Reviewer 1

Comment: *The paper by Koenig et al, addresses an important topic concerning the determination of the tropopause altitude from high resolution data using low resolution kernels. These have to be applied e.g. when comparing high resolution vertical data to satellite observations, when filtering oder interpolating data sets of different vertical resolution. The authors provide a systematic analysis using SHADOZ sonde data. They test for the effect of different resolutions (kernels) on the WMO lapse rate tropopause and cold point tropopause altitudes. They find, that the tropopause altitudes of the kernel weighted fine scale profiles differ significantly from the tropopause altitudes of the original data. Importantly they conclude that there is no correction scheme applicable to account for the displacement.*
The paper is sound and I regard it as highly relevant and recommend it for publication almost as it is. The authors could easily expand the scientific importance of their work by extending their analysis adding a systematic analysis of the cold point temperatures as well for different resolutions. Since the temperatures are crucial for the analysis of humidity transport, this would add another important aspect to this nice and important study.

Reply: Excellent idea!

Action: A new Section on water vapour saturation mixing ratios has been included, along with a table and a figure.

Comment: *Specific: It would be interesting to plot the cold point temperature in the same way as Fig. 7. Which implications does this have for the water vapor saturation mixing ratio?*

Reply: Agreed.

Action: Plots and tables of related cold point temperature have been included. Implications for the water vapor saturation mixing ratio are now discussed.

Comment: *Why are the differences mostly negative with decreasing resolution (Fig. 3, especially Fig. 7)?*

Reply: For the lapse rate tropopause, this is because weaker gradients are found already at lower altitudes if the resolution is coarser. And the tropopause search goes bottom up. For the cold point tropopause, we have no general explanation. This depends on the particular wiggles of each individual temperature profile.

Action: An explanation has been added to the text.

Comment: *Technical: Are there differences in the kernels used in Fig.1 and Fig.6? If not, is Fig.6 necessary for the paper?*

Reply: The main difference is the sampling of the degraded profiles. It is the sampling of the radiosonde profiles in Fig 6 (smoother curves) while it is a 1 km grid in Fig 1. Please note that the figure numbering has changed, it is now Fig. 5.

Action: We have changed the line styles to make this difference better discernable.

Comment: *p. 11, line 6. "coaErse"*

Reply: Thanks for spotting.

Action: This typo has been corrected.

Reviewer 2:

Comment: *General remarks:*
The manuscript by König et al. presents interesting aspects for the tropopause determination from temperatures profiles of restricted vertical resolution. This topic is not only crucial for satellite based temperature sounding but also for frequent analyses and applications of meteorological data sets. The mathematical background based on Rodgers (2000) is described in detail and allows a nearly correct and elegant description of the problem. In addition, it allows an accurate description and quantification of resulting error sources. The paper is well organized and written, and the scientific and technical objectives will fit to the scope of AMT.

Reply: We thank the reviewer for this encouraging evaluation.

Action: none

Comment: *However, I have strong concerns that the paper in the current form is adequately addressing the scientific and technical standards of AMT, I am generally missing a more in depth analysis and larger statistics (e.g. a larger set of radiosonde stations) to draw robust and meaningful conclusions. I can only recommend the publication of the study of König et al. after some major revisions and improvements. More detailed suggestions for improvements and comments are specified in the following sections.*

Reply: Please see our answers below.

Action: Those of the answers below.

Comment: *Major comments:*
The title of the paper promises more than the analysis and the final results can

deliver.

Reply: We do not quite see the point. The title is "Tropopause altitude determination from temperature profiles of reduced altitude resolution" and this is exactly what we critically assess in the paper.

Action: As stated below, we have reworded the title: "Tropopause altitude determination from temperature profile measurements of reduced vertical resolution". Since we have now found a correction scheme at least for midlatitudes, the reviewer might find the title more appropriate now.

Comment: *The tropopause (TP) determination of reduced altitude profiles - like announced - is only analyzed for one very specific instrument (MIPAS)[...]*

Reply: This is not true. We investigate into this effect for a series of idealized instruments with altitude resolutions of 1 to 5 km. This analysis is applicable to a wide range of instruments. Analyses based on averaging kernels from real instruments suffer from the fact that the averaging kernels have to be taken as they are, and the dependence of the effect on variation of the altitude resolution thus cannot be systematically assessed. Thus the focus of the study is on the idealized averaging kernels where the altitude resolution can be varied. MIPAS results are presented in addition as an illustrative case study.

Action: none

Comment: *[...] but is a quite general problem and especially important for many studies taken into account meteorological data sets like ERA-Interim or MERRA. Very similar problems occur for these type of data, if you like to quantify the error in the TP determination for the relatively coarse vertical sampling around the TP compared the typical fine resolution of the radiosonde data. TP heights are not part of the meteorological data sets. It would be by far more interesting to apply the methodology in a more general approach , for example to the problem outlined above. Take these criticisms into account the author should change the title accordingly.*

Reply: This study was performed with application to satellite measurements in mind. The applicability of the methodology presented is an added value but for a paper in Atmospheric Measurement Techniques we find it adequate to restrict the study to atmospheric measurements.

