. *Since COSMO 5.1, ice sedimentation is also taken into account in the 1-moment schemes.*

   Thanks to Reviewer 1 for providing this correction, we have added the following details in the text:

   In terms of terminal velocities, in the version of COSMO that is being used (5.04), neither ice crystals nor cloud droplets are sedimentating. In more recent versions (starting from 5.1) however, ice crystals have a bulk non-diameter dependent terminal velocity, that depends on their mass concentration.

2. *Page 4, line 19: Add two more references for the 2-moment scheme, because the addition of the separate hail class came after Seifert (2006): Blahak (2008), Noppel et al. (2010)*

   We have added these references in the corresponding sentence.

   A more advanced two-moment scheme with a sixth hydrometeor category, hail, was developed for COSMO by Seifert and Beheng (2006) and extended by Blahak (2008) and Noppel et al. (2010).

3. *Page 5, line 6 (Table 1): N_0 Rain: missing free after 2529. Also check the value 2529 (which units???), because the N_0-μ-relation of Ulbrich (1983) is applied, with a base value of 8000 $m^3 mm^1$ for $\mu = 0$ and increasing with increasing μ. Specify the units of N_0 in the table caption.*

   We fixed the term "free" after "2529", and performed several modifications (see point 3 of Reviewer 2). We also checked the value 2529, and this value is clearly false. Fortunately, it was just a typo. We checked the code, and the value used there is 1253 $\mathrm{m}^{-3}\mathrm{mm}^{-1-\mu}$. To get this value, we used the formulas defined in the following document: `http://www.cosmo-model.org/content/model/releases/histories/cosmo_4.21.htm`. They specify that

   $$N_0 = \mathrm{rain\_n0\_factor} \cdot N_{00} \exp(3.2\mu)$$

   Note that in the document it is written lambda, but we expect it to be mu, it doesn't make sense otherwise (this document is a bit confusing). Also $N_{00} = 8e6$ m^{-4}. With $\mu = 0.5$ and rain_n0_factor $= 1.0$, which is the value we used and it is the default used by MeteoSwiss, this gives $N_0 = 39624259.39$, which must be in units of $\mathrm{m}^{-4-\mu}$. However we prefer to have it units of $\mathrm{m}^{-3}\mathrm{mm}^{-1-\mu}$, because we work with diameters in mm, so we divide by $1000^{1+\mu}$ and this gives 1253.03 $\mathrm{m}^{-3}\mathrm{mm}^{1-\mu}$. We added the units in the description of the table.

4. *Page 10, line 3: $\frac{dn}{dh}$ = const, not cst*

   This has been fixed, thanks.

5. *Change mathematical presentation of your formulas (10) and (11). To reflect that in your ansatz the parameters of the Normal- and generalized gamma distribution depend on diameter D, you dont have to use the awkward superscripts. In the second formula, I think a_r has to be replaced by $1/a_r$, if I look at your Figure 5 and if Im not mistaken: [...]. You can eliminate the offset l from the formula and text. Just set it to 1.*

   The equations have been corrected and edited according to the Reviewer 1's recommendations:

$$o : g_o(o, D) = \mathcal{N}(0, \sigma_o(D)) \tag{15}$$

$$\frac{1}{a_r} : g_{1/a_r}(1/a_r, D) = \frac{(\frac{1}{a_r} - 1)^{\Lambda_{a_r}(D)-1} \exp\left(-\frac{\frac{1}{a_r}-1}{M(D)}\right)}{M(D)^{\Lambda_{a_r}(D)} \Gamma(\Lambda_{a_r}(D))} \tag{16}$$

where Λ_{a_r} and M are the *shape* and *scale* parameters of the gamma aspect-ratio probability density function and σ_o is the *standard deviation* of the Gaussian canting angle distribution. These parameters depend on the diameter D. Technically Λ, M and σ_o have been fitted separately for each single diameter bin of MASC, then their dependence on D has been fitted by power-laws for each parameter, which also allows further integration over the canting angle and aspect-ratio distributions for all particle sizes. Note also that the gamma distribution is rescaled with a constant shift of 1, to account for the fact that the smallest possible inverse of aspect-ratio is 1 and not 0.

$$\begin{aligned} \sigma_o(D) &= 58.07\ D^{-0.11} \quad [°] \\ \Lambda_{a_r}(D) &= 6.33\ D^{-0.4} \quad [-] \\ M(D) &= 0.06\ D^{-0.71} \quad [-] \end{aligned} \tag{17}$$

Note that using the properties of the inverse distribution, the distribution of aspect-ratios can easily be obtained from the distributions of their inverses:

$$g_{a_r}(a_r, D) = \frac{1}{a_r^2} g_{1/a_r}(1/a_r, D) \tag{18}$$

6. *Delete the sentence starting with The superscript [D] . . . . In the next sentence, correct. . . constant factor $l = 1$, . . . $\Rightarrow$ constant shift of 1, ... Replace also the next sentence The relationship . . . by These parameters depend on the diameter D. Technically, Λ_{a_r}, M and σ_o first have been fitted separately for each single diameter bin of MASC, then their dependence on D has been fitted by power laws for each parameter, At this point, you can insert the power laws from Figure 5 as equations in the text, they deserve it! When you do so, please indicate all units. Then continue with Note that these power laws allow to estimate the parameters for any arbitrary maximum diameter. This also allows integration over the canting angle...*

   Thanks to Reviewer 1 for this comment, we have adapted this part of the paper and added the numerical expression for the best-fits, please see the previous point.

7. *Page 20, line 23: Homogenize notation of the probability density functions $p(\beta)$, $p(a_r)$ with Eq. (10) and (11)*

   Thanks, we have fixed these equations and their description:

$$C^{b,(j)}(D) = \frac{1}{2\pi} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_0^1 c^{b,(j)}(D, a_r, \alpha, o)\ \cos(o)\ g_o(o, D)\ g_{a_r}(a_r, D)\ d\alpha\ do\ da_r \tag{23}$$

   And for rain and hail, where a_r is constant for a given diameter:

$$C^{b,(j)}(D) = \frac{1}{2\pi} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} c^{b,(j)}(D, \alpha, o)\ \cos(o)\ g_o(o, D)\ d\alpha\ do \tag{24}$$

   where $c^{b,(j)}(D, \alpha, o)$ are the scattering properties for a fixed diameter, canting angle o and yaw Euler angle (azimuthal orientation) α. $g_o(o)$ and g_{a_r} are the probabilities of o and a_r for a given diameter D as obtained from Equations 15 and 18. Note that the final scattering properties are averaged over all azimuthal angles α, which are all considered to be equiprobable. The $\cos(o)$ in the equation is the *surface element* which arises from the fact that the integration over α and o is a surface integration in spherical coordinates. The procedure for S^f is exactly the same.

8. *Page 27, line 21: Missing backslash in front of sigmaθ.*

   Fixed, thanks.

9. *Sentence is garbled, delete will be performed.*

   We have corrected the sentence according to the suggestion.

10. *Page 37, line 21: "$(x^{rot}, y^{rot}, z^{rot})$" the same as (x_m, y_m, z_m) ?*

    Yes, thanks to Reviewer 1. for pointing this out. We have changed all superscripts $^{\mathrm{rot}}$ to the subscript $_m$, in order to be consistent.

11. *Page 41, line 1 and 2: The factor 2 has to be removed from the equations because k_H and k_V are already two-way attenuation coefficients. And the + sign should be .*

    Please note that this part of the appendix has been rewritten to be more mathematically correct. The attenuation is now considered in linear Z units, so it becomes a multiplication and not an addition.