

## ***Interactive comment on “Atmospheric bending effects in GNSS tomography” by Gregor Möller and Daniel Landskron***

**Gregor Möller and Daniel Landskron**

gregor.moeller@tuwien.ac.at

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We would like to thank referee #2 for his/her valuable suggestions. In the following, you will find our responses, separately for each comment/concern. We are confident, that we can provide a revised version of the manuscript, which addresses all of your points of the first review and looking forward to your positive feedback.

Comment: The paper outlines a method for including ray bending in GNSS tomography. My main concern relates to the iterative retrieval technique, and the possible confusion between "a priori" and "first guess".

Author's response: In the revised version of the manuscript, the terms "a priori" and "first guess" will be used according to its definition in 'Inverse Methods for Atmospheric

C1

Sounding Theory and Practice' by Clive Rogers, section 5.6.2 as "the best estimate of the state before the measurement is made" (a priori) and as 'the starting point of an iteration' (first guess).

Comment: Equation 7 should not be used in iterative form, and therefore a solution from eq. 7 should not be used as "a priori" for the next iteration (lines 7-8, page 7). This is covered in section 5.6.2 ("A popular mistake") in Inverse Methods for Atmospheric Sounding Theory and Practice by Clive Rogers. I suggest that this issue should be clarified before moving to the discussion phase, because it impacts all of the results.

Author's response: We agree, by using the result of a previous iteration as first guess for the next iteration provides in a least squares sense not an optimal estimation. In consequence, we adapted our approach slightly – following the non-linear iterative approach suggested by Iyer and Hirahara, Seismic tomography: Theory and practice, 1993. Therefore, we keep the initial a priori field as first guess for each iteration but use the result of each iteration for improving the reconstruction of the signal paths. Therewith, and according to Fermat's principle (first order changes of the ray path lead to second order changes in travel time) the estimation error can be significantly reduced. Besides, the aim should be always to make use of the best available a priori field, especially if low elevation observations are involved. If this is not possible, the proposed non-linear iterative approach will allow for a more exact reconstruction of the signal paths, even if the used a priori refractivity field deviates significantly from the true atmospheric conditions.

Author's changes in manuscript: Due to the adaption in the tomography approach, several parts of the manuscript will be revised. This includes: Section 3.2.1 The a priori refractivity field, page 7 line 7-14, Section 4 Impact of atmospheric bending on the tomography solution, page 10 line 12-22, Section 4.1 Validation with radiosonde data, page 12 line 1-3, Section 5 Conclusions, page 12 line 6-7, page 13 line 4-7 and line 16-18 and Figure 3, 7 and 8.

C2

Minor points Comment: I do not understand why the authors use singular value decomposition in Eq. 7. Normal matrix inversion should suffice.

Author's response: Solving Eq. (6) for  $Nw$  requires the inversion of matrix  $A$ . In GNSS tomography, matrix  $A$  is mostly singular; in consequence, a straightforward inversion is not possible. Thus, for our solution we make use of singular value decomposition methods for solving the ill-posed inversion problem. Together with a proper singular value selection method like L-curve technique (see Moeller, 2017), it allows for solving the equation system and for retrieving as much signal as possible from the observations without introducing too large artefacts.

Author's changes in manuscript: We will add a more detailed explanation of our approach, in particular why we used SVD, to the revised version of the manuscript below Eq. 7.

Comment: The meaning of the weighting matrices should be explained (they are the inverse of covariance matrices I think) and they should be clearly defined.

Author's response: The weighting matrices  $P$  and  $Pc$  are the inverse of the variance-covariance matrices  $C$  and  $Cc$ . They are defined separately for the SWDs, i.e. the observations ( $C$ ) and the first guess ( $Cc$ ). The diagonal elements of  $C$  were computed as function of elevation angle:  $\sin(e)^2 \cdot \text{sig\_ZTD}^2$ , whereby  $\text{sig\_ZTD} = 2.5$  mm reflects the accuracy of the estimated zenith total delays. The diagonal elements of  $Cc$  were derived from a weighting model based on height-dependent error curves for pressure, temperature and specific humidity in form of standard deviations (see Steiner A. K. et al., Error characteristics of refractivity profiles retrieved from CHAMP radio occultation data, 2006). Both error matrices were introduced into the tomography solution for proper weighting of the individual observations against the first guess.

Author's changes in manuscript: An explanation of the weighting method will be added to Chapter 1 after Eq. 7.

### C3

Comment: The work might benefit from investigating how ray-bending is handled in GPS radio occultation measurements. EG, Burrows, C. P., Healy, S. B., and Culverwell, I. D.: Improving the bias characteristics of the ROPP refractivity and bending angle operators, *Atmos. Meas. Tech.*, 7, 3445-3458, <https://doi.org/10.5194/amt-7-3445-2014>, 2014.

Author's response: According to the reviewer's suggestion, we did a literature study on ray-tracing methods, in particular for assimilation of radio occultation measurements into numerical weather prediction systems.

While wave-theoretic approaches can help to increase the vertical resolution of the refractivity profile derived from radio occultation measurements, nowadays approaches based on the principles of geometric optics are still valid for operational analysis of radio occultation measurements. The main reason is that necessary assumption in signal processing, like the symmetric assumption or limitations in the GPS signal structure are still the dominating factors (Melbourne W. G., Radio Occultation Using Earth Satellites: A Wave Theory Treatment, 2004). Thus, and for highest consistency, also ray-tracing approaches based on the principles of geometric optics are widely used for reconstruction of the signal geometry, especially for assimilation of radio occultation measurements.

In contrast to radio occultation, the analysis of ground-based observations, including the weighting of ground-based observations in GNSS data processing, is based on different geometrical parameters. While in radio occultation, the bending angle is described as function of impact parameter  $a$ , for ground-based observations, elevation and azimuth angle are used for characterizing the observation geometry. In consequence, the optimal ray-tracing approach for ground-based observation and its conversion differs from approaches used for radio occultation measurements.

Nevertheless, in both cases the quality of the a priori field, but also how refractivity is interpolated between the given model levels, will have an impact on the ray-traced

### C4

signal paths. Latter is clearly shown in the paper of Burrows C. P. et al. (2014), as suggested by the reviewer.

In the best case, the interpolation between model levels is carried out on state parameter level, i.e. separately for pressure, temperature and water vapour pressure as obtained from the a priori field. However, since this approach is not applicable in iterative tomography processing, we assume exponentially varying refractivity. We are aware, that thereby the bending angle is slightly underdetermined. Nevertheless, in relation to the total bending effect, its contribution is fractional. Thus, we will not further modify our ray-tracing approach but we will address this affect in section '5 Conclusions'.

Author's changes in manuscript: In the manuscript, we will add a brief review of existing ray-tracing approaches in section '3 Reconstruction of GNSS signal paths' and will highlight the differences between radio occultation and ground-based observations and its consequences on the ray-tracing approach. Furthermore, we will add a paragraph to section '5 Conclusion' to address the interpolation problem in the iterative non-linear tomography approach to highlight its impact on the tomography solution.

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