

Title: **Dual-Wavelength Radar Technique Development for Snow Rate Estimation: A Case Study from GCPEX**

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Authors' Response to Interactive Comments from Anonymous Referee #1 and Anonymous Referee #2

This document combines previously posted authors' responses to comments from both Referees, as well as a marked-up revised manuscript version.

Authors' Response to Interactive Comment from Anonymous Referee #1

This article describes a dual-frequency method to estimate the amount of snowfall from radar measurements obtained by NASA's D3R radar during the GCPEX experiment in 2012. The proposed method hinges on a representation of the Z-SR relationship conditional to the DWR. The Authors demonstrate the superiority of their DWR-based algorithm when compared to traditional power-law relationships to retrieve the liquid-equivalent snow rate.

The article provides a nice illustration of the use of in situ microphysical data, with radiative-transfer models (T matrix with various assumptions about the mass-size relationship) constrained by remote sensing observations. However, before I can recommend this article for publication, the Authors should revise a few key points detailed below. The writing is of unequal quality with some paragraph extremely well written when others are filled with typos and unclear sentences. It would also be possible good to reduce the length of the manuscript by removing 1 or 2 figures and the overly long part that details the processing of the 2DVD images. Lastly, I couldn't find much about the efforts of the Authors to avoid or mitigate the effects of attenuation on the radar measurements, particularly at Ka band. Failing to do so can significantly bias the retrievals performed using the radar observations.

Our Response:

We would like to thank the reviewer for the positive general comments and detailed technical and editorial comments (corrections, suggestions, and questions). We have responded below to all of the comments (except those that are entirely complimentary) and included changes in the revised manuscript. In addition, we have tried to make the quality of writing more uniform in the revised manuscript.

Detailed comments and suggestions (technical questions in italic)

- General comment

- Please decide whether to use “gauge” (preferred) or “gage” and use this consistently throughout the manuscript. Similarly for mis-/miss-/mismatch

Response:

We have replaced “gage” with “gauge” and “miss-“ with “mis-“ on all occasions in the revised manuscript.

- Please provide a table of acronyms and symbols.

We have included a table of acronyms and symbols in the revised manuscript, as *Appendix: List of Acronyms and Symbols*. The table might not be exhaustive, but it should increase the readability for the readers who are not fully familiar with GCPEX, 2DVD, and DWR retrievals.

- Introduction o Page 2 Line12 (P2L12): “In this study, we” ...

- P2L29: Please introduce the “Dual- λ ” notation (e.g. in line 26) before using it.

Response:

We have replaced “The dual-wavelength reflectivity ratio (DWR) radar-based QPE was proposed by ...” by “The dual-wavelength reflectivity ratio (DWR; the ratio of reflectivity from two different bands) radar-based QPE was proposed by ...” in the revised manuscript.

- P3L4: “ratio of 4th moment to 3rd moment...”: “ D_m ” the mean mass-weighted diameter will be the ratio of the 4th to the 3rd moment only if the exponent of the m-D relationship is 3, which is seldom the case when dealing with snow... please reformulate.

Response:

The PSD we discussed here is in term of liquid-equivalent size (or melting size). So the 3rd moment of PSD is proportional to mass.

- P3L14: “estimate the mass of”

Response:

Corrected.

- P3L16: “based on a particle’s mass”

Response:

Changed to “based on particles’ mass”.

- Section 2

- P3L29: “dominates”

Response:

Corrected.

- P4L16: please include a space between “ δ_0 ” and “in Bohn”;

Response:

Corrected.

- P4L19: “parameterization error”: Is this an error (i.e. producing wrong results) or a different convention?

Response:

It is an error due to use of an equation to represent real data.

- P5L15: “circumscribed circle”?

Response:

Replaced by “circumscribed circle” in the revised manuscript.

- P5, last paragraph to the end of Section 2.2: This paragraph provides waaaay too much detail on the operation of the 2DVD. Please refer to literature or move to an appendix. Similarly, I’m not sure that Figure 2 is really needed...

We believe that this paragraph and Figure 2 are really important for the paper and the explanation of geometric and fall speed measurements as a key component of the proposed and presented methodology. We have therefore opted to keep them in the revised manuscript.

- P5L21&28: “optical planes”?

Response:

Yes. The optical plane refers to the plane which optical system (light source and line-scan camera) is observing.

- P5L31 to 34: is it miss-match or mismatch?

Response:

We have corrected “miss-“ to “mis-“.

- P6L2: “manufacturer’s matching”

Response:

Corrected.

- P6L8-10: “The Huang... because... size”. I do not understand the causality in this sentence. Please rephrase.

Response:

This has been rephrased to “The Huang and Bringi approach (Huang et al. 2015) is referred to as HB ...” in the revised manuscript.

- P6L26: “particles, which ... Hence, the”

Response:

Corrected.

- P6L4: “Here, we use”

Response:

Corrected.

- P6L22: There is a typo in the units reported (200 and 400 \mu m ?)

Response:

The units are “microns”. We have corrected this typo.

- P6L24: “spheroids”

Response:

Corrected.

- Fig3: Since both quantities being plotted are positive, would a plot in log-log scale be more appropriate/revealing?

Response:

The fall speed versus size plots are commonly presented in linear scale.

- Section 3

- P8L8-9: Is Figure 4 necessary? It is already in Skofronick et al 2015... Please consider deleting this Figure otherwise.

Response:

We think this figure is necessary to orient the reader to GCPEX and the spatial configuration of the instrumentation sites and important natural features.

- P8L10: "with an extensive"

Response:

Corrected.

- P8L14: "similar to a 'vertical pointing"

Response:

Corrected.

- P8L20: "Echo tops... high-altitude radar was were"

Response:

Corrected.

- Compared to Section 3.1, Section 3.2. is really well written. The Authors provide a clear description of their thorough QC efforts of the DWR data.

- What do the Authors do to address attenuation especially at Ka band? This could have a strong effect on the DWR, right?

Response:

In general, the attenuation of Ku- and Ka-band cannot be ignored. The case we analyzed is a low liquid equivalent precipitation rate, dry snow event. Based on the HB method, the density-size power-law relationship for this case is $\rho = 0.19 \cdot D_{app}^{-0.8}$. This relationship is very close to Ikeda-Brandes (2007) relationship which represents typical Colorado dry aggregated snow. The density of large snowflakes (> 2 mm) is less than 0.1 g/cm³. The imaginary part of the dielectric constant for such snowflakes will be close to 0. Therefore, the attenuation can be ignored. Also we computed the k-Z relationship using three methods, and the highest attenuation for Ka-band is 0.03 dB/km at 40 dBZ and most of our data set has $k < 0.015$ dB/km using the LM method.

- P10L29: "amount of"

Response:

Corrected.

- P11L22: "particles which, in principle, could"

Response:
Corrected.

- P12L3: "Figure 11a,b compares"

Response:
Corrected.

- P12L8: "falls off as"

Response:
Corrected.

- P13L1: For the airborne radar data, please refer to APR2 and cite appropriate references.

Response:

We have added in the revised manuscript a mention of APR-2 when talking about airborne radar data, and have cited Skofronik-Jackson et al. 2015 as an appropriate reference.

- P13L2: "Skolfronik"?

Response:
Corrected.

- P13L7: "mis-match" please decide between "miss-match, mis-match and mismatch" for the entire article...

Response:

Corrected "miss-" to "mis-".

- P13L11: "found not to be not size dependent"

Response:
Corrected.

- P13L12: "factor may be size"

Response:
Corrected.

- P13L20: "It is obvious from Fig. 13 that there is considerable scatter", how about in a log-log scale, is there still a significant scatter?

Response:

Yes. The log-log plot will make it look better. However, we want to represent the error of Z-SR relationship.

- P13L21-22: "... ranging from 55 to 70%... from 40 to 45%"

Response:
Corrected.

- P14L1: Please re-write all "Ku" or "Ka" with a capital "K" even in the subscripts

Response:
Corrected.

- P14L14: "don't do not use"

Response:
Corrected.

- P14L16-L17: There is a truly remarkable agreement between measured and simulated accumulated snowfall!
- P14L18: "Figure 15b is the same as..."
Response:
Corrected.
- P14L20: why italics in "accumulations"?
Response:
Corrected.
- P15L3: "at the same time while maintaining"
Response:
Corrected.

- Section 4

- P15L6: "is to develop a technique development for ... using a scanning"
Response:
Corrected.
- P15L26: "a large-scale synoptic ... site of CARE during GCPEX."
Response:
Corrected.
- P15L30: "explained by a possible"
Response:
Corrected.
- P16 1st paragraph: Could the discrepancies that you couldn't explain be due to attenuation, which could strongly affect the Ka-band measurements?
Response:
According to our scattering computation, the most of our data set has attenuation less than $k < 0.015$ dB/km. The maximum range of D3R is 30 km. So the worst round trip PIA is 0.9 dB which is less than the error of reflectivity measurement (~1dB). Considering that the Ku-band reflectivity is also attenuated, the impact on DWR should be even less. However, we agree that for heavy snow, more compact snow types (i.e., graupel), or melting snow, an attenuation correction would be needed.
- P16L13: "and LM gave (1.94 mm)."
Response:
Corrected.
- P16L19-21: Is the smoothing of DWR actually a smoothing of Ka or is it necessary to first form DWR and then smooth only this parameter?
Response:
Since Ku- and Ka-band reflectivities are two independent measurements, it does not matter whether smoothing of Z is done first or DWR is formed first and smoothed.

- Could you also provide some perspective as to how to extend your method to other dual-frequency datasets? Would the methods presented here still work if applied to a Ku-W or Ka-W data set? What would be the expected differences/commonalities? Similarly, would there be some potential or added value for airborne or spaceborne dual-frequency radars? Lastly, how would you extend your method to handle more frequencies (3,4,...)?

Response:

Theoretically, our method is independent of the frequency band pair. However, reflectivity oscillates when the particle's size is in the Mie region, thus the DWR also oscillates. So, a different frequency band pair would have a different dynamic range of DWR.

The second problem is due to the T-matrix method. By using the HB method, in our experience, T-matrix will have convergence problem when frequency is higher than Ka-band. The LM and HW methods use Liao's model (Liao et al. 2013) for the scattering computations. They compute scattering coefficients up to 183 GHz, and their equivalent diameter limits are 2.5 mm. Therefore, this size may be too small for large snowflakes/graupel or highly rimed snowflakes.

- References

- P17L24: "for scattering"

Response:

Corrected.

- P17L31: "Orinetations"

Response:

Corrected.

- Figures

- Figure 10: legend: "that were directly measured..."

Response:

Corrected.

- Figures 13 and 14:

- Please use the same units for SR between the two plots (Figs 13 and 14) to help the reader compare plots;

Response:

This was a mistake. The unit of SR in the figures should be mm/hr. We have corrected Figure 13.

- In Fig. 13, would a logarithmic scale work better?

Response:

We believe that it is better to use linear scale.

Authors' Response to Interactive Comment from Anonymous Referee # 2

A method is proposed to estimate snowfall rates from the D3R dual-frequency radar measurements. The method is developed and assessed during a snowfall event in 2012. 2D-video disdrometer and gauge data are used to link observed radar reflectivity and particles physical properties. The dual-frequency estimator is shown to estimate snowfall rates with more accuracy than the conventional single frequency Z-S approach.

The paper is well written. The detailed discussion on the assumptions, methodology and techniques is appreciated. Dual-frequency estimation of snowfall rates addresses the critical need for improved snow estimation from ground- and space-based remote sensing. The work presented is worthy of publication after some minor aspects have been addressed.

Our Response:

We would like to thank the reviewer for the positive general comments, as well as the corrections, suggestions, and questions regarding minor aspects. We have responded below to all of the comments (except those that are entirely complimentary) and included changes in the revised manuscript.

1. This study relies on a set of observations that is unique. This is probably the reason why it is applied on a single event. It is recommended to discuss the representativeness of the results, i.e. to what extent the Z-S and DWR-S relations developed in this study can be applied to other precipitation events, regions, or environmental conditions.

Response:

The case we have analyzed during GCPEX is a common synoptic snowfall event (large scale forcing is described in brief in Section 3) that occurs in the area surrounding the CARE site and is not unique per se, but dual-wavelength scanning radar observations combined with measuring instruments sited inside a DFIR wind shield are somewhat unique. The Z-SR power law derived for this event is expected to be applicable to similar synoptic forced snowfall in similar climatology under similar environmental conditions (eg Temperature and RH profiles from sounding) but not for example to lake effect snowfall as the microphysics are quite different. Regarding the DWR-SR relations, they should be considered for now as 'condition' specific and analysis of more events are needed before any firm conclusions can be drawn as to applicability to other regions or environmental conditions.

We have added some sentences in the conclusions at the very end as the ending para.

"The snow rate estimation algorithms developed here are expected to be applicable to similar synoptic forced snowfall under similar environmental conditions (e.g., temperature and relative humidity) but not for example to lake effect snowfall as the microphysics are quite different. However, analysis of more events are needed before any firm conclusions can be drawn as to applicability to other regions or environmental conditions."

2. What are the perspectives in terms of implementing such approach to other instruments on the ground or in space (i.e. GPM dual-frequency radar)?

Response:

Substantial work exists and is on-going using airborne particle size distribution probes and downward pointing radars to develop Z-SR relations stratified by temperature, for example. Multiple-wavelength downward pointing airborne radars at the GPM frequencies (Ku, Ka-bands) have been used in many field programs including comparisons with GPM overpasses. The GPM dual-wavelength technique for measuring snowfall near the surface is an active area of research. As such our experience using scanning dual-wavelength radar (D3R) is bound to be useful as ground-validation for GPM-based algorithms.

3. P.2 ll.10-11: “it is shown that a physically consistent representation of the geometric, microphysical, and scattering properties needed for radar-based QPE can be achieved” and following discussion on Ze-SR relations. For information this has been also been shown in a recent contribution involving dual-polarization ground-based radars: Bukovci Ć, P., A. Ryzhkov, D. Zrni Ć, and G. Zhang, 2018: Polarimetric Radar Ć Relations for Quantification of Snow Based on Disdrometer Data. J. Appl. Meteor. Climatol., 57,103–120, <https://doi.org/10.1175/JAMC-D-17-0090.1>.