Action: To avoid wrong expectations we change the title to "Tropopause altitude determination from temperature profile measurements of reduced altitude resolution." We mention that the range of applicability of our concept may be wider but that this is beyond the scope of our paper.

Comment: *The abstract is extremely short and includes even repetitions ('3* *km vertical resolution'). The reader may ask, are there as little results? This is also true for the conclusions and unfortunately my final impression of the presented study, although there seems a high potential in the formalism.*

Reply: The additional analysis performed in reply to both reviews added length to the abstract and the short sections.

Action: The additional results have been included in the abstract.

Comment: *The data base and the statistical analyses have a couple of limitations, which need improvements in a new version of the paper: a) The number of 30 radiosonde profile for only one station (Nairobi) is far too small for significant conclusions based on the presented analyses and statistics.*

Reply: We wonder in which sense the reviewer uses the term 'significant' here. The context of sample size suggests that it is meant as a statistical technical term, while the context of 'conclusion' suggests that the term is used in a colloquial sense. In statistics only differences can be significant. In our case, the differences are considerably larger than the uncertainties of even a single profile (which has nothing to do with the standard error of the mean or other related statistical quanties. It is the mere measurement error which is relevant here). Thus we can say even for a single profile that the effect is significant. For any given profile, the effect is deterministic. Note that in the original manuscript we did not recommend any inductive correction based on the mean tropopause altitude displacement on the basis of this limited data set.

Action: We have increased the sample size considerably to gain a better idea on the representativeness of our results.

Comment: *b) The selection of only one station representative for the tropics seems also critical. Are there any references for this simplification? I guess continental and coastal area can have quite different temperature profile (wave activity), also regions with strong and low convection activity. If the study likes to stay with its focus on the tropics, then more stations and coincidences with MIPAS should be taken into account.*

Reply: We do not see how a statistic should become more robust by making the sample more inhomogeneous. Nevertheless we take this suggestion and include analyses of other sites but we do not think that it would be a good idea to merge all this into one statistic.

Action: We have included one further tropical station and two stations in midlatitudes.

Comment: *c) More stations and profiles (by taken a longer time period) would also help to bypass the very coarse coincidence criteria applied in this study. A*

± 1000 km in longitudinal direction and ±500 km in latitude is by far too coarse to define a proper coincidence. In addition, I am missing a miss-time criterion in the manuscript?

Reply: This reads as if the reviewer has misunderstood our approach. Coincidence criteria would indeed be far too coarse if we had compared MIPAS profiles to radiosonde profiles but we did not do that. We only have applied MIPAS averaging kernels to the sonde profiles and compare the original sonde profiles to those where the averaging kernel has been applied. MIPAS averaging kernels are only weekly or at best moderately temperature-dependent, and thus this issue is, in this application, a higher order effect. The only problem left is geolocation-dependent priori information.

Action: We say now clearly in the text that application to MIPAS is meant only as an illustrative case study. We have reduced the number of MIPAS cases in order to guarantee fairly close collocations. We have removed all related statistics in order not to lead the reader astray towards misinterpretation of these results in a statistical sense.

Comment: *d) All statistic plots suffer on the general problem of the study of rather low number of profiles/coincidences. For me it makes no sense to fit Gauss distributions to histograms or to present box-whisker plots for such low ensemble numbers.*

Reply: Even in the original manuscript no conclusions were drawn from the fitted Gaussian distributions. All statistics came (and comes) directly from the data without any detour over Gaussian pdfs.

Action: Sample sizes have been increased. The Gaussian curves were removed.

Comment: *Overall, I would recommend to apply the methodology not only to the tropics, because TP determination it is a general problem at all latitudes, which would give the study a much broader scientific relevance.*

Reply: Agreed.

Action: Two sites in midlatitudes have been included.

Comment: *In addition, the authors should think about to apply the formalism to temperature profiles of meteorological analyses, which would give a much broader scientific community a tool or reference to quantify uncertainties in the tropopause determination (e.g. for tropopause related coordinates, definition of the transport barrier).*

Reply: We are glad if the theoretical part helps also for these applications but we think that for a paper in a journal on measurement techniques it is justified to restrict the analysis to measurements.

Action: We mention the possible applicability of our concept to other types of data but state that this is beyond the scope of this paper.

Comment: *Minor comments:*
The authors should reference in the introduction to other limb based remote sensing analyses in former publications or to more general publications highlighting the difficulties and importance of an accurate tropopause determination (e.g. Pan and Munchak, 2011, Peevy et al., 2012, or Spang et al., 2015).

Reply: Agreed.

Action: These and some more references have been included.

Comment: *What is the effect of the higher resolved and vertically more structured a priori profiles (e.g. ERA interim) on the MIPAS temperature retrievals and finally on the MIPAS TP determination. Can you quantify this effect with your methodology?*

Reply: This depends on the relation between the amplitudes of the fine structure and those resolved by the instrument. No general statement can be made on this.

Action: Some discussion has been added.

Comment: *Why do the authors include Figures 6 with no additional information compared to Figure 1, what is new or has to be highlighted here? Tropopause heights (radiosonde and potentially for different degraded resolutions) should be superimposed in both Figures.*

Reply: The different characteristic (sampling of the smoothed profiles) was simply not visible with the line style chosen in the original manuscript.

Action: The line style of the smoothed profiles has been changed to solid lines to make the sampling characteristic better visible.

Comment: *Section 4 on the feasibility of correction schemes is missing a detailed analysis and the description is too short. This section has currently not the substance for a full section in a paper, it's just a result for a paragraph. Again the number of profiles is not sufficient. I am wondering why the author made the analysis with such a limited data set of radiosonde profiles and MIPAS profiles. It will be easy - but of coarse additional work - to extend the complete study to a larger database and to draw better and more profound conclusions.*

Reply: Again, we suspect that there is a misunderstanding because no MIPAS profiles have been used for this purpose. As expected, for Nairobi the scatter is, even after increasing the sampling size considerably, still way too large to recommend a correction. This does not come as a surprise, because the standard deviation does not converge towards zero for increasing sample size but converges towards the true standard deviation. Since the point of interest is the correction of individual profiles, it is the scatter which is relevant, not the standard error of the mean. The scatter of a sample with a size larger than approximately 30 commonly is typically fairly robust such that even the use of a student's t distribution instead of a Gaussian is considered unnecessary.
For the midlatitudinal stations we have found a fairly robust correction scheme which is now discussed in this section and adds length.

Action: Additional results have been included in this section.

By the way: This article was intended to be a "Technical Note". The manuscript was then published as a regular discussion paper because the format "Technical Note" is not supported by AMT. At the AMT editorial board meeting in Vienna 2019 one of us suggested to the AMT Executive Editors to introduce "Technical Note" as a new article format in AMT. Their reply was that this is not necessary because length is no criterion for AMT articles.

Tropopause altitude determination from temperature ~~profiles~~ profile measurements of reduced ~~altitude~~ vertical resolution

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Abstract. Inference of the lapse rate tropopause or the cold point from temperature profiles of finite vertical resolution entails an uncertainty of the tropopause altitude. For tropical radiosonde profiles the tropopause altitude inferred from coarse grid profiles was found to be lower than that inferred from the original profiles. The mean (median) displacements of the lapse rate tropopause altitude when inferred from a temperature profile of 3 km vertical resolution and a Gaussian kernel ~~is -240 m~~are -130 m, -420 m, -730 m, and -510m (-70 m, -230 m, -390 m, and -280 m) for Nairobi, Hilo, Munich, and Greifswald, respectively. In case of a MIPAS averaging kernel the displacement of the lapse rate tropopause altitude is -640 m. The mean (median) displacement of the cold point tropopause inferred from a temperature profile of 3 km vertical resolution (Gaussian kernels) was found to be ~~-500 m. The~~ -510 m, -620 m, -530 m, and -410 m (-460 m, -520 m, -370 m, and -280 m) for the stations mentioned above. Unsurprisingly, the tropopause altitude displacement is larger for coarser resolutions. The effect of the tropopause displacement on the water vapour saturation mixing ratio is roughly proportional to the vertical resolution. In tropical latitudes the resulting error is about one to two parts per million by volume per vertical resolution in km. The spread of the ~~results~~ tropopause displacements within each sample seems too large as to recommend a correction scheme for tropical temperature profiles, while for midlatitudinal temperature profiles of vertical resolutions of 1 to 5 km a lapse rate of -1.3 K/km reproduces tropopause altitudes determined from high-resolution temperature profiles with the nominal lapse rate criterion of -2 K/km fairly well.

1 Introduction

The tropopause constitutes a vertical separation in the atmosphere that segregates the lower, weather active region, *viz.*, the troposphere, from an upper, steadier region, the stratosphere. High altitude temperature soundings that became possible at the end of the 19th century showed an – at that time – unexpected temperature behaviour, where temperatures would stagnate or even increase with height (see Hoinka, 1997, for a historical overview). Once it was established that this observation was no measurement error, and that above the troposphere another region of the atmosphere ~~exist~~exists, namely the stratosphere, an unambiguous definition for the height of the boundary, the tropopause, had to be agreed. The earliest comprehensive definition provided by the British Meteorological Office was based on either the existence of a temperature inversion, or an abrupt transition to a temperature gradient below 2 K/km. If the first two criteria were not met, a more general vertical temperature gradient criterion was applied: "at the point where the mean fall of temperature for the kilometre next above is 2 K or less provided that it does not exceed 2 K for any subsequent kilometre" Dines (1919, cited after Hoinka, 1997). A similar definition, focusing solely on the lapse rate of 2 K/km was adapted by the World Meteorological Organization (WMO) in later years (World Meteorological Organization, 1957). Since then additional definitions of the tropopause have emerged, focusing on the behaviour of dynamical quantities (e.g. Hoerling et al., 1991) or of trace gas changes (e.g. Pan et al., 2004). However, the most commonly used method to define the position of the tropopause still is the WMO criterion.

In tropical latitudes, another useful reference for distinguishing the tropopause from the stratosphere is the cold point (where the temperature minimum occurs). It relates to the existence of a temperature inversion in the original definition as described above and the corresponding lapse rate tropopause lies commonly a few hundred meters below the cold point (e.g., Figure 8 in Kim and Son, 2012).