Response:

We are aware of this paper which uses dual-polarization radar to estimate snowfall with algorithms developed using 2DVD. To the best of our knowledge, the derived algorithms were not tested with independent snow gage measurements. We have cited this reference in our revised version on page 2, lines 10-11.

4. P.3 ll.8&10: Dm is not measured; it is actually estimated from measurements.

Response:

We have changed “measurements” to “estimation”.

5. p.3 ll.1-11: this paragraph seems too technical in the introduction section. You can consider including it in the methodology section.

Response:

After much consideration we feel that keeping the paragraph the same i.e., in the Introduction, is reasonable.

6. Please correct Skolfronik-Jackson et al. (2015) to Skofronick-Jackson et al. (2015) throughout the paper.

Response:

This typo has been corrected throughout the paper.

7. p.13 ll.20 – p.14 l.10: Fig. 13 that there is considerable scatter at Ku-band for all three methods with the normalized standard deviation (NSTD) ranging from 55- 70%”. Are the errors in table 3 assumed to be normally distributed? Kirstetter et al. (2015) proposed a probabilistic Z-S QPE approach showing that uncertainty is characterized by non-

symmetric distributions: Kirstetter, P.E., J.J. Gourley, Y. Hong, J. Zhang, S. Moazamigoodarzi, C. Langston, A. Arthur, 2015: Probabilistic Precipitation Rate C2 Estimates with Ground-based Radar Networks. Water Resources Research, 51, 1422-1442. doi:10.1002/2014WR015672 “To reduce error, we may take the geometric mean of these two estimators”: do you mean to reduce the bias? Does the non-linear least square fitting approaches assumes normally distributed uncertainty? Can this assumption be discussed?

Response:

As stated in the text the Z-SR power law prefactor and exponent are determined using weighted total least squares method where Z is in mm^6m^{-3} and SR in mm/h. We do not fit a straight line in log-log space. As in all least squares methods the residual errors are assumed to be normally distributed. Also, the estimated NSTD values reflect the parameterization errors only (see Chapter 7 of Bringi and Chandrasekar 2001) in an additive error model. We use the geometric mean to show the central tendency and avoid giving too much weight to the outliers. It is not intended for reducing the bias. We do not believe that it is necessary to include this in the revised text.

We have read the article by Kirstetter et al. which is based on mapping the radar measurement of Z to measured snow rate at the ground; thus their uncertainty is due to various error sources such as radar bias in Z as well as errors in snow gage measurements due to horizontal wind etc. They find 50% bias (underestimate) when using the climatological Z-SR relationship and the relative uncertainty expressed in terms of the interquartile range normalized by the mean of 50-75%. We agree that the errors (bias+random) can be unsymmetric and interquartile ranges are more appropriate. We have added a few sentences from Kirstetter et al. in our revised manuscript just before Section 4 on Summary and Conclusions.

“Note that the error model used here is additive with the parameterization and measurement errors modeled as zero mean and uncorrelated with the corresponding error variances estimated either from data or via simulations (as described in Chapter 7 of Bringi and Chandrasekar 2001). This is a simplified error model since it assumes that radar Z and snow gage measurements are unbiased based on accurate calibration. A more elaborate approach of quantifying uncertainty in precipitation rates is described by Kirstetter et al. (2015).”

8. p.14 l.4: “the SR (ZKu ,DWR) using LM method has the smallest NSTD (28.49%) but the other two methods have similar values of NSTD ($\approx 30\%$)”. Is this difference in NSTD significant?

Response:

The differences in NSTD are not statistically different. That sentence has been modified to:

“As can also be seen from Fig. 14 and Table 4, the SR(ZKu,DWR) using LM method results in the lowest NSTD of 28.49%, but the other two methods have similar values of NSTD ($\approx 30\%$) and, as such, these differences are not statistically significant.”

Dual-Wavelength Radar Technique Development for Snow Rate Estimation: A Case Study from GCPEX

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Abstract. Quantitative Precipitation Estimation (QPE) of snowfall has generally been expressed in power-law form between equivalent radar reflectivity factor (Z_e) and liquid equivalent snow rate (SR). It is known that there is large variability in the pre-factor of the power law due to changes in particle size distribution (PSD), density, and fall velocity whereas the variability of the exponent is considerably smaller. The dual-wavelength radar reflectivity ratio (DWR) technique can improve SR accuracy by estimating one of the PSD parameters (characteristic diameter) thus reducing the variability due to the pre-factor. The two frequencies commonly used in dual-wavelength techniques are Ku- and Ka-bands. The basic idea of DWR is that the snow particle size-to-wavelength ratio is such as to fall in the Rayleigh region at Ku-band but in the Mie region at Ka-band.

We propose a method for snow rate estimation by using NASA D3R radar DWR and Ka-band reflectivity observations collected during a long-duration synoptic snow event on 30–31 January 2012 during the GCPEX (GPM Cold Season Precipitation Experiment). Since the particle mass can be estimated using 2D-video disdrometer (2DVD) fall speed data and hydrodynamic theory, we simulate the DWR and compare directly with D3R radar measurements. We also use the 2DVD-based mass to compute the 2DVD-based SR . Using three different mass estimation methods, we arrive at three respective sets of Z - SR and $SR(Z_h, DWR)$ relationships. We then use these relationships with D3R measurements to compute radar-based SR . Finally, we validate our method by comparing the D3R radar-retrieved SR with accumulated SR directly measured by a well-shielded Pluvio gauge for the entire synoptic event.

1 Introduction

The detailed understanding of the geometric, microphysical, and scattering properties of ice hydrometeors is a vital prerequisite for development of radar-based quantitative precipitation estimation (QPE) algorithms. Recent advances in surface and airborne optical imaging instruments and the wide proliferation of dual-polarization and multi-wavelength radar

systems (ground-based, airborne or satellite) have allowed for observations of the complexity inherent in winter precipitation via dedicated field programs (e.g., ~~Skolfronik~~Skofronick-Jackson et al. 2015; ~~Petaja~~Petäjä et al. 2016). These large field programs are vital given that the retrieval problem is severely under-constrained due to large number of geometrical and microphysical parameters of natural snowfall, their extreme sensitivity to subtle changes in environmental conditions, and co-existence of different populations of particle types within the sample volume (e.g., ~~Szyrmer~~Szyrmer and Zawadzki 2014).

The surface imaging instruments that give complementary measurements and used in a number of recent studies include (i) 2D-Video Disdrometer (2DVD; ~~Schöenhuber~~Schönhuber et al. (2008), (ii) Precipitation Imaging Package (PIP; von Lerber et al. 2017), (iii) Multi-Angle Snowflake Camera (MASC; Garrett et al. 2012). When these instruments are used in conjunction with a well-shielded GEONOR or PLUVIO gauge, it is shown that a physically consistent representation of the geometric, microphysical, and scattering properties needed for radar-based QPE can be achieved (Szyrmer and Zawadzki 2010; Huang et al. 2015; von Lerber et al. 2017; ~~Bukovčić~~Bukovčić et al. 2018). In this study, we use the 2DVD and PLUVIO ~~gage~~gauge located within a double fence international reference (DFIR) wind shield to reduce wind effects.

Radar-based QPE has generally been based on Z_e - SR (Z_e : reflectivity; SR : liquid equivalent snow rate) power laws of the form $Z_e = \alpha(SR)^\beta$ where the pre-factor and exponent are estimated based on, (i) direct correlation of radar measured Z_e with snow gauges (Rasmussen et al. 2003; Fujiyoshi et al. 1990; Wolfe and Snider 2012), or (ii) using imaging disdrometers such as 2DVD or PIP (Huang et al. 2015; von Lerber et al. 2017). Recently, Falconi et al. (2018) developed Z_e - SR power laws at three frequencies (X, Ka and W-band) by direct correlation of radar and PIP observations. These studies have highlighted the large variability of α due to particle size distribution (PSD), density, fall velocity, and dominant snow type, whereas the variability in β is considerably less. Similarly, both methods, (i) and (ii), have been used to estimate ice water content (IWC) from Z_e using power laws of the form $Z_e = \alpha(IWC)^\beta$ based on airborne particle probe data, direct measurements of IWC , and airborne measurements of Z_e (principally at X, Ka, and W-bands) (e.g., Heymsfield et al. 2005; Hogan et al. 2006; Heymsfield et al. 2016). The advantage of airborne data is that a wide variety of temperatures and cloud types can be sampled (Heymsfield et al. 2017).

The dual-wavelength reflectivity ratio (DWR ; the ratio of reflectivity from two different bands) radar-based QPE was proposed by Matrosov (1998; 2005) to improve SR accuracy by estimating the PSD parameter (median volume diameter D_0) with relatively low dependence on density if assumed constant. There has been limited use of dual- λ techniques for snowfall estimation, mainly using vertical-pointing ground radars or nadir pointing airborne radars (Liao et al. 2005, 2008, 2016; Szyrmer and Zawadzki 2014; Falconi et al. 2018). The dual- λ method is of interest to us due to the availability of the NASA D3R scanning radar (Vega et al. 2014), which, to the best of our knowledge, has not been exploited for snow QPE to date.

The *DWR* is defined as the ratio of the equivalent radar reflectivity factors at two different frequency bands. The main principle in *DWR* is that the particle's size-to-wavelength ratio falls in Rayleigh region at a low frequency band (e.g., Ku-band) but in Mie region at a high frequency band (e.g., Ka-band) (Matrosov 1998; Matrosov et al. 2005; Liao et al. 2016). Previous studies have shown that the *DWR* can be used to estimate D_m , where D_m is defined as the ratio of 4th moment to 3rd moment of the PSD expressed in terms of liquid-equivalent size or mass (Liao et al. 2016). In this sense the *DWR* is similar to differential reflectivity (Z_{dr}) in dual-polarization radar technique where Z_{dr} is used to estimate D_m (but the physical principles are, of course, different; Meneghini and Liao 2007). The *SR* is obtained by 'adjusting' the coefficient α in the Z_e -*SR* power law based on the ~~measure~~estimation of D_m provided by the *DWR*. The pre-factor α depends on the intercept parameter of PSD (von Lerber et al., 2017) and not on D_m directly. However, because of apparent negative correlation between D_m and PSD intercept parameter for a snowfall of a given intensity (~~Delanoë~~Delanoë et al., 2005; Tiira et al., 2016) measurements of D_m can be used to "adjust" Z_e -*SR* power law.

This paper is organized as follows. In Section 2, we introduce the approach and methodologies proposed and used in this study which may be considered as technique development. We briefly explain how to estimate the mass of ice particles using a set of aerodynamic equations based on Böhm (1989) and Heymsfield and Westbrook (2010). We also give a brief introduction of the scattering model based on ~~particle's~~particles' mass. Section 3 provides a brief overview of instruments installed at the test site and the dual-wavelength radar used in this study (D3R: Vega et al. 2014). We analyze surface and D3R radar data from one synoptic snowfall event during GCPEX and compare *SR* retrieved from *DWR*-based relation with *SR* measured by a snow gauge. The conclusions and possibilities for further improvement of the proposed techniques are discussed in Section 4. The acronyms and symbols are list in Appendix.

2 Methodology

2.1 Estimation of Particle Mass

The direct estimation of mass of an ice particle is difficult and at present there is no instrument available to do this automatically. The conventional method is to use a power law relation between mass and the maximum dimension of the particle of the form $m=aD^b$ where the pre-factor a and exponent b are computed via measurements of particle size distribution $N(D)$ from aircraft probes and independent measurements of total ice water content as an integral constraint (Heymsfield et al. 2010). A similar method was used by Brandes et al. (2007) who used 2DVD data for $N(D)$ and a snow gauge for the liquid equivalent snow accumulation over periods of 5 min. These methods are more representative of an average relation when one particle type (e.g., snow aggregates) ~~dominated~~dominates the snowfall with large deviations possible for individual events with differing particle types (e.g., graupel).

To overcome these difficulties a more general method was proposed by Böhm (1989) based on estimating mass from fall velocity measurements, geometry and environmental data if the measured fall velocity is in fact the terminal velocity (i.e., in the absence of vertical air motion or turbulence and in more or less uniform precipitation). The methodology has been described in detail by (Szyrmer and Zawadzki 2010; Huang et al. 2015; von Lerber et al. 2017) and we refer to these articles for details. The essential feature is the unique non-linear relation between the Davies (1945) number (X) and the Reynolds number (Re) where X is the ratio of mass to area or $m/A_r^{0.25}$ ($A_r=A_e/A$ is the area ratio, where A_e is the effective projected area normal to the flow and A is the area of the minimum circumscribing circle or ellipse that completely contains A_e) and the Re is the product of terminal fall speed and characteristic dimension of the particle. We have neglected the environmental parameters (air density, viscosity) as well as boundary layer depth of Abraham (1970) and the inviscid drag coefficient. The procedure is to (i) compute Re from fall velocity measurements and characteristic dimension of the particle (usually the maximum dimension), (ii) compute the Davies number X which is expressed as a non-linear function of Re , and boundary layer parameters ($C_0 = 0.6$ and $\delta_0 = 5.83$; Böhm 1989), (iii) estimate particle mass from X and A_r . Heymsfield and Westbrook (2010) proposed a simple adjustment (based on field and tank experiments) by defining a modified Davies number as proportional to $m/A_r^{0.5}$ along with different boundary layer constants ($C_0=0.292$; $\delta_0=9.06$) from Böhm. Their adjustment was shown to be in very good agreement with recent tank experiments by Westbrook and Sephton (2017) especially for particles like pristine dendrites with low A_r and at low Re . Note that the difference of C_0 and δ_0 in Böhm and Heymsfield-Westbrook equations is mainly due to differences in the shape correcting factor (A_r) to find the optimal relation between drag coefficient (or equally said, Davies number; X) and Reynolds number (Re). This is the main parameterization error in this set of equations.

2.2 Geometric and Fall Speed Measurements

One source of uncertainty in applying the Böhm or Heymsfield and Westbrook (HW) method is calculating the area ratio (A_r) using instruments such as 2DVD or precipitation instrument package (PIP) as they do not give the projected area normal to the flow (i.e., they do not give the needed top view rather the 2DVD gives two side views on orthogonal planes as illustrated in Fig. 1). This is reasonable for snow aggregates which are expected to be randomly oriented. The other source of uncertainty is in the definition of characteristic dimension used in Re which in the HW method is taken to be the diameter of the circumscribing circle that completely encloses the projected area, the maximum dimension (D_{max} ; this is what we use for the 2DVD in our application of the HW method). For the Böhm method we use the procedure in Huang et al. (2015) which used D_{app} defined as the equal-volume spherical diameter.

The two-dimensional video disdrometer (2DVD) used herein is described in Schönhuber et al. (2000) and calibration and accuracy of the instrument are detailed in Bernauer et al. (2015). The 2DVD is equipped with two line-scan cameras (referred to as camera A and B) which can capture the particle image projection in two orthogonal planes (two side views). As mentioned earlier the area ratio (A_r) should be obtained from the projected image in the plane normal to the flow i.e., (top

or bottom view). However, to the best of our knowledge, there are no ground-based instruments that can automatically and continuously capture the horizontal projected views (i.e., in the plane orthogonal to the flow) of precipitation particles (however, 3D-reconstruction based on multiple views can give this information; Kleinkort et al. 2016). Compared with other optical-based instruments, such as HVSD (Hydrometeor Velocity Size Detector; Barthazy et al. 2004) or SVI (Snow Video Imager; Newman et al. 2009) which only captures the projected view in one plane, the 2DVD offers views in two orthogonal planes giving more geometric information. Figure 1 shows a snowflake observed by a 2DVD from two cameras. The thick black line is the contour of the particle and the thin black lines show the holes inside the particle. The effective projected area A_e in the definition of area ratio is easy to compute by counting total pixels from the particle's image, and then multiplying by horizontal and vertical pixel-width. The blue line is the minimum circumscribed ellipse. The area of the ellipse is A in the definition of area ratio. The size of particle measured by 2DVD is called apparent diameter (D_{app}) which is defined as the diameter of the equivalent volume sphere (Schönhuber et al. 2000; Huang et al. 2015). The D_{app} is used when computing Re as mentioned earlier. The area ratio and D_{app} are the geometric parameters that are used in our implementation of the Böhm method.

In our application of the HW method the A is based on the diameter of the circumscribed circle that completely encloses the projected pixel area (A_e) which is easy to calculate from the contours in Fig. 1. Thus the area ratio is A_e/A while the characteristic dimension in Re is the diameter of the circumscribing circle. Note that the area ratio and characteristic dimension in Re depend on the type of instrument used (e.g., advanced version of snow video imager by von Lerber et al. 2017; the HVSD by Szyrmer and Zawadzki 2010). These instruments give projected view in one plane only and thus geometric corrections are used as detailed in the two references.

The two optic planes of the 2DVD are separated by around 6 mm and the accurate distance is based on calibration by dropping 10 mm steel balls at three corners of the sensing area (details of the calibration as well as accuracy of size, fall speed, and other geometric measures are given in Bernauer et al. 2015). During certain time periods, more than one precipitation particle falls in the 2DVD observation area. Since two cameras look at different directions, the particles observed by camera A and camera B need to be paired. This pairing procedure is called “matching”, and it is illustrated in Fig. 2. The time period $[t_1, t_2]$ is dependent on the assumed reasonable fall speed range. Assuming that the minimum and maximum reasonable fall speeds are v_{min} and v_{max} , respectively, the distance between two optic planes is D_d , and that camera A observed a particle at t_0 , we have $t_1 = t_0 + D_d/v_{max}$ and $t_2 = t_0 + D_d/v_{min}$. After matching, the fall speed can be calculated as $D_d/\Delta t$, where Δt is the time difference between two cameras observing the same particle. Because the fall speed of the 2DVD is dependent on matching, the geometric features and fall speeds will be in error when **missmis**-match occurs. Huang et al. (2010) analyzed snow data from the 2DVD and found that the 2DVD manufacturer's matching algorithm for snow resulted in significant **missmis**-matching problem (see, also, Bernauer et al. 2015). In the Appendix of Huang et al. (2010), they showed that **missmis**-match will cause the volume, vertical dimension, and fall speed of particles to be over-estimated.

Subsequently, the mass of particles will also be over-estimated mainly because of fall speed. To get the best estimation of mass, they used 2DVD single camera data (before applying the manufacturer's matching ~~algorithm~~) and re-did matching based on a weighted Hanesch criteria (Hanesch 1999). The disadvantage of using single camera data, as in Huang et al. (2015), is that the particles' contour data is not available (i.e., the manufacturer's code does not provide line scan data from single camera). Without contour data, both D_{app} and A can only be estimated by maximum width of the scan line and height of the particle as detailed in Huang et al. (2015). Moreover, the diameter of the circumscribing circle or ellipse cannot be obtained without contour data. The only quantity included in single camera data is A_e in terms of number of pixels. The Huang- and Bringi ~~approach (Huang et al. (2015)-approach~~ is referred to as HB because both PSD (particle size distribution) and reflectivity (Z_e) are computed using D_{app} as the measure of particle size.

In order to obtain contour data, we have to use the manufacturer's matching algorithm. To avoid over-estimating mass due to ~~missmis~~-match, we need to filter out those particles with unreasonable v_t . The vertical dimension of the particle's image before match is expressed as a number of scan lines (i.e., how many scan lines are masked by the particle). After match (so v_t is known), the vertical pixel width is v_t/f_s , where f_s is the scan frequency of a camera (~ 55 kHz), and the vertical size of the particle is the vertical pixel width multiplied by the number of scan lines. Because two optical planes of the 2DVD are parallel, theoretically, the number of scan lines from cameras A and B should be the same. Considering the distance from the particle to the two cameras (projective effect of a camera), digital error of a camera and particle rotation in two planes, the difference in the number of scan lines between two cameras may not always be the same but should be very close. Hanesch (1999) gave a set of criteria for matching, the most important being the tolerance of the number of scan lines between the two cameras (see Table 1). To obtain reliable fall speeds, we examined all matched particles (given by the manufacturer's matching algorithm) and removed those particles which did not satisfy the Hanesch scan line criteria. We used the fall speed of these filtered particles to compute their mass for both Böhm and Heymsfield-Westbrook methods, and then, dividing the mass by apparent volume ($=\pi D_{app}^3/6$), to get the particle density. Since the maximum density of ice particles is around 0.9 g cm^{-3} , we further remove particles whose density comes out as larger than 1 g cm^{-3} . After this two-step filtering, the particles we use for further analysis are shown in Fig. 3 as blue points. The filtering will eliminate particles, which will reduce the liquid equivalent snow accumulation. Hence, the Pluvio ~~gagagegauge~~ accumulation is used as an integral constraint, i.e., the concentration in each bin is increased by a constant factor to match the 2DVD accumulation to the Pluvio accumulation.

2.3 Scattering Model

The scattering computation of ice particles is difficult because of their irregular shapes with large natural variability of shapes (e.g., snow aggregates or rimed crystals). The most common scattering method used in the meteorological community is the Discrete Dipole Approximation (DDA; Draine and Flatau 1994). However, DDA is very time consuming and not suitable for large numbers of particles especially at W-band (e.g., Chobanyan et al. 2015). On the other hand, the T-matrix

method (Mishchenko et al. 2002) is more time efficient and commonly used in radar meteorology but it requires that the irregular particle shape be simplified to an axis-symmetric shape (e.g., spheroid). Ryzhkov et al. (1998) have shown that, in Rayleigh region, the radar cross-section is mainly related to particle's mass-squared and less to the shape. For Mie scattering, however, the irregular snow shape plays a more significant role. Here, we use two scattering models, one based on the soft spheroid (Huang et al. 2015) with fixed axis ratio and quasi-random orientation. The apparent density is calculated as the ratio of mass to apparent volume. There is considerable controversy in the literature on the applicability of the soft spheroid model with fixed axis ratio especially at Ka and higher frequencies such as W-band (e.g., Petty and Huang 2010; Botta et al. 2010; Leinonen et al. 2012; Kneifel et al. 2015)). However, Falconi et al. (2018) used the soft spheroid scattering model using T-matrix to compute Z_e (at X, Ka and W-bands) and showed that an effective optimized axis ratio of (oblate) spheroid could be selected that matches directly measured Z_e by radar (their optimal axis ratio, however, varied with the frequency band, i.e., 1 for X-band, 0.8 for Ka and 0.6 for W). They also found some differences in the optimal axis ratios for fluffy snow versus rimed snow. Nevertheless, they compared DDA calculations of complex-shaped aggregates to the soft spheroid model at W-band and concluded that the axis ratio can be used as a 'tuning' parameter. They also showed the importance of size integration to compute Z_e , i.e., the product of $N(D)$ and the radar cross-section for the soft spheroid versus complex-shape aggregates. Their result implied that smaller particles had a larger value for the product when using soft spheroid of 0.6 axis ratio relative to complex aggregates and vice-versa for larger particles leading to α -compensation when Z_e is computed by size integration over all sizes. Thus, the soft spheroid model with axis ratio at 0.8 used by Huang et al. (2015) and which is used herein at Ku- and Ka-bands is a reasonable approximation.

The second scattering model we used herein is from Liao et al. (2013) who use an 'effective' fixed density approach to justify the oblate spheroid model. To compare the scattering properties of a snow aggregate with its simplified equal-mass spheroid, Liao et al. (2013) used 6-branch bullet rosette snow crystals with a maximum dimension of 200 μm and 400 μm , respectively, as two basic elements to simulate snow aggregation. They computed the backscattering coefficient, extinction coefficient, and asymmetry factor for simulated snowflakes, by the DDA, and for the corresponding spheres and ~~spheroids~~spheroids with the same mass but density fixed at 0.2 or 0.3 g cm^{-3} , and hence the "apparent" sphere volume equals the mass divided by the assumed fixed density. They showed that when the frequency was lower than 35 GHz (Ka-band), the Mie scattering properties of spheres with a fixed density equal to 0.2 g cm^{-3} were in a good agreement with the scattering results for the simulated complex-shaped aggregate model with the same mass using the DDA (see, also, Kuo et al. 2016). They also showed this agreement with a spheroid model with a fixed axis ratio of 0.6 and random orientation. Here, we use the Liao et al. (2013) equivalent spheroid model with a fixed 'effective' density of 0.2 g cm^{-3} at Ku- and Ka-bands (note that we estimate the mass of each particle from 2DVD measurements as described in Section 2.1). Note that this fixed-density spheroid scattering model is not based on microphysics (where the density would fall off inversely with increasing size) but on scattering equivalence with a simulated (same-mass) complex-shaped aggregate snowflake (Liao et al. 2016).

3 Case Analysis

3.1 Test Site Instrumentation and the Synoptic Event

The GPM Cold-season Precipitation Experiment (GCPEX) was conducted by the National Aeronautics and Space Administration (NASA), U.S.A., in cooperation with Environment Canada in Ontario, Canada from 17 January to 29 February, 2012. The goal of GCPEX was "... to characterize the ability of multi-frequency active and passive microwave sensors to detect and estimate falling snow ..." (SkofronikSkofronick-Jackson et al. 2015). The field experiment sites were located north of Toronto, Canada between Lake Huron and Lake Ontario. The GCPEX had five test sites, namely, CARE (Centre for Atmospheric Research Experiments), Sky Dive, Steam Show, Bob Morton, and Huronia. The locations of five sites are shown in Fig. 4. The CARE site was the main test site for the experiment, located at 44°13'58.44" N, 79°46'53.28" W and equipped with [an](#) extensive suite of ground instruments. The 2DVD (SN37) and OTT Pluvio² 400 used for observations and analyses in this paper were installed inside a DFIR (Double Fence Intercomparison Reference) wind shield. The dual-frequency, dual-polarized Doppler Radar (D3R) was also located at the CARE site (Vega et al. 2014) near the 2DVD. The instruments used in this paper are depicted in Fig. 5. Because the radar and the instrumented site were nearly collocated, we can effectively view the set up as similar to [a](#) 'vertical pointing' radar as described in more detail in Section 3.2.

We examine a snowfall event on 30-31 January 2012 that occurred across the GCPEX study area between roughly 22 UTC 30 January through 04 UTC 31 January. Details of this case using King City radar and aircraft spiral descent over the CARE site is given in SkofronikSkofronick-Jackson et al. (2015). This event resulted in liquid accumulations of roughly 1-4 mm across the GCPEX domain with fairly uniform snowfall rates throughout the event. At the CARE site the accumulations over an 8 h period were <3.5 mm. Echo tops as measured by high altitude airborne radar ~~waswere~~ 7-8 km. The precipitation was driven by a shortwave trough moving from southwest to northeast across the domain. Figure 6 displays the 850 hPa geopotential heights (m), temperature (K), relative humidity (%), and winds (m s^{-1}) at 00 UTC 31 January, during the middle of the accumulating snowfall. A trough axis is apparent just to the west of the GCPEX domain (green star in Fig. 6). Low level warm air advection forcing upward motion is coincident with high relative humidity on the leading edge of the trough, over the GCPEX domain (Fig. 6). Temperatures in this layer were around -10 to -15 °C throughout the event, supporting efficient crystal growth, aggregation, and potentially less dense snowfall as this is in the dendritic crystal temperature zone (e.g., Magono and Lee 1966). Aircraft probe data during a descent over the CARE site between 23:15-23:43 UTC showed the median volume diameter (D_0) of 3 mm, with particles up to maximum of 8 mm (aggregates of dendrites) at 2.2 km MSL with a large concentration of smaller sizes < 0.5 mm (dendritic and irregular shapes; SkofronikSkofronick-Jackson et al. 2014). At the surface, photographs of the precipitation types by the University of Manitoba showed small irregular particles and aggregates (< 3 mm) at 2330 UTC on 30 January.