Aspirational targets exist for knowing the altitude distribution of the thermal tropopause with an uncertainty of 100 m globally (see the Observing Systems Capability Analysis and Review Tool at https://www.wmo-sat.info/oscar/variables/view/81). However, it is obvious that deriving the altitude of a lapse rate tropopause will depend to some extent on the resolution of the temperature profile that is used to calculate the vertical gradient. The same holds true for the cold point tropopause. Thus, it seems important to understand how the derived altitude of the tropopause depends on the vertical resolution of the temperature data. Knowledge of the exact tropopause altitude is essential in particular, when distributions of atmospheric state variables such as mixing ratios of trace species are transformed to a tropopause-related vertical coordinate system in order to investigate chemical, transport and mixing processes in the upper troposphere and lowermost stratosphere . (e.g. Tuck et al., 1997; Pan et al., 2004; Birner, 2006; Tilmes et al., 2010; Pan and Munchak, 2011). Tropopause altitudes inferred from limb measurements have been reported by, e.g., Peevey et al. (2012) or Spang et al. (2015).

The goal of this paper is to analyze the possible dependence of a derived tropopause altitude on the vertical resolution of the temperature profile and to evaluate possibilities to potentially correct tropopause altitudes inferred from coarsly resolved temperature profiles. After presenting the formal concept used for this study (Section 2), we first asses the impact of finite vertical resolution on the determination of the tropopause altitude in quantitative terms (Section 3). We do this separately for lapse rate tropopause altitudes (Section 3.1) and cold point tropopause altitudes (Section 3.2). Then, focusing on low latitudes, The implication for the saturation mixing ratio of water vapour is analyzed in Section 4. Then we investigate if related altitude errors can be corrected by a slight modification of the tropopause definition which, when applied to temperature profiles of finite vertical resolution, reproduces the tropopause altitude according to the WMO definition when applied to the original data (Section 5). Finally we discuss the applicability of our results to various types of constrained temperature retrievals from satellite data (Section ??) and conclude what the upshot of this study is from a data user perspective (Section 6).

2 The formal concept

The altitude resolution of a vertical profile such as temperature can be characterized by the $n \times n$ averaging kernel matrix $\mathbf{A}$ (Rodgers, 2000). It consists of the partial derivatives $\frac{\partial \tilde{x}_i}{\partial x_j}$ of the elements $\tilde{x}_i$ of the degraded profile with respect to the variation of the element x_j of the true profile. Its columns represent the relative response of a degraded profile $\tilde{\boldsymbol{x}}$ to a delta perturbation of the true profile $\boldsymbol{x}$. Conversely, the jth column represents the weights with which the elements of the true profile contribute to the jth element of the degraded profile. The averaging kernel of a profile without degradation is the identity matrix $\mathbf{I}$. It goes without saying that effects on a finer scale than that reproducible in the n-dimensional grid remain undetected, unless some prior information on the profile shape between the gridpoints is used, as suggested by, e.g., Reichler et al. (2003). This is to say, the averaging kernel does not characterize the degradation with respect to the fully resolved true profile but only the degradation with respect to the profile represented in a vector of n gridpoints.

Typically, the reduction of altitude resolution is caused by one of the following three mechanisms: (1) the atmosphere is remotely sensed by an instrument of finite vertical resolution. In this case, the atmospheric state is often sampled on a grid finer than that corresponding to the altitude resolution of the measurement system; (2) A high-resolution profile is resampled on a coarser grid. This resampling goes along with a degradation of the altitude resolution; (3) a filter function is applied, which reduces the vertical resolution.

2.1 Remotely sensed vertical profiles

Often the degradation, i.e., the loss of vertical resolution, is caused by the use of a constraint in the retrieval of atmospheric state variables from remote measurements $\boldsymbol{y}$. The estimated state $\hat{\boldsymbol{x}}$ depends on the measurement $\boldsymbol{y}$ and the prior information $\boldsymbol{x}_{\mathrm{a}}$ as

$$\hat{\boldsymbol{x}} = \boldsymbol{x}_{\mathrm{a}} + \left(\mathbf{K}^T \mathbf{S}_{\mathrm{y}}^{-1} \mathbf{K} + \mathbf{R}\right)^{-1} \mathbf{K}^T \mathbf{S}_{\mathrm{y}}^{-1} \left(\boldsymbol{y} - \boldsymbol{f}(\boldsymbol{x}_{\mathrm{a}})\right) \tag{1}$$

where $\mathbf{K}$ is the Jacobian matrix $\frac{\partial y_i}{\partial x_j}$, ^{T} indicates a transposed matrix, $\mathbf{S}_{\mathrm{y}}$ is the covariance matrix characterizing measurement noise, $\mathbf{R}$ is a regularization matrix, and $\boldsymbol{f}$ is the radiative transfer function (von Clarmann et al., 2003a). Using an inverse a priori covariance matrix $\mathbf{S}_{\mathrm{a}}^{-1}$ as regularization matrix, this formalism renders a maximum a posteriori retrieval as described by Rodgers (2000). Other widely used choices of $\mathbf{R}$ are squared lth order difference matrices (see, e.g. von Clarmann et al., 2003b). The latter are often used in order to stabilize the profile by smoothing without pushing the values towards an a priori profile (e.g. Steck and von Clarmann, 2001).