3.2 D3R Radar Data

The D3R is a Ku- and Ka-band dual-wavelength, polarimetric scanning radar. It was designed for ground validation of rain and falling snow from GPM satellite-borne DPR (dual-frequency precipitation radar). The two frequencies used in the D3R are 13.91 GHz (Ku) and 35.56 GHz (Ka). These two frequencies were used for scattering computations in this research as well. Some parameters of the D3R radar relevant for this paper are shown in Table 2. The range resolution of the radar is adjustable but usually set to 150 m and the near field distance is ~300 m; the practical minimum operational range is around 450 m. The minimum detectable signal of the D3R is -10 dBZ at 15 km. This means that when Z_h is -10 dBZ at 15 km, the Signal to Noise Ratio (SNR) is 0 dB. Therefore, the SNR at any range, r , can be computed as:

$$SNR(r) = Z_h(r) + 10 + 20 \log_{10} \left(\frac{15}{r} \right) \text{ [dB]}. \quad (1)$$

The SNR is a very important indicator for radar data Quality Control (QC); the other important parameter for QC (in terms of detecting ‘meteo’ versus ‘non-meteo’ echoes) being the texture of the standard deviation (std) of the differential propagation phase (ϕ_{dp}). We randomly selected 20 out of 85 RHI sweeps from 31 January, 2012 and computed the std of Ku-band ϕ_{dp} for each beam over 10 consecutive gates where $SNR \geq 10$ dB. According to the histogram of the std of ϕ_{dp} , 90% of the values were less than around 8° . Radar data at a range gate m is identified as a ‘good’ data (i.e., meteorological echoes) only if the standard deviation of ϕ_{dp} from the $(m - 5)^{th}$ gate to the $(m + 4)^{th}$ gate is less than 8° . This criterion sets a ‘good’ data mask for each beam at Ku-band. On the other hand, the ϕ_{dp} at Ka-band was determined to be too noisy and hence not used herein. The ‘good’ data mask for the Ka-band beam is set by the mask determined by the Ku-band criteria, with the additional requirement that the Ka-band $SNR > 3$ dB for the range gate to be considered as ‘good’. Note that both radars are mounted on a common pedestal so that the Ku and Ka-band beams are perfectly aligned.

There are four scan types that can be performed by the D3R, namely, PPI (Plan Position Indicator), RHI (Range Height Indicator), surveillance, and vertical pointing. Figure 7 shows the scan strategies of the D3R on 31st January, 2012, which consisted of a fast PPI scan (surveillance scan; 10° per second) followed by four RHI scans (1° per second) except from 01:00 UTC to 02:00 UTC. The RHI scans with an azimuth angle of 139.9° point to the Steam Show site and those at 87.8° point to the Sky Dive site. There were no RHI scans pointing to the Bob Morton site, and Huronia (52 km) was beyond the operational range (maximum 30 km) of the D3R. During the most intense snowfall the D3R scans did not cover the instrument clusters at Sky Dive and Steam Show sites. So we were left with analysis of D3R radar data at close proximity to the 2DVD or ‘effectively vertical pointing’ equivalent using RHI data from 75 to 90° at the nearest practical range of 600 m. PPI scan data at low elevation angle (3°) -were also used from range gate at 600 m. The assumption is that there is little evolution of particle microphysics from about 600 m height to surface and that the synoptic scale snowfall was uniform in azimuth (confirmed by SkolfronikSkolfronick-Jackson et al. 2015). The snowfall was spatially uniform around the CARE site so we selected data at 600 m range to compare with the 2DVD and Pluvio observations (this range was selected based on

the minimum operational range of 450 m; see Table 2) to which 150 m was added based on close examination of data quality. For RHI scans, the Z_h at each band was averaged over the beams from 75° to 90° . The 75° is obtained from $600 \cdot \cos(75^\circ) \approx 155$ m which is close to the range resolution. For the fast PPI scan, Z_h was averaged over all azimuthal beams at 600 m range.

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Figure 8 shows the time profile of the averaged Z_e at Ku- and Ka-bands. There are two problems indicated in this figure. First, theoretically, the Ku-band Z_e should be greater than or equal to the Ka-band Z_e . The smaller Ku-band Z_h indicates that a Z offset exists at both bands. The other problem is that, compared with the Ka-band, there are many dips in the Ku-band Z_h . Comparing Fig. 8 with Fig. 7, we found that these dips occur only at RHI scans with azimuth angle larger than 300° . We examined those RHI scans beam by beam from 90° to 75° . We further found that when the elevation angle is smaller than 78° , the unreasonably low Z_h disappears. Therefore, the RHI scans with azimuth angles larger than 300° were averaged over the 75° to 78° elevation angles. To compute the DWR , we need to know the Z offset between the two bands. The measured Z_h includes three components (neglecting attenuation):

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$$Z_h^{meas} = Z_h^{true} \pm error(Z_h) + Z_{offset}, \quad (2)$$

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where $error$ refers to measurement fluctuations (typically with standard deviation of ~ 1 dB). The DWR is obtained as the difference between Ku-band Z_h and Ka-band Z_h , with Z_h being in units of dBZ. The measured DWR is:

$$DWR^{meas} = DWR^{true} \mp error(DWR) + \Delta Z_{offset}, \quad (3)$$

20

where $error(DWR)$ is now increased since the Ku and Ka-band measurement fluctuations are uncorrelated (standard deviation of around 1.4 dB). The ΔZ_{offset} is determined by selecting data where the scatterers (snow particles) are sufficiently small in size so that Rayleigh scattering is satisfied at both bands, i.e., $DWR^{true} = 0$ dB. The criteria used here is to select gates where Ku-band $Z_h < 0$ dBZ along with spatial averaging to reduce the measurement fluctuations in DWR to estimate ΔZ_{offset} in Eq. (3). Figure 9 shows the averaged Z_h for the two bands from 20 RHI scans which satisfy the conditions above. After removing three extreme values (outliers) from Fig. 9, ΔZ_{offset} was estimated as -1.5 dB which is used in the subsequent data processing.

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3.3 2DVD Data Analysis

The 2DVD used in this study was also located at the CARE site. The particle-by-particle mass estimation is based on three methods, respectively, as follows:

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- 1) Following the procedure in Huang et al. (2015) use 2DVD single-camera data and apply the weighted Hanesch matching algorithm (Hanesch 1999) to re-match snowflakes and PSD adjustment factor is computed by comparing the amount of matched particles with single camera observed particles (details refer to Huang et al. 2015). Compute mass from fall speed, D_{app} and environmental conditions using Böhm (1989). The ‘apparent’ density of the snow (ρ) is

defined as $6m/\pi D_{app}^3$. A mean power-law relation of the form $\rho = a D_{app}^\beta$ is derived for the entire event as in Huang et al. (2015) as well as 1-min averaged $N(D_{app})$ is calculated. Note that the scattering model is based on the soft spheroid model with fixed axis ratio=0.8 and apparent density ρ . The results obtained by this method are denoted as “HB” method in the figures and in the rest of the paper.

- 2) Use the manufacturer’s (Joanneum Research, Graz, Austria) matching algorithm and filter ~~missmis~~-matched snowflakes as described in Section 2.2. The mass is computed from Böhm’s equations. Following Liao et al. (2013) as far as the scattering model is concerned, the density is fixed at 0.2 g/cc and the volume is computed from mass=density*volume. The effective equal-volume diameter is D_{eff} and the corresponding PSD is denoted as $N(D_{eff})$ which is different from $N(D_{app})$ in 1) above. Henceforth, this method is denoted as “LM”.
- 3) Use Joanneum matching and filtering method as in 2) but compute mass using Heymsfield-Westbrook equations as well as revised D_{eff} and $N(D_{eff})$. This method is denoted as “HW”. Thus, the only difference with 2) is in the estimation of mass and the difference in D_{eff} and $N(D_{eff})$. The scattering model follows Liao et al. (2013).

The 2DVD measured liquid equivalent snow rate (SR) can be computed directly from mass as:

$$SR = \frac{3600}{\Delta t} \sum_{i=1}^N \sum_{j=1}^M \frac{V_j}{A_j}; \quad [mm \ hr^{-1}], \quad (4)$$

where Δt is the integral time (typically 60 s), N is the number of size bins (typically 101 for the 2DVD), M is the number of snowflakes in the i^{th} size bin, and A_j is the measured area of the j^{th} snowflake. Further, V_j is the liquid equivalent volume of the j^{th} snowflake, so it is directly related to the mass. Figure 10 compares the liquid equivalent accumulation computed using the three methods above based on 2DVD measurements with the accumulation directly measured by the collocated Pluvio snow gauge. The Pluvio-based accumulation at the end of the event (0330Z) was 1.9 mm while the 2DVD-measured accumulations using the three methods are 1.27 mm (HB), 1.45 mm (LM), and 1.24 mm (HW), respectively. It is expected that the PSDs of LM and HW should be under-estimated because of eliminating ~~missmis~~-matched particles which, in principle, could be re-matched. Re-matching of ~~missmis~~-matched particles is a research topic on its own and is beyond the scope of this paper. We used a simple way to adjust the PSD which is similar to the procedure in Huang et al. (2015) by scaling the PSD by a constant so that the final accumulation from HB, LM and HW methods match the Pluvio data. Specifically, the PSD adjustment factors are 1.3 for LM and 1.52 for HW. Note that PSD adjustment of HB is not done by forcing 2DVD accumulated SR to agree with Pluvio. They are 1.54 for 00:00-00:45 UTC and 1.11 for 00:45-04:00 UTC. From the Pluvio data in Fig. 10 the SR is nearly constant at 0.7 mm/h between relative times of 1.5 to 3 h (or actual time from 01:00 on 30 Jan to 02:30 UTC).

The radar ~~reflectivity~~reflectivities at the two bands are simulated by using the T-matrix method assuming spheroid shape with axis ratio of 0.8 consistent with Falconi et al. (2018). The orientation angle distribution is assumed to be ~~quasi-random~~

with Gaussian distribution for the zenith angle [mean=0°, $\sigma=45^\circ$] and uniform distribution for the azimuth angle. However, other studies have assumed $\sigma=10^\circ$ (Falconi et al. 2018). The recent observations of snowflake orientation by Garret et al. (2015) indicate that substantial broadening of the snow orientation distribution can occur due to turbulence. Figure 4a,b compares the time series of D3R-measured Z_h with the 2DVD-derived Z_e for the entire event (2000-03-30 20:00-03:30 UTC) at (a) Ku and (b) Ka-band. The Z_e for both bands computed by the three methods generally agree with the D3R measurements to within 3-4 dB. Overall, LM gives the highest Z_e and HB gives the lowest especially evident at Ka-band. This is consistent with scattering calculations by Kuo et al. (2016) of single spherical snow aggregates using constant density (0.3 g/cc) giving higher radar cross-sections and size-dependent density i.e., density falls off as inverse size (giving lower cross-sections). This feature is consistent with the scattering models referred to herein as LM and HB, respectively.

From 00:45 UTC to 01:30 UTC on 31 January 2012, the three 2DVD-derived Z_e simulations deviate systematically from the D3R results for both bands. The other period is from 23:00 UTC to 23:30 UTC on 30 January 2012, when the Ku-band Z_e has significant deviation from the D3R observations but the Ka-band Z_e generally agrees with the D3R. Note that this synoptic event started at around 21:00 UTC on the 30th January and stopped at 03:30 UTC. We checked the D3R data and found that before 22:30 UTC, the RHI scans were from 0° to 60°, so there were no usable data available for comparison with the 2DVD and Pluvio at the CARE site. We note that at 00:30 UTC the King City C-band radar recorded Z_h in the range 15-20 dBZ around the CARE site which is in reasonable agreement with the D3R radar observations (Skofronick-Skofronick-Jackson et al. 2015).

Figure 12a compares the time series of DWR simulated from 2DVD observations with the D3R measurements whereas Fig. 12b shows the scatterplot. In general, HB appears qualitatively in better agreement (better correlated and with significantly less bias) with D3R measurements relative to both LM and HW (significant underestimation relative to D3R). The scatterplot in Fig. 12b is an important result since in the HB method the soft spheroid scattering model is used with density varying approximately inverse with D_{app} (density- D_{app} power law where the larger snow particles have lower density). Hence for a given mass the D_{app} is larger (relative to Ka-band wavelength) and enters the Mie regime which lowers the radar cross-section at Ka-band (relative to same mass but constant density radar cross-section in LM and HW). Whereas at Ku-band the difference in radar cross-sections is less between the two methods (Rayleigh regime). The significant DWR bias in LM and HW relative to DWR observations is somewhat puzzling in that the Liao et al. (2013) scattering model radar cross-sections agree with the synthetic complex shaped snow aggregates of the same mass at Ka-band whereas the HB model underestimates the radar cross-section relative to the synthetic complex shaped aggregates. On the other hand, Falconi et al. (2018) demonstrate that the soft spheroid model is adequate at X (close to Ku-band) and Ka-band and by inference adequate for DWR calculations with the caveat that different ‘effective’ axis ratios may need to be used at Ka- and W-bands.

We also refer to airborne (Ku,Ka) band radar data at 00:30 UTC which showed DWR measurements of 3-6 dB about 1 km height MSL around the CARE site but nearly 0 dB above that all the way to echo top (Skofronick-Skofronick-Jackson et al. 2015). The latter is not consistent with aircraft spirals over the CARE site about an hour earlier where maximum snow sizes reach ~8 mm. In spite of the difficulty in reconciling the observations from the different sensors, the appropriate scattering model in this particular event appears to favor the soft spheroid model used in HB based on better agreement with *DWR* observations. The other factor to be considered is the PSD adjustment factor which is assumed constant and independent of size which may not be the case, especially for the LM and HW methods as considerable filtering is involved due to mismatch (as discussed in Section 2.2). Note that a constant PSD adjustment factor will not affect *DWR* but it will affect Z_e . For the HB method Huang et al. (2015) determined the PSD adjustment factor for 4 events by comparing the 2DVD PSD to that measured by a collocated SVI (snow video imager which was assumed to be ‘truth’) for each size bin. The PSD adjustment was found not to be ~~not~~ size dependent for the HB method. On the other hand, because of the filtering of mis-matched particles by the LM and HW methods, the PSD adjustment factor may be size dependent in which case the *DWR* will also change. More case studies are clearly needed to understand the applicability of the LM and HW methods of simulating *DWR*.

3.4 Snow Rate Estimation

To obtain radar-*SR* relationships, we use the 2DVD data and simulations. Since we employ a constant PSD adjustment factor and it will have similar influence on both Z_e and *SR*, the 2DVD PSD used for scattering computation and *SR* need not be adjusted. Figure 13 shows the scatter plot of the 2DVD-derived Z_e versus 2DVD-measured *SR* along with a power-law fit as $Z = a SR^b$. The fitting method used is based on Weighted Total Least Square (WTLS) so the power law can be inverted without any change. The coefficients and exponents of the power-law *Z-SR* relationship for both bands and three methods are given in Table 3. It is obvious from Fig. 13 that there is considerable scatter at Ku-band for all three methods with the normalized standard deviation (NSTD) ranging from 55- to 70%. Whereas at Ka-band the scatter is significantly lower with NSTD from 40- to 45%. The errors in Table 3 are generally termed as parameterization errors.

By using dual-wavelength radar, we can estimate *SR* using Z_e at two bands as:

$$\begin{cases} SR_{Ku} = a'_1 * Z_{Ku}^{b'_1} \\ SR_{Ka} = a'_2 * Z_{Ka}^{b'_2} \end{cases}, \quad (5)$$

where $a' = (1/a)^{b'}$ and $b' = 1/b$. To reduce error, we may take the geometric mean of these two estimators as:

$$\overline{SR} = (SR_{Ku} * SR_{Ka})^{1/2} = c * Z_{Ku}^d * DWR^e, \quad (6)$$

where $c = (a'_1 a'_2)^{1/2}$, $d = (b'_1 + b'_2)/2$, and $e = -b'_2/2$. Note that the *DWR* in Eq. (6) is in linear scale i.e., expressed as a ratio of reflectivity in units of $\text{mm}^6 \text{m}^{-3}$. Using Table 3 to set the initial guess of (c,d,e) , non-linear least squares fitting was used

to determine the optimized (c,d,e) with the cost function being the squared difference between the 2DVD-based measurements of SR and $c \frac{Z_{ku}^d Z_{Ku}^d}{DWR^e}$, where $Z_{ku} Z_{Ku}$ and DWR are from 2DVD simulations. Figure 14 shows the SR computed from the 2DVD simulations of Ku-band Z_e and the DWR using Eq. (6) versus the 2DVD-measured SR . The (c,d,e) values for the three methods are given in Table 4. As can also be seen from Fig. 14 and Table 4, the $SR(Z_{Ku}, DWR)$ using LM method ~~has results in the smallest lowest~~ NSTD ~~(of 28.49%)~~%, but the other two methods have similar values of NSTD ~~($\approx 30\%$)-%) and, as such, these differences are not statistically significant. Although $SR(Z_{Ku}, DWR)$ has smaller parameterization error than Z_e - SR , the $SR(Z_{Ku}, DWR)$ estimation is biased high when $SR < 0.2$ mm/hr (see Fig. 14). When SR is small, the size of snowflakes is usually also small and falls in the Rayleigh region at both frequencies resulting in DWR very close to 1 (when expressed as a ratio). This implies that there is no information content in the DWR so including it just adds to the measurement error. Hence, for small SR or when $DWR \approx 1$, we use the Z_e - SR power law.~~

So far the single frequency SR retrieval algorithms were based on 2DVD-based simulations with a PSD adjustment factor using the total accumulation from Pluvio as a constraint. The algorithm we propose for radar-based estimation of SR is to use Eq. (6) when $DWR > 1$ and $SR > 0.2$ mm/h else we use the Z_{Ka} - SR power law (note: we ~~don't do not~~ use the Z_{Ku} - SR power law as the measurement errors of Z_{Ku} seem to be on the high side, Fig. 9). The precise thresholds used herein are *ad hoc* and may need to be optimized using a much larger data set. Figure 15a shows the radar-derived accumulation using Z_{Ka} - SR versus the Pluvio accumulation versus time. The total accumulation from the Pluvio is 2.5 mm and the three radar-based total accumulations-, respectively for HB, LM and HW methods amount to- [2.6, 1.8, 2.6 mm]. Except for the underestimate in the LM method (-28 %) the other two methods agree with the Pluvio accumulation in this event. Figure 15b is ~~the~~ same as Fig. 15a except the combination algorithm mentioned above is used. For this case, $\sim 33\%$ of data used the $-Z_{Ka}$ - SR power law due to threshold constraints given above. The event accumulations-, respectively for HB, LM and HW methods amount to- [2.4, 1.9, 2.2 mm] consistent with the algorithm that uses only the Z_{Ka} - SR power law. However, the criteria of relative bias error in the total accumulation (in events with low accumulations such as this one) ~~is are~~ not necessarily an indication that the DWR-based ~~algorithm~~ is not adding value. Rather, the criteria should be snow rate inter-comparison which could not be done due to the low resolution (0.01 mm/min) of the Pluvio² 400 ~~gagageauge~~ along with the low event total accumulation of only 2.5 mm. A close qualitative examination of Fig. 15b shows that the HB method more closely 'follows' in time the ~~gagageauge~~ accumulation relative to HB in Fig. 15a. In Fig. 15, the time grid is different for the radar-based data and the ~~gagageauge~~ data. It is common to linearly interpolate the ~~gagageauge~~ data to the radar sampling time and if this is done, the rms error for the HB method reduces from 0.1 mm (when using only the Z_{Ka} - SR power law) to 0.045 mm for the DWR algorithm, which constitutes a significant factor of two reduction.

The total error in the radar estimate of SR is composed of both parameterization errors as well as measurement errors with measurement errors dominating since the DWR involves the ratio of two uncorrelated variables. From Section 8.3 of Bringi and Chandrasekar (2001) the total error of SR in Eq. (6) is around 50% (ratio of standard deviation to the mean). The

assumptions are, (a) the standard deviation of the measurement of Z_e is 0.8 dB, (b) the standard deviation of the DWR (in dB) measurement is 1.13 dB, and (c) the parameterization error is 30% from Table 4. However, considering the Z_e fluctuations in Fig. 9, the measurement standard deviation probably exceeds 0.8 dB, especially at Ku-band. Thus, sufficient smoothing of DWR is needed to minimize as much as possible the measurement error ~~at the same time~~while maintaining sufficient spatial resolution.

Note that the error model used here is additive with the parameterization and measurement errors modeled as zero mean and uncorrelated with the corresponding error variances estimated either from data or via simulations (as described in Chapter 7 of Bringi and Chandrasekar 2001). This is a simplified error model since it assumes that radar Z and snow gage measurements are unbiased based on accurate calibration. A more elaborate approach of quantifying uncertainty in precipitation rates is described by Kirstetter et al. (2015).

4 Summary and Conclusions

The main objective of this paper is ~~to develop a technique~~development for snow estimation using scanning dual-wavelength radar operating at Ku and Ka-bands (D3R radar operated by NASA). We use the 2D-video disdrometer and collocated Pluvio ~~gage~~gauge to derive an algorithm to retrieve snow rate from reflectivity measurements at the two frequencies as compared to the conventional single-frequency Z_e - SR power laws. The important microphysical information needed is provided by the 2DVD to estimate the mass of each particle knowing the fall speed, apparent volume, area ratio and environmental factors from which an average density-size relation is derived (e.g., Huang et al. 2015; von Lerber et al. 2017; ~~Bohm~~Böhm 1989; Heymsfield and Westbrook 2010).

We describe in detail the data processing of 2DVD camera images (in two orthogonal planes) and the role of particle mis-matches that give erroneous fall speeds. We use the Huang et al. (2015) method of re-matching using single camera data but also use the manufacturer's matching code with substantial filtering of the mis-matched particles since the apparent volume and diameter (D_{app}) are more accurate. To account for the filtering of the mis-matched particles, the particle size distribution is adjusted by a constant factor using the total accumulation from the Pluvio as a constraint.

Two scattering models are used to compute the Z_{Ku} and Z_{Ka} termed as the soft spheroid model (Huang, Bringi et al. 2015; HB method) and the Liao-Meneghini (LM) model which uses the concept of "effective" density. In these two methods the particle mass is based on Böhm (1989). The method of Heymsfield and Westbrook (2010) is also used to estimate mass which is similar to Böhm (1989) but is expected to be more accurate (Westbrook and Sephton 2017); along with the LM model for scattering this method is termed as HW.

The case study chosen is a large-scale synoptic snow event that occurred over the instrumented site of CARE during GCPEx. The Z_{Ku} and Z_{Ka} were simulated based on 2DVD data and the three methods i.e., HB, LM and HW yielded similar values within ± 3 dB. When compared with D3R radar measurements extracted as a time series over the instrumented site, the LM and HW methods were closer to the radar measurements with HB method being lower by ≈ 3 dB. Some systematic deviations of simulated reflectivities by the three methods from the radar measurements were explained by a possible size dependence of the PSD adjustment factor.

The direct comparison of DWR (ratio of Z_{Ku} to Z_{Ka}) from simulations with DWR measured by radar showed that the HB method gave the lowest bias with the data points more or less evenly distributed along the 1:1 line. The simulation of DWR by LM and HW methods underestimated the radar measurements of DWR quite substantially even though the correlation appeared to be reasonable. The reason for this discrepancy is difficult to explain since a constant PSD adjustment factor (slightly different for the 3 methods) would not affect the DWR . From the scattering model viewpoint, the LM method takes into account the complex shapes of snow aggregates via an ‘effective’ density approach whereas the HB method uses soft spheroid model with density varying approximately inversely with size. We did not attempt to classify the particle types in this study.

The retrieval of SR was formulated as $SR = c * Z_{Ku}^d * DWR^e$ where $[c, d, e]$ were obtained via non-linear least squares for the three methods. The total accumulation from the three methods using radar-measured Z_{Ku} and DWR were compared with the total accumulation from the Pluvio (2.5 mm) to demonstrate closure. The closest to Pluvio was the HB method (2.4 mm), next was the HW method (2.24 mm) and LM ~~gave~~ (1.94 mm). At such low total accumulations, the three methods show good agreement with each other as well as with the Pluvio gagage. The poor resolution of ~~the gagage~~ combined with the relatively low total accumulation ~~in~~ this event precluded direct comparison of snow rates. The combined estimate of parameterization and measurement errors for snow rate estimation was estimated around 50%. From variance decomposition, the measurement error variance as a fraction of the total error variance was 58% and the parameterization error variance fraction was 42%. Further, the DWR was responsible for 90% of the measurement error variance which is not surprising since it is the ratio of two uncorrelated reflectivities. Thus, the DWR radar data has to be smoothed spatially (in range and azimuth) to reduce this error which will degrade the spatial resolution but is not expected to pose a problem in large scale synoptic snow events.

The snow rate estimation algorithms developed here are expected to be applicable to similar synoptic forced snowfall under similar environmental conditions (e.g., temperature and relative humidity) but not for example to lake effect snowfall as the microphysics are quite different. However, analysis of more events are needed before any firm conclusions can be drawn as to applicability to other regions or environmental conditions.

Appendix: List of Acronyms and Symbols

<u>Acronyms & Symbols</u>	<u>Description</u>	<u>Remarket</u>
<u>A</u>	<u>Minimum circumscribing circle or ellipse that completely contains A_e</u>	
<u>A_e</u>	<u>Particle's effective projected area normal to air flow.</u>	
<u>A_r</u>	<u>Area Ratio. The ratio of A_e to A (A_e/A)</u>	
<u>C_0</u>	<u>Inviscid drag coefficient.</u>	
<u>D_{app}</u>	<u>Apparent Diameter. It is equivalent-volume spherical diameter commonly used in 2DVD.</u>	
<u>DDA</u>	<u>Discrete Dipole Approximation</u>	
<u>D_d</u>	<u>Distance between two optical planes of 2DVD</u>	
<u>D_{eff}</u>	<u>Effective diameter.</u>	<u>Liao et al. 2013</u>
<u>DFIR</u>	<u>Double Fence International Reference</u>	
<u>D_m</u>	<u>Mass-weighted mean diameter.</u>	
<u>D_{max}</u>	<u>Maximum dimension of a particle.</u>	
<u>DWR</u>	<u>Dual-Wavelength Ratio</u>	<u>Eq. 3</u>
<u>D_0</u>	<u>Median volume diameter</u>	
<u>D3R</u>	<u>Dual-wavelength Dual-polarization Doppler Radar</u>	
<u>f_s</u>	<u>Scanning frequency of 2DVD line-scan camera</u>	
<u>GCPEX</u>	<u>GPM Cold-season Precipitation Experiment</u>	
<u>HB</u>	<u>Huang and Bringi method</u>	<u>Huang et al. 2015</u>
<u>HVSD</u>	<u>Hydrometeor Velocity Size Detector</u>	
<u>HW</u>	<u>Heymsfield and Westbrook method</u>	<u>Heymsfield et al. 2010</u>
<u>IWC</u>	<u>Ice Water Content</u>	
<u>LM</u>	<u>Liao and Meneghini method</u>	<u>Liao et al. 2013</u>
<u>$N(D)$</u>	<u>Concentration of PSD function of size</u>	
<u>PIP</u>	<u>Precipitation Imaging Package</u>	
<u>PPI</u>	<u>Plan Position Indicator</u>	
<u>PSD</u>	<u>Particle Size Distribution</u>	

<u>QPE</u>	<u>Quantitative Precipitation Estimation</u>	
<u>Re</u>	<u>Reynolds number</u>	
<u>RHI</u>	<u>Range Height Indicator</u>	
<u>SNR</u>	<u>Signal-to-Noise ratio</u>	<u>Eq. 1</u>
<u>SR</u>	<u>Liquid equivalent Snow Rate.</u>	<u>Eq. 4 and Eq.5</u>
<u>$V_{min_or_max}$</u>	<u>Minimum or maximum possible terminal fall speed</u>	
<u>v_t</u>	<u>Terminal fall speed</u>	
<u>X</u>	<u>Davies number also called Best number</u>	
<u>Z_{dr}</u>	<u>Differential Reflectivity</u>	
<u>Z_e</u>	<u>Equivalent Reflectivity Factor. In this paper, we refer to the reflectivity factor based on scattering computation.</u>	
<u>Z_h</u>	<u>Radar horizontal reflectivity factor</u>	<u>Eq. 1</u>
<u>$Z_{Ka\ or\ Ku}$</u>	<u>Radar reflectivity factor at Ka- or Ku-band</u>	<u>Eq. 5</u>
<u>Z_{offset}</u>	<u>The offset of radar reflectivity measurement</u>	
<u>δ_0</u>	<u>A dimensionless coefficient relate to boundary layer thickness.</u>	
<u>ϕ_{dp}</u>	<u>Differential propagation phase</u>	
<u>2DVD</u>	<u>2D-Video Disdrometer</u>	

Data Availability

The data used in this study can be made available upon request to the corresponding author.