In all cases, the averaging kernel matrix is

$$\mathbf{A}_{\mathrm{retrieval}} = \left(\mathbf{K}^T \mathbf{S}_{\mathrm{y}}^{-1} \mathbf{K} + \mathbf{R}\right)^{-1} \mathbf{K}^T \mathbf{S}_{\mathrm{y}}^{-1} \mathbf{K}, \tag{2}$$

and with this the state estimate can be separated into two components, which are the contribution of the true atmospheric state and the contribution of the prior information

$$\begin{aligned} \hat{\boldsymbol{x}} \quad = \quad & \mathbf{A}_{\mathrm{retrieval}} \boldsymbol{x}_{\mathrm{true}} \\ & + (\mathbf{I} - \mathbf{A}_{\mathrm{retrieval}}) \boldsymbol{x}_{\mathrm{a}} + \epsilon_{x;total} \end{aligned} \tag{3}$$

where, as its index suggests, $\boldsymbol{x}_{\mathrm{true}}$ represents the true temperature profile, and $\epsilon_{x;total}$ is the actual realization of the retrieval error.

The altitude resolution of the retrieval can be determined from the averaging kernel matrix. Common conventions are to either use the halfwidths of its rows or the gridwidths divided by the diagonal elements. It goes without saying that the altitude resolution of a retrievend profile can be altitude-dependent.

2.2 Resampling on a coarser grid

Other causes for degraded profiles are representation on a grid not sufficiently fine to represent all structures or application of a numerical filter to the original profile. The averaging kernel matrix is the adequate tool for dealing with all these cases.

The effect of a coarse grid is best understood by construing the coarse-grid profile as a result of an interpolation of the profile from a finer grid (see Rodgers 2000, Sect 10.3.1, where a slightly different notation is used). Let $\tilde{\boldsymbol{x}}$ be the profile represented on the coarse grid, and $\boldsymbol{x}$ the profile in the original representation where all fine structure is resolved. In this case we use an interpolation matrix $\mathbf{V}$ and get

$$\tilde{\boldsymbol{x}} = \mathbf{V}\boldsymbol{x} \tag{4}$$

For an interpolation from a fine grid to a coarse grid $\mathbf{V}$ is often chosen as

$$\mathbf{V} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T, \tag{5}$$

where $\mathbf{W}$ is the interpolation from the coarse to the fine grid. A definition of $\mathbf{A}$ based on $\mathbf{V}$ gives us an asymmetric averaging kernel matrix which represents the dependence of the profile values on the coarse grid on the “true” values on the fine grid.

$$\mathbf{A}_{\text{coarse}} = \mathbf{V} \tag{6}$$

Contrary to the averaging kernel matrix introduced by Eq (2), $\mathbf{A}_{\text{coarse}}$ is not quadratic.

To characterize the loss of resolution due to coarse sampling the averaging kernel on the fine grid, $\mathbf{A}_{\text{interpolation}}$, is needed. It is

$$\mathbf{A}_{\text{interpolation}} = \mathbf{W}\mathbf{V}. \tag{7}$$

If the profile on the fine grid is in itself a degraded profile, e.g., because it was generated by a constrained retrieval, we need a combined averaging kernel matrix

$$\mathbf{A}_{\text{combined}} = \mathbf{W}\mathbf{V}\mathbf{A}_{\text{retrieval}}. \tag{8}$$

2.3 Application of filter functions

Application of a linear filter corresponds to the convolution of the original profile with a filter function and is best formulated as a matrix product involving a filter matrix $\mathbf{T}$ whose lines correspond to the moving discretized filter functions at its actual position.

$$\tilde{\boldsymbol{x}} = \mathbf{T}\boldsymbol{x} \tag{9}$$

In this case the averaging kernel matrix is identical to the $\mathbf{T}$ matrix.

$$\mathbf{A}_{\mathrm{filter}} = \mathbf{T} \tag{10}$$

3 The dependence of the estimated tropopause altitude on vertical resolution of the underlying temperature profile

To analyze the impact of smoothing effects on the estimated tropopause altitude we use temperature profiles measured by radiosondes launched from Nairobi ~~, 1.21~~(1.3°S, 36.8°E~~. These data~~), Hilo (19.4°N 155.4°W), Munich (47.8°N,10.9°E), and Greifswald (54.1°N, 13.4°E). Data from Nairobi and Hilo were available via the Southern Hemisphere Additional Ozonesondes (SHADOZ) network (https://tropo.gsfc.nasa.gov/shadoz/, retrieved on 20 May 2017 ~~, Witte et al. 2017; Thompson et al. 2017).~~ Witte et al. 2017; Thompson et al. 2017), while data from Munich and Greifswald were obtained from the German Weather Service (available via ftp://ftp-cdc.dwd.de/). All radiosonde data sets cover the period 2007-2018. All available Nairobi and Hilo radiosonde profiles within this time period were used. For Munich and Greifswald one profile per week was selected. Details of the sonde profiles used in our study are compiled in Table ~~??~~1. ~~We focus~~ One focus of our study lies on tropical temperature profiles, because of the importance of the tropical tropopause in the climate system. Obviously, the area of the tropics exceeds that of other latitude bands; the tropical tropopause is the entry point of air into the stratosphere (Fueglistaler et al., 2009); and finally the tropical tropopause region plays a ~~distintive~~ distinctive role in the radiative budget of the Earth (Riese et al., 2012). Nairobi was chosen as an example of a continental station, while Hilo (Hawaii) is a maritime station. As a contrast we have also used the two midlatitudinal stations Munich (close to the Alps) and Greifswald (close to the Baltic Sea).

Various averaging kernels $\mathbf{A}$ are applied to the original radiosonde profiles $\mathbf{x}$ to get degraded temperature profiles $\tilde{\boldsymbol{x}}$. These are used for the determination of the lapse rate tropopause altitude and the results are then compared to the tropopause altitudes determined from the original sonde data.

3.1 The Lapse Rate Tropopause

The lapse rate tropopause is the lower boundary of the lowermost layer where the temperature gradient is larger (more positive) than -2 K/km provided that the average lapse rate between this level and all higher levels within 2 km does not exceed 2 K/km (World Meteorological Organization, 1992). It is often determined from data resampled on a grid corresponding to a wider range of vertical spacings from below 1 km (e.g., significant pressure levels) and not always from the raw radiosonde data (see, e.g., Reichler et al. 2003, and references therein).

From the radiosonde profiles the lapse rate tropopause altitudes were determined and served as our benchmark. Only in those cases when the lapse rate tropopause determination failed, the cold point tropopause was used instead. In subsequent steps, the profiles were systematically degraded using averaging kernels of different shapes and altitude resolutions, in order to investigate a possible vertical displacement of the apparent tropopause.

3.1.1 Gaussian averaging kernels

Table 1. ~~Radiosonde launches from Nairobi~~ Statistics for lapse rate tropopause displacements

Nairobi	
Latitude	1.3°S
Longitude	36.8°E
Sample Size	452
Years	2007-1018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
~~09-06-2010~~ 1.0	~~15-06-2010~~ 0.22	~~23-06-2010~~ 0.83	~~30-06-2010~~ -2.19	0.10	5.32
~~07-07-2010~~ 2.0	~~14-07-2010~~ -0.00	~~21-07-2010~~ 0.52	~~28-07-2010~~ -2.19	0.02	3.11
~~03-08-2010~~ 3.0	~~11-08-2010~~ -0.13	~~18-08-2010~~ 0.41	~~25-08-2010~~ -2.51	-0.07	0.73
~~08-09-2010~~ 4.0	~~15-09-2010~~ -0.18	~~23-09-2018~~ 0.42	~~29-09-2010~~ -2.50	-0.14	0.75
~~06-10-2010~~ 5.0	~~13-10-2010~~ -0.22	~~19-10-2010~~ 0.42	~~27-10-2010~~ -2.44	-0.20	0.85
~~03-11-2010~~ height					

Hilo	~~10-11-2010~~	~~17-11-2010~~	~~24-11-2010~~
~~01-12-2010~~ Latitude	~~08-12-2010~~ 19.4°N	~~15-12-2010~~	~~22-10-2010~~
~~29-12-2010~~ Longitude	155.4°W		
Sample Size	534		
Years	2007-2018		

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.26	0.75	-3.46	-0.07	2.90
2.0	-0.36	0.76	-3.67	-0.15	1.04
3.0	-0.42	0.79	-3.81	-0.23	1.50
4.0	-0.47	0.79	-3.86	-0.30	1.31
5.0	-0.48	0.78	-3.85	-0.34	1.25

Munich	
Latitude	47.8°N
Longitude	10.9°E
Sample Size	297
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.59	0.98	-4.81	-0.36	3.00
2.0	-0.69	1.02	-6.46	-0.38	2.83
3.0	-0.73	1.05	-6.43	-0.39	0.27
4.0	-0.77	1.20	-6.38	-0.37	0.37
5.0	-0.76	1.25	-6.29	-0.33	0.57

Greifswald	
Latitude	54.1°N
Longitude	13.4°E
Sample Size	277
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.33	0.88	-4.84	-0.26	4.54
2.0	-0.47	0.82	-5.40	-0.27	2.64
3.0	-0.51	0.88	-6.21	-0.28	0.20
4.0	-0.52	0.95	-6.19	-0.25	0.24
5.0	-0.47	0.94	-6.13	-0.22	0.33

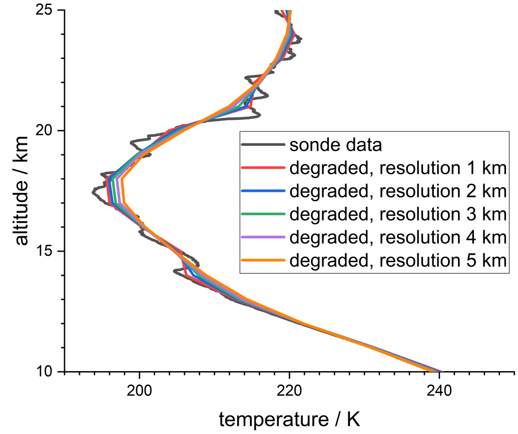

Figure 1. A radiosonde temperature profile measured at Nairobi, ~~1.21~~1.3°S 36.8°E on 18 August 2010, 7:35 UT, along with a set of degraded profiles (Gaussian kernel) with resolutions ~~of~~ from 1 to 5 km.