Competing Interests

- The authors declare that they have no conflict of interest.

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Table 1: Hanesch 2DVD scan line criteria.

Max. of Total Scan Lines	Difference of Scan Lines
≤ 20	< 3
21-44	15%-11%
45-181	11%
≥ 182	20

Table 2: Some D3R parameters relevant for this study. Full D3R specifications can be found in Chandrasekar et al. 2010.

	Ku	Ka
Frequency (GHz)	13.91	35.56
Min. Detectable Signal	-10 dBZ at 15 km	
Range (km)	0.45 - 30	
Range Resolution (m)	150	
Ant. Beam Width	~ 1°	

Table 3: Coefficients and exponents of the power-law Z - SR relationship for “HB”, “LM”, and “HW” methods and Ku- and Ka-bands, respectively.

Method	Band	a	b	STD (mm/hr)	NSTD (%)
HB	Ku	140.52	1.48	0.2156	70.99
	Ka	60.17	1.18	0.1366	44.97
LM	Ku	129.27	1.64	0.2235	55.89
	Ka	99.85	1.25	0.1614	40.35
HW	Ku	106.25	1.58	0.1889	55.30
	Ka	66.96	1.42	0.1473	43.11

Table 4: Coefficients and exponents of the $SR(Z_{Ku}, DWR)$ relation [see Eq. (10)] for three methods.

Method	c	d	e	STD (mm/hr)	NSTD (%)
HB	0.0632	0.6537	-0.9155	0.0986	32.45
LM	0.0995	0.5648	-1.3415	0.1139	28.49
HW	0.1017	0.5426	-1.1772	0.1076	31.52

Figures

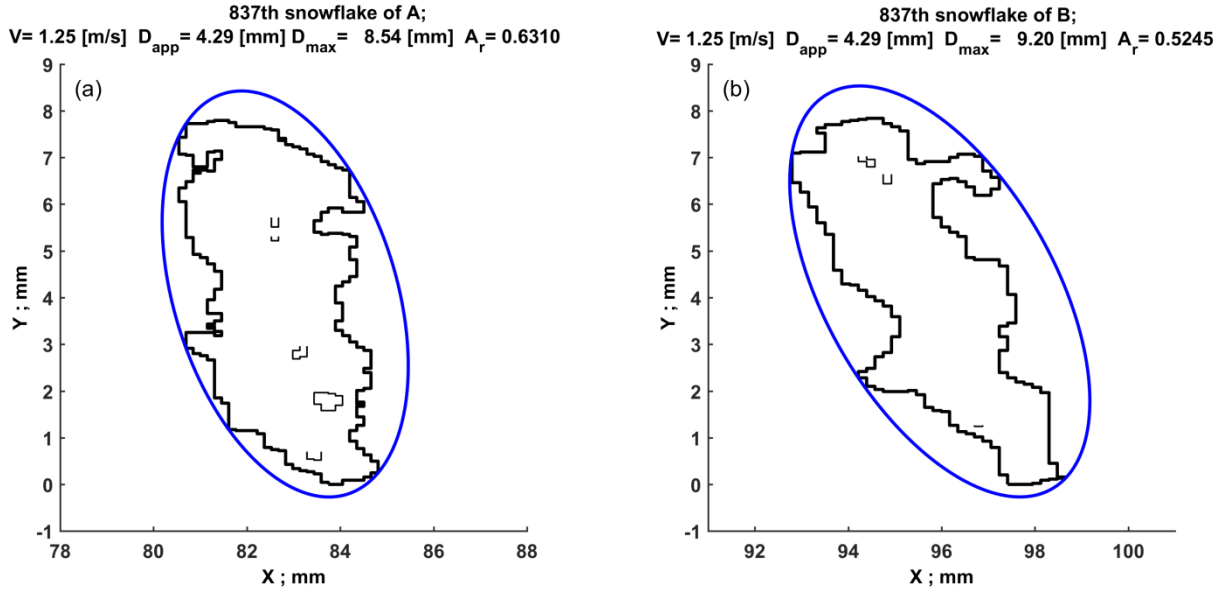


Figure 1: A snowflake observed by a 2DVD from two views. The thick black line is the contour of the snowflake and the thin black lines show the holes inside the snowflake. The effective area, A_e , equals the area enclosed by the thick black curve minus the area enclosed by thin lines. The blue line represents the minimum circumscribed ellipse, whose enclosed area is denoted by A .

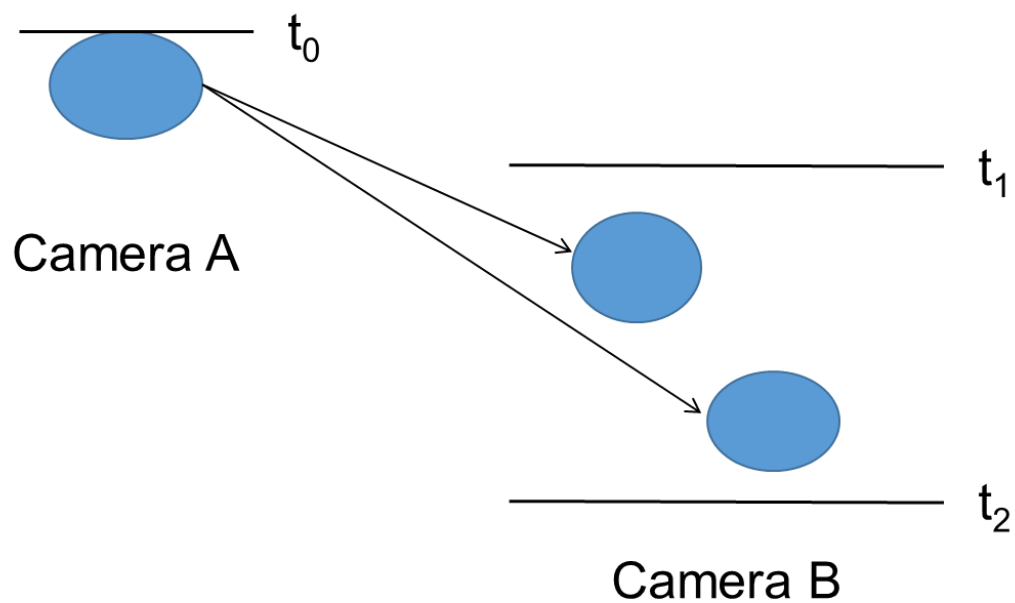


Figure 2: Illustration of the matching procedure. In the situation shown, it is assumed that camera A observed a particle at time t_0 , and afterwards during a certain time period t_1 to t_2 , camera B observed two particles. The matching procedure decides which particle observed by camera B is the same particle observed by camera A.

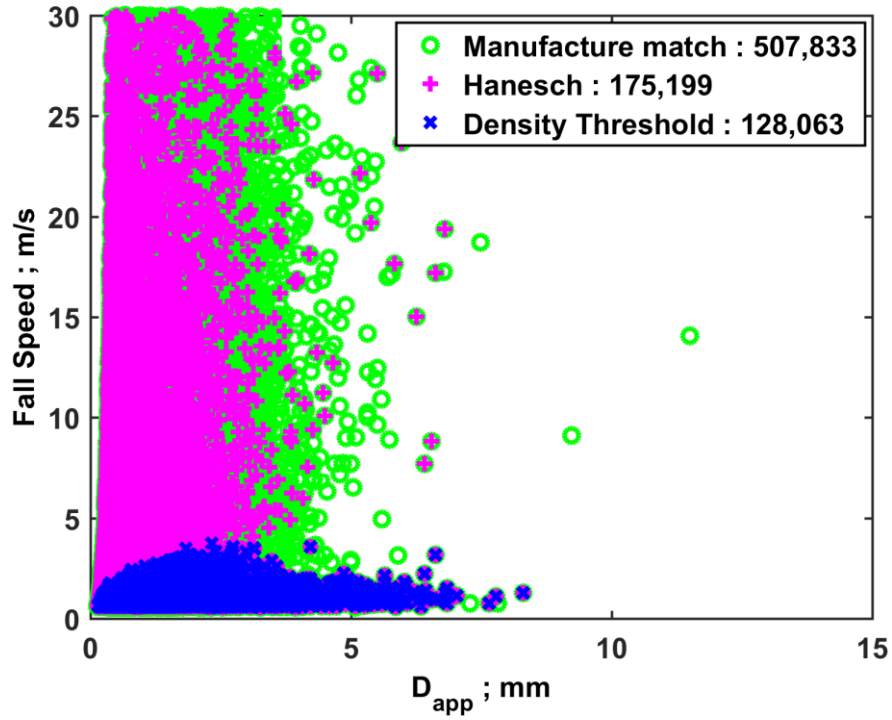


Figure 3: Fall speed versus D_{app} for the synoptic case on January 31, 2012 at the CARE site. The green circles represent the results of the manufacturer's matching algorithm. The magenta crosses are the matched particles which satisfy Hanesch scan line criteria. The blue "x"s are the density of particles (mass computed by Böhm's equations) lower than 1 g cm^{-3} .

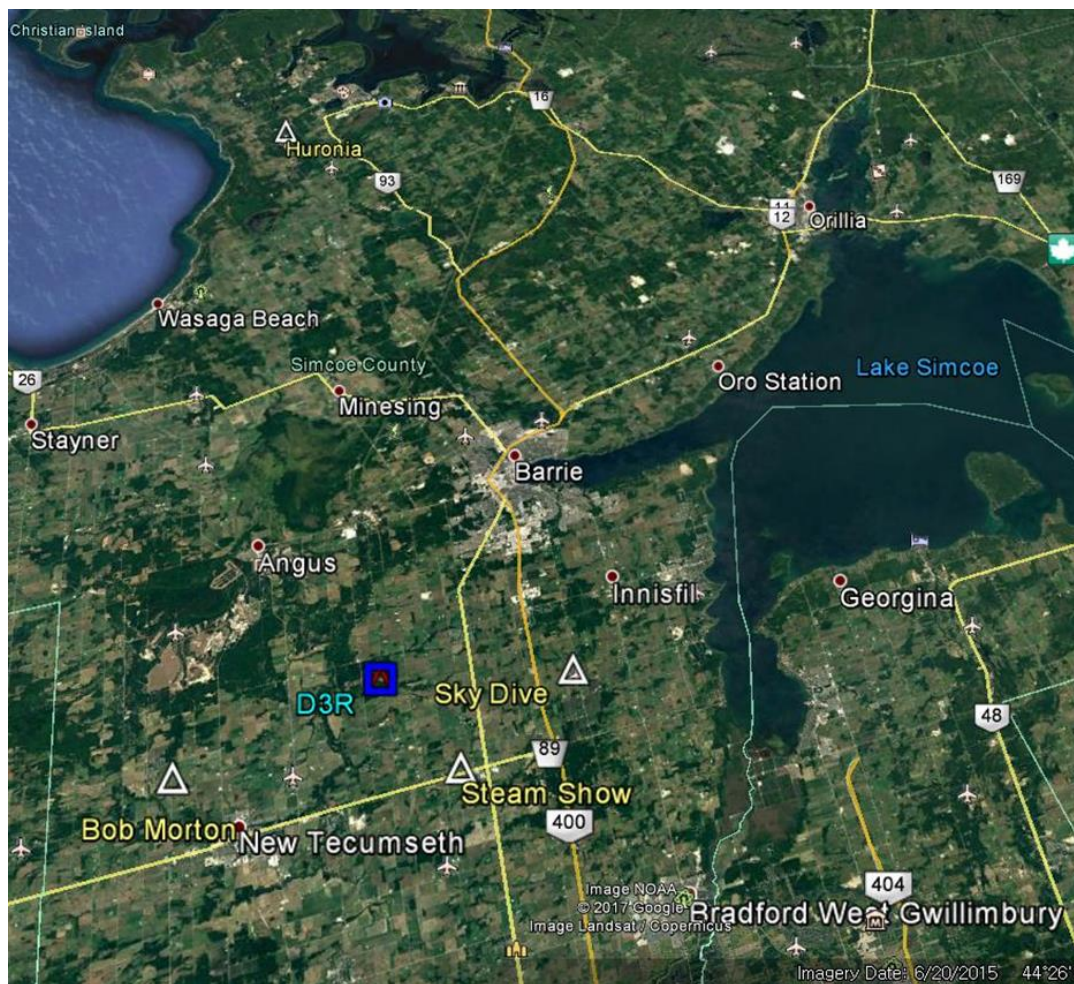


Figure 4: A map of the GCPEX field campaign. The five test sites are CARE, Sky Dive, Steam Show, Bob Morton, and Huronia. The ground observation instruments, namely, 2DVD, D3R, and Pluvio, used in this research were located at CARE.



Figure 5: Instruments used in this study (from left to right): 2DVD (SN37), D3R (Dual-wavelength Dual-polarized Doppler Radar), and OTT Pluvio² 400 precipitation gauge.