~~In a first test series~~ To test the dependence of apparent tropopause altitudes on the vertical resolution of the temperature profile, we use Gaussian-shaped averaging kernels to smooth the original radiosonde profiles. For this purpose we use the radiosonde data on their native grid. The smoothed profiles are sampled on a 1-km altitude grid. ~~Half widths (Full width~~ Vertical resolutions in terms of full widths at half maximum ~~)~~ of 1 to 5 km were tested. Since we are not interested in the contribution by any a priori profile but only in the degradation of the vertical profiles by a degraded altitude resolution, we use Eq. (9), with the necessary modification to cope with the irregular input grid.

An example of an original radiosonde profile and a set of smoothed profiles are shown in Figure 1. These smoothed profiles were used to determine the tropopause altitudes according to the lapse rate criterion. Histograms of resulting vertical tropopause ~~dispacements~~ displacements for all profiles listed in Table ~~??~~ 1 are shown in Figures 6. They are all well-behaved in a sense that they have no pronounced secondary modes~~and are fairly symmetric. Apparent asymmetries are caused by the wide bins which are necessary to have enough data points per bin.~~. The histograms indicate an underlying left-skewed distribution. These asymmetries are attributed to the shape of the temperature profiles itself. A less resolved profile typically has a less negative lapse rate at lower altitudes than the better resolved profile. The tropopause determination scheme procedes from bottom upwards and the threshold will thus first be met already at lower altitudes.

The average tropopause ~~displacement~~ displacements as a function of vertical resolution ~~is~~ are reported in Table 1 and shown in Fig. 3. Tropopause altitudes inferred from coarser reduced data tend to be lower than those determined from the original sonde data. For a resolution of 3 km the mean (median) tropopause altitude displacement was found to be ~~-240~~ -130 to -730 m (~~-110~~ -70 to -390 m). ~~The~~ In most cases larger displacements were found for the midlatitudinal than for the tropical data, and the median is less affected than the mean at all resolutions. The 5 to 95 percentile range increases for coarser resolutions and