2012 12 31 00 UTC Analysis

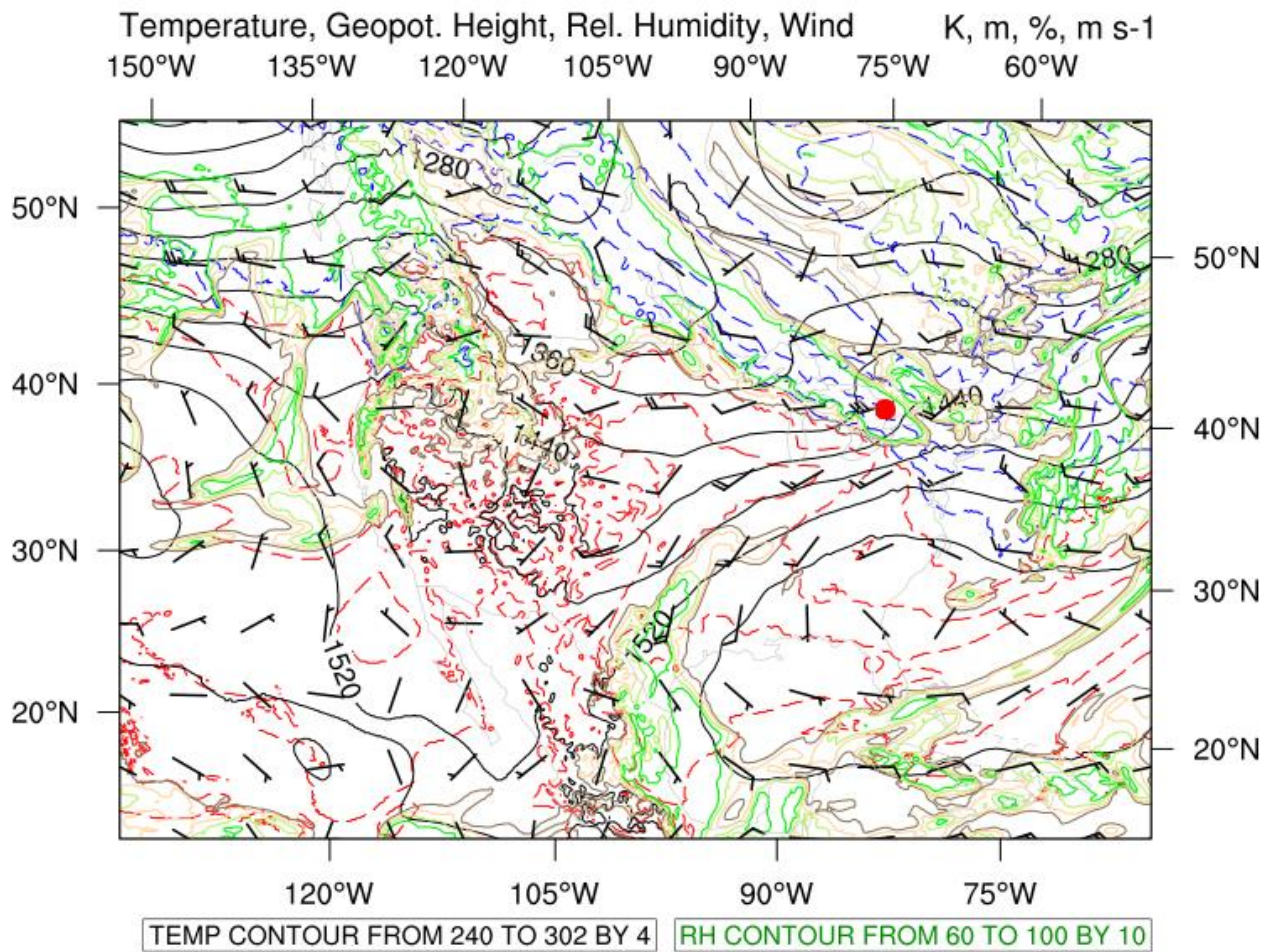


Figure 6: The 00 UTC 31 January 2012 850 hPa geopotential heights (m, black solid contours), temperature (K, red - above freezing, blue -below freezing), relative humidity (%), and wind (m s⁻¹, wind barbs). The red dot in the center right portion of the figure denotes the general location of the GCPEX field instruments.

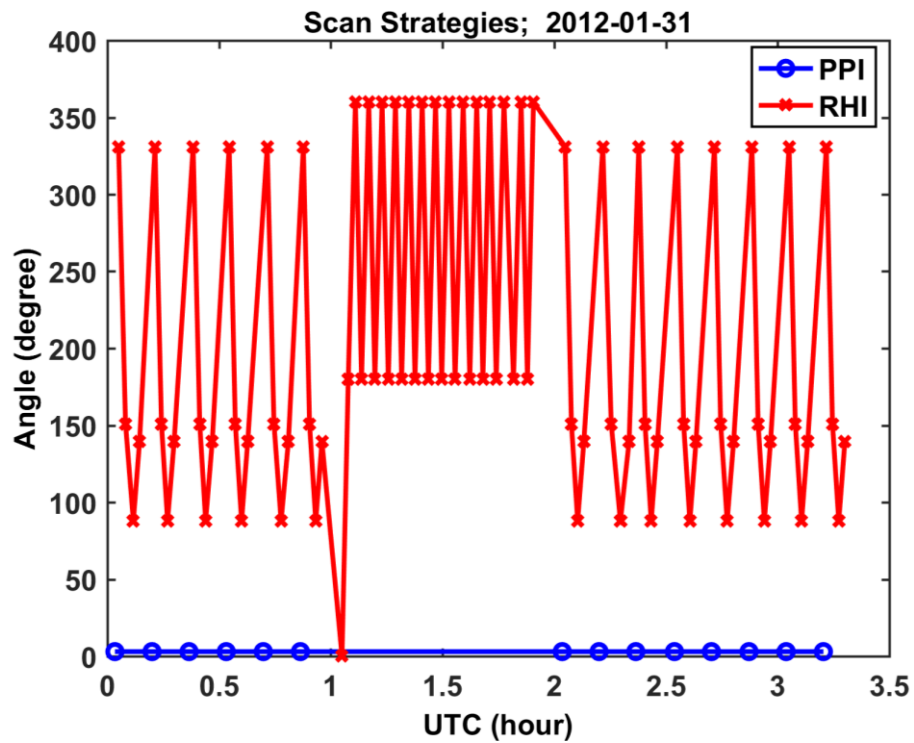


Figure 7: D3R scan strategies on 31 January 2012. The Y-axis is azimuth angle (RHI; red "x") or elevation angle (PPI; blue "o"). The scan rate of RHI was 1°/s and 10°/s for PPI.

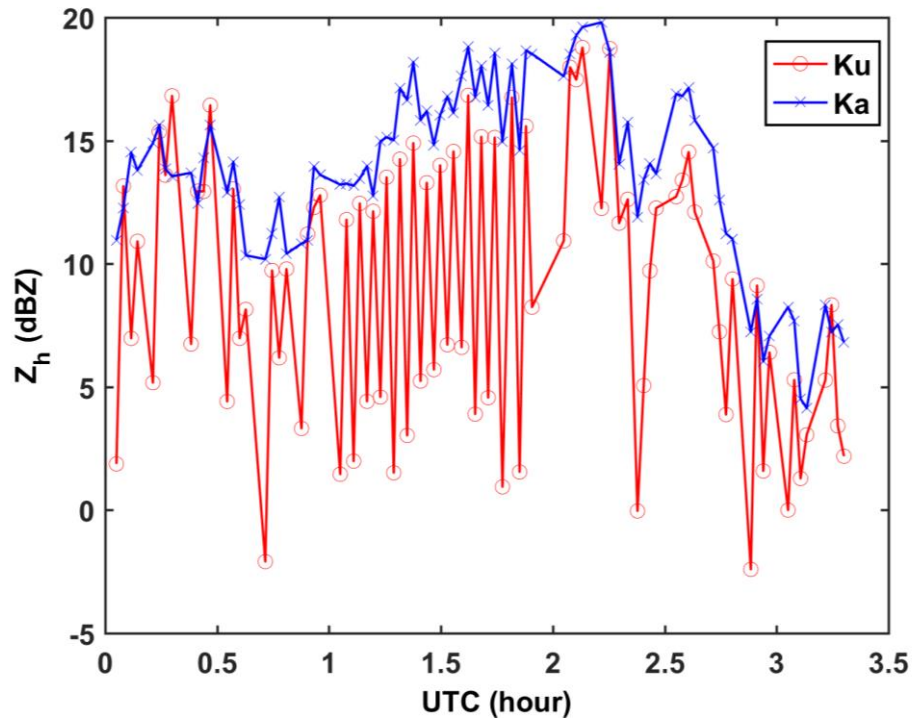


Figure 8: The time series of averaged raw Z_h at the CARE site. There are two problems indicated in this figure: (i) The Ku-band Z_h is smaller than the Ka-band Z_h on average. (ii) Compared with the Ka-band, there are many too small values of the Ku-band Z_h .

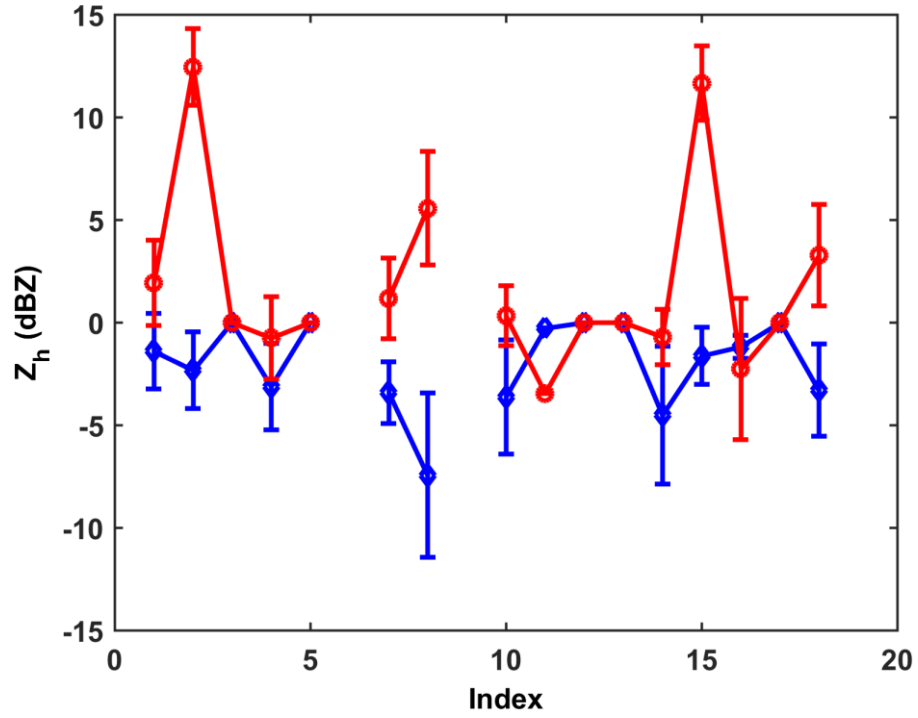


Figure 9: The averaged raw Z_h for Ku- and Ka-bands. The Z_h was randomly selected from 20 of 85 RHI scans with Ku-band $Z_h < 0$ dBZ, range < 1 km, and Ka-band SNR > 3 dB.

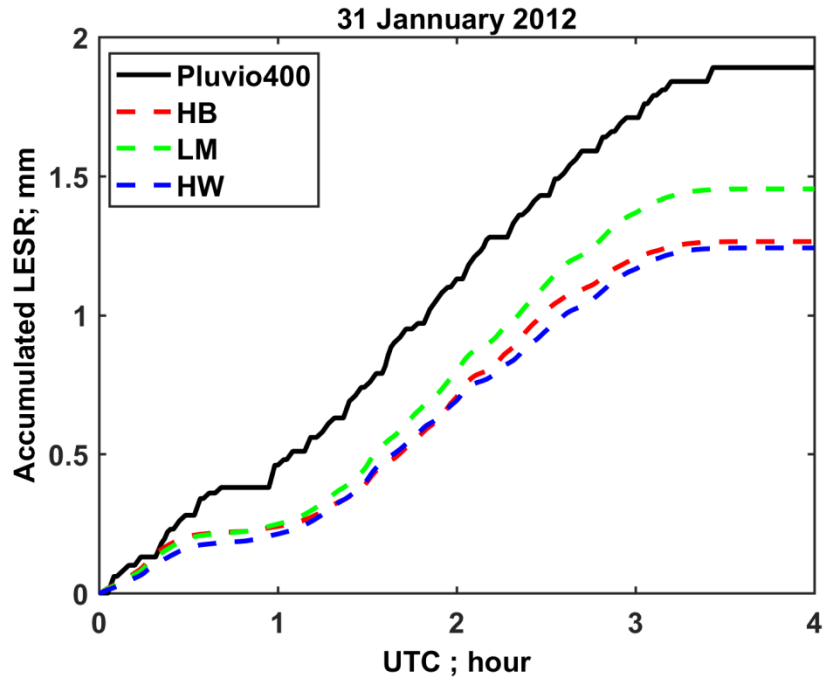


Figure 10: Comparison of liquid equivalent accumulations computed using HB, LM, and HW methods, respectively, based on 2DVD measurements and that were directly measured by the collocated Pluvio snow gauge. We used the total accumulation to estimate the PSD adjustment factor for the “LM” and “HW” methods.