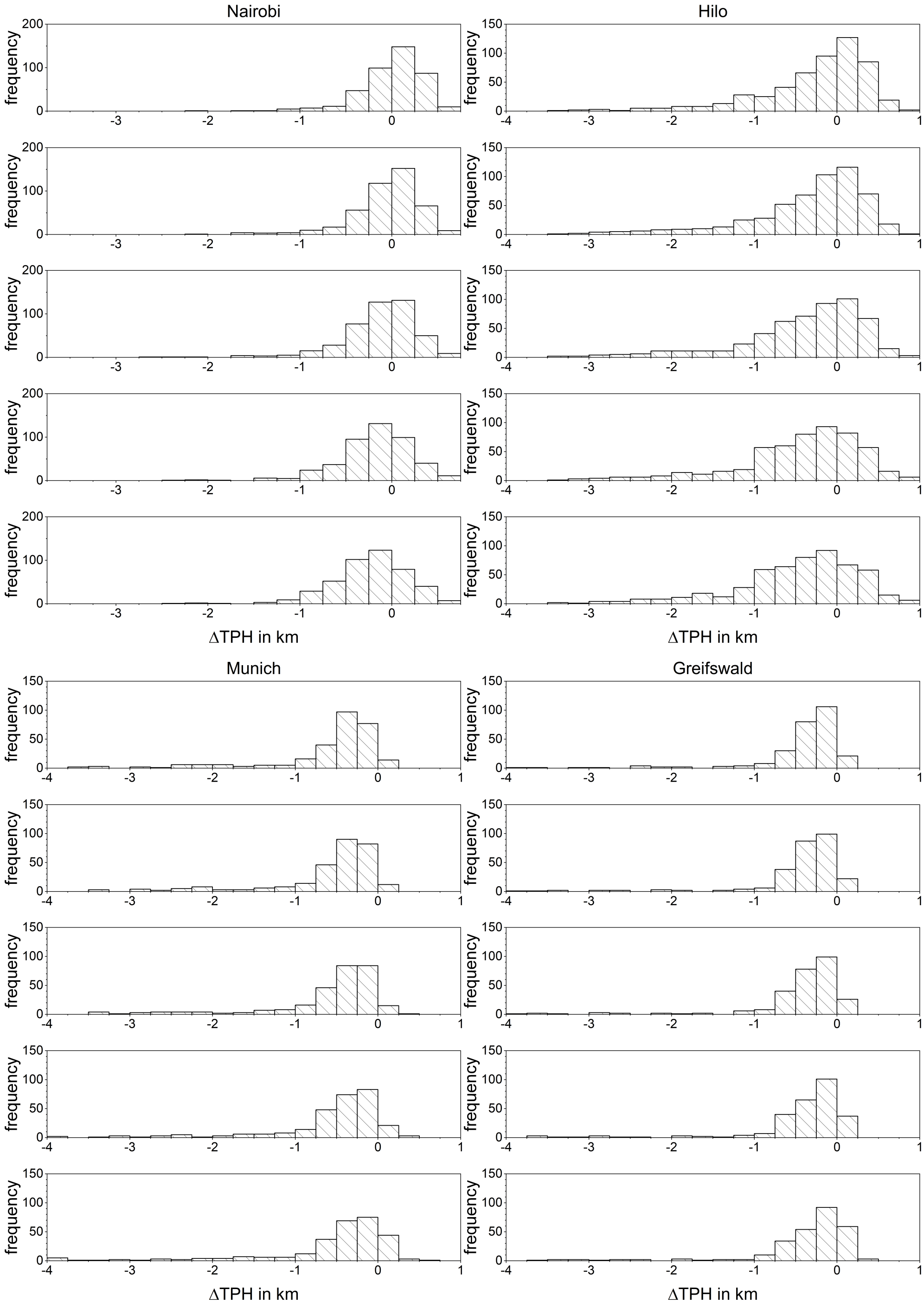

Figure 2. Histograms and of mean tropopause height (ΔTPH) offsets (ΔTPH) of degraded temperature profiles (Gaussian kernel) with resolutions of 1 to 5 km in steps of 1 km (from top to bottom) for Nairobi (top left), Hilo (top right), Munich (bottom left) and Greifswald (bottom right).Note that the apparent asymmetry of some of the histograms is accentuated by the symmetric choice of relatively large bins to accomodate a reasonable amount of results per bin.

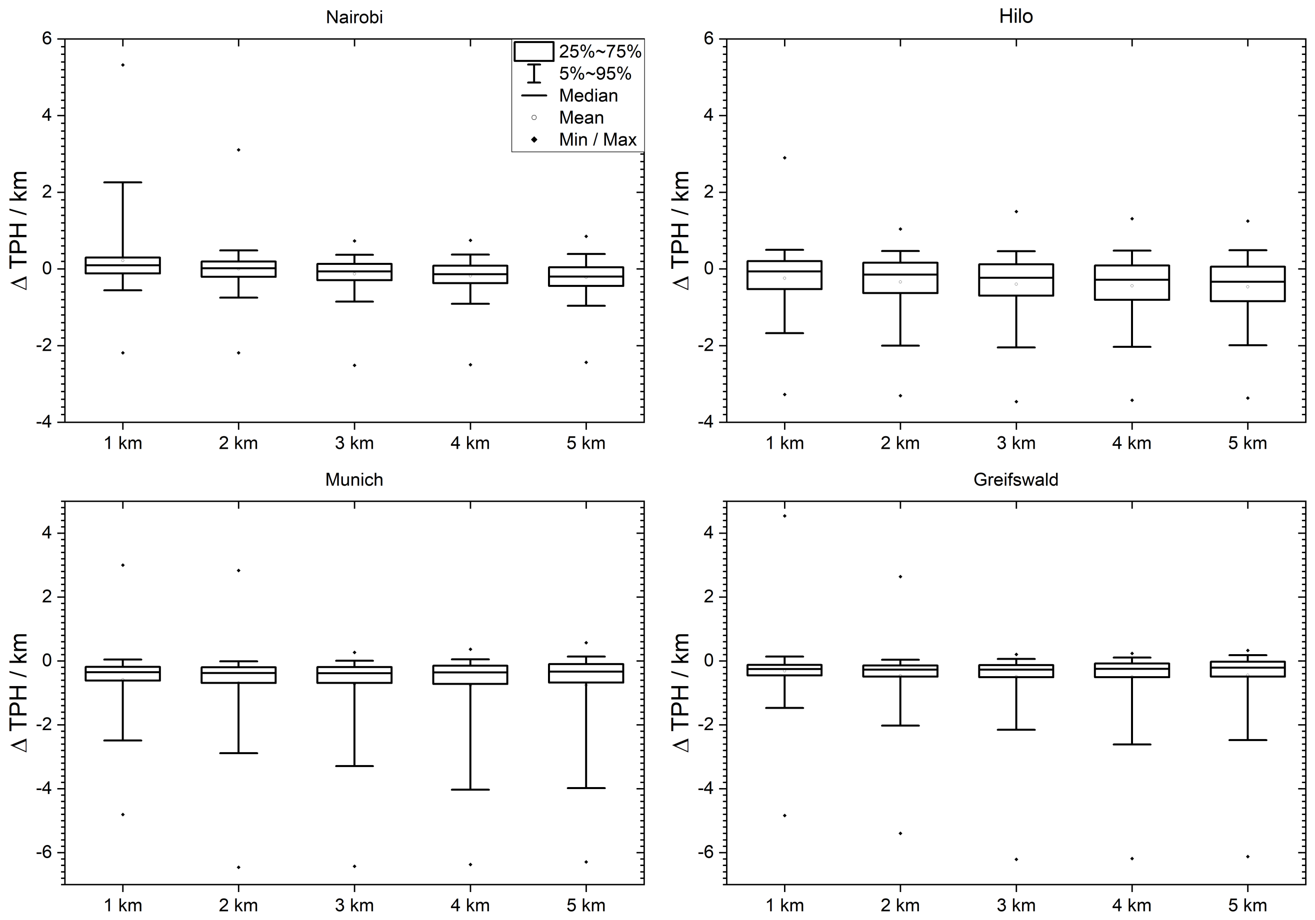

Figure 3. The lapse rate tropopause displacement for four geolocations as a function of vertical resolution for Gaussian kernels. Boxes represent both of the second and the third quartile. The