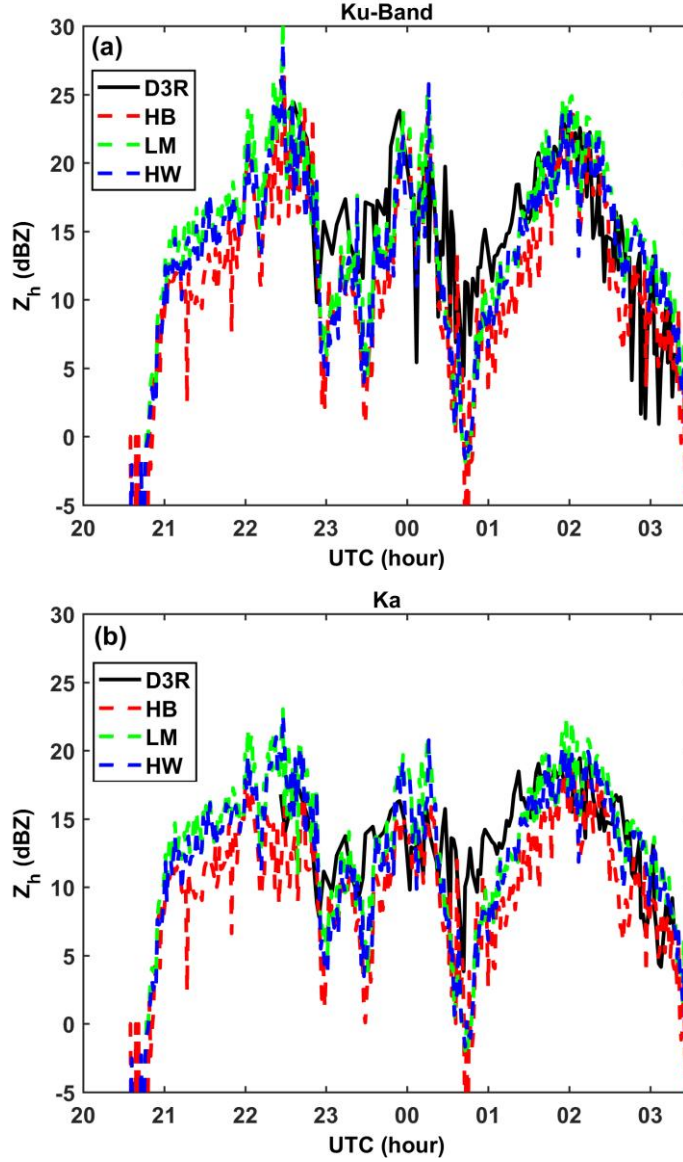


Figure 11: Comparison of the 2DVD derived Z_h with D3R measurements for the entire event, for Ku-band (a) and Ka-band (b). This synoptic system started at around 2100Z on 30 January and ended at around 0330Z on 31 January 2012. Z_e by "LM" is close to "HW" and slightly higher, whereas "HB" method gives the lowest Z_e . Z_e results computed by all methods generally agree with D3R measured Z_h .

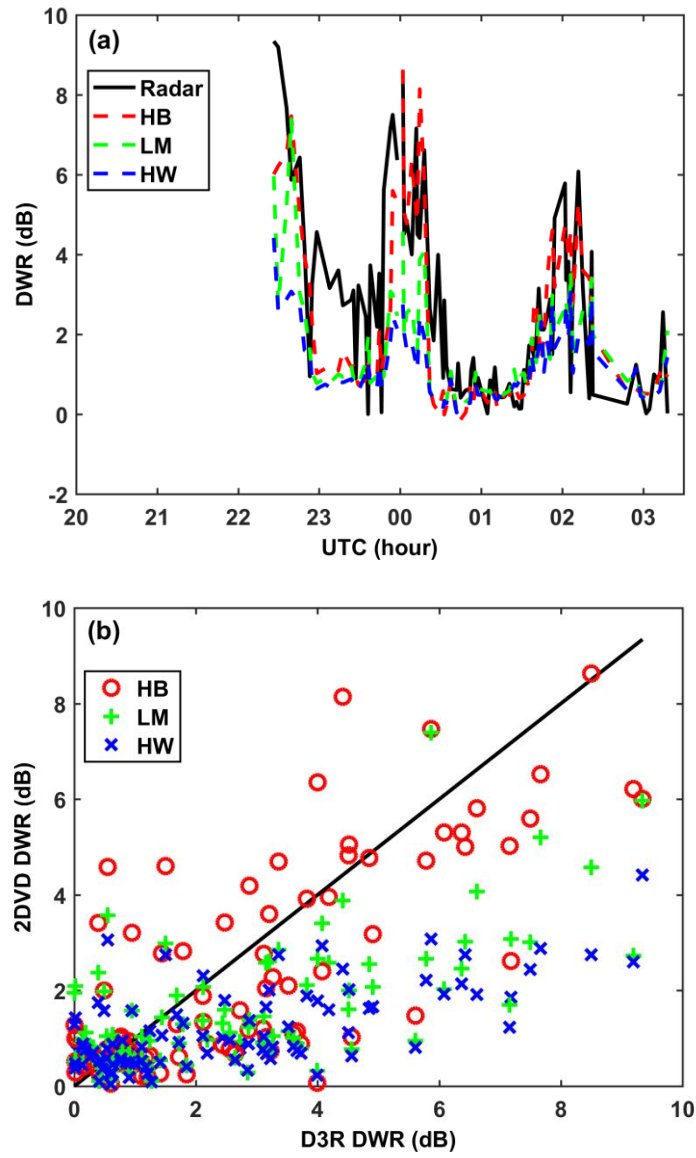


Figure 12: Comparison of the 2DVD derived DWR using “HB”, “LM”, and “HW” methods, respectively, with the D3R measured *DWR*. (a) shows the time profile of the D3R, and (b) shows the scatter plot.

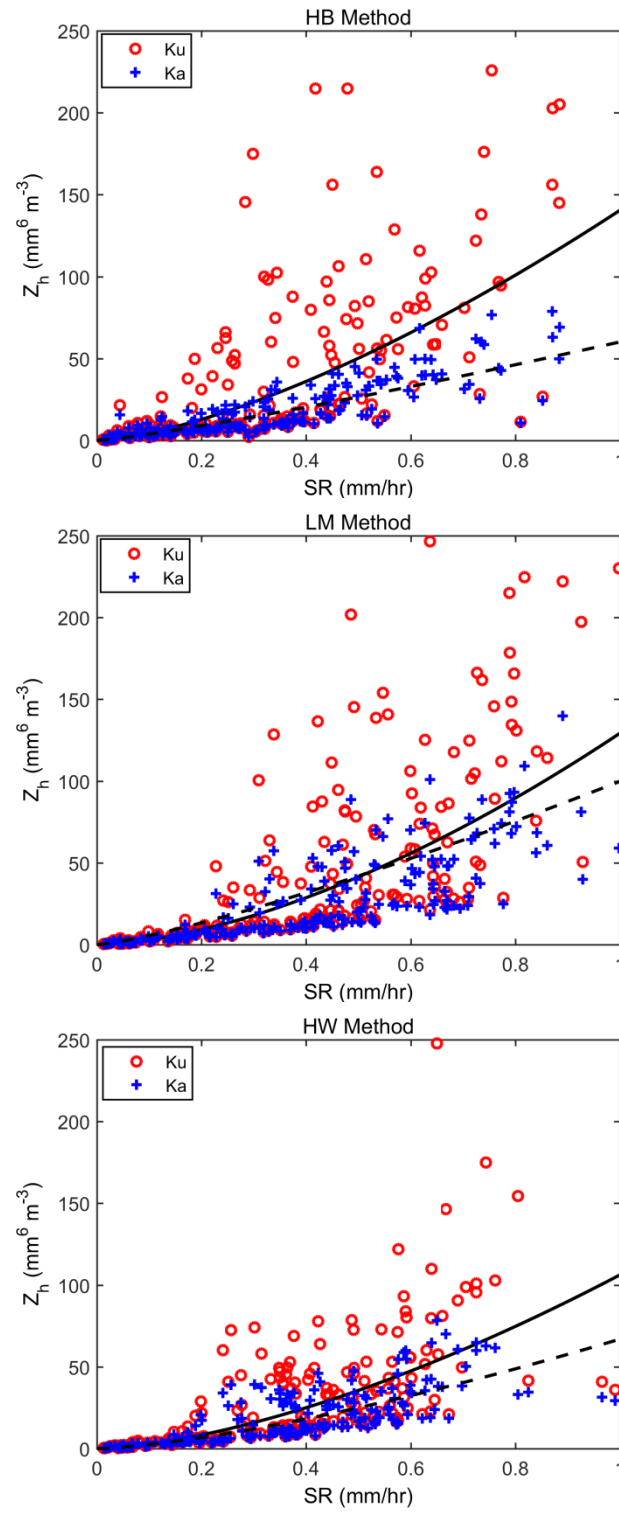


Figure 13: 2DVD derived Z_h versus 2DVD measured SR scatter plots, with Z -SR power-law fits, for Ku- and Ka-bands and “HB” method (top panel), “LM” method (central panel), and “HW” method (bottom panel).

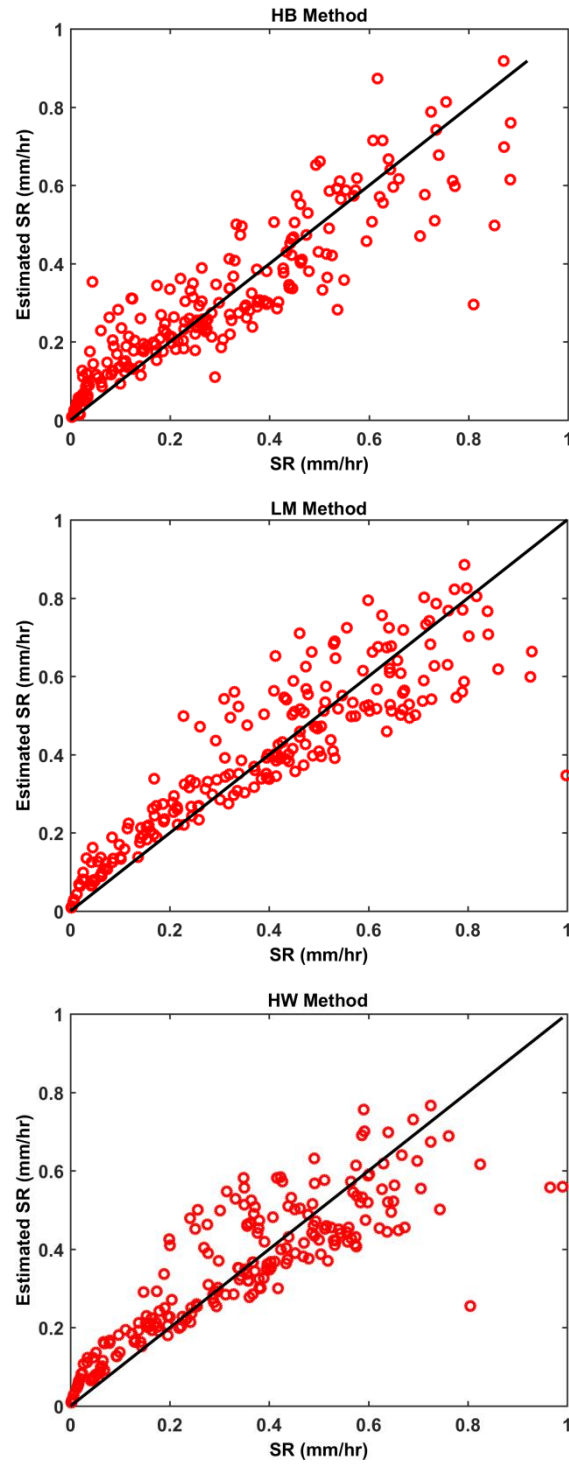


Figure 14: Estimated SR using Z_e and DWR of the 2DVD and Eq. (10) versus 2DVD SR scatter plot, for “HB” method (top panel), “LM” method (central panel), and “HW” method (bottom panel).

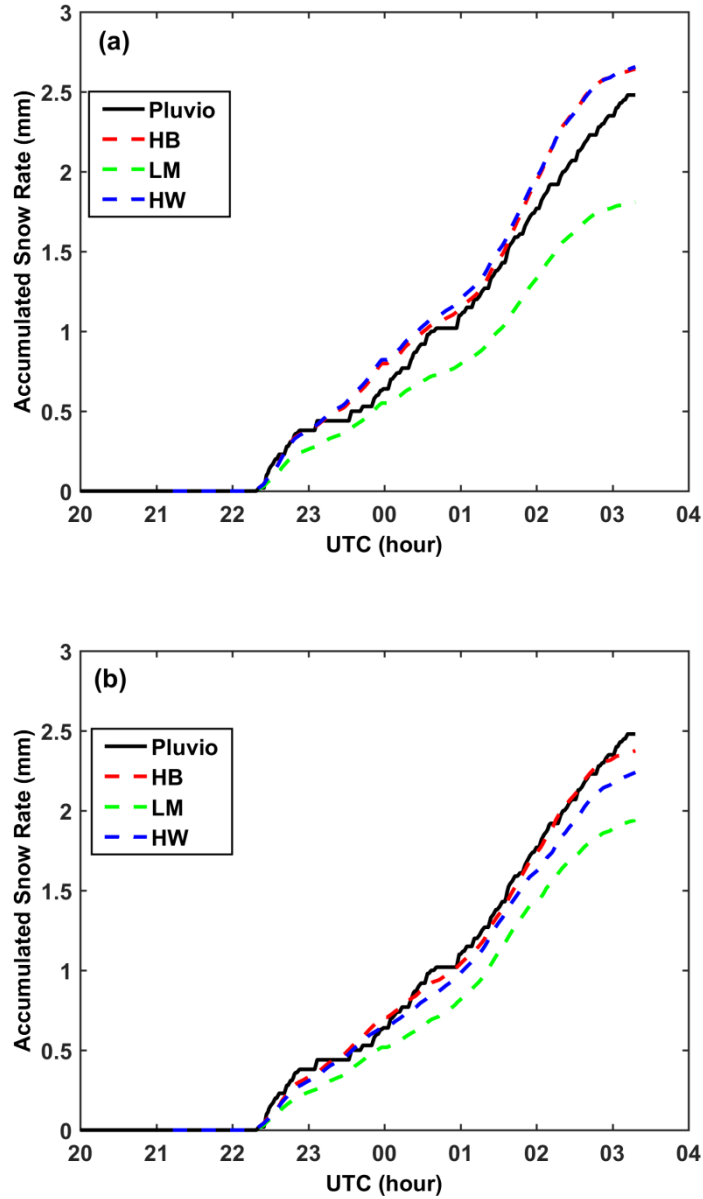


Figure 15: Comparison of the radar derived accumulated SR using “HB”, “LM”, and “HW” methods, respectively, with Pluvio gauge measurement. (a) The radar SR is computed by Z_{Ka} - SR relationships. The Pluvio accumulated SR on 03:18 UTC is 2.48 mm. The radar accumulated SR s for “HB”, “LM”, and “HW” are 2.64, 1.81 and 2.66 mm respectively. (b) The radar SR is computed by combining $SR(Z_{Ku}, DWR)$ and Z_{Ka} - SR as described in the text. The accumulated SR derived from the radar using “HB” method is 2.38 mm, using “LM” is 1.94 mm, and using “HW” is 2.24 mm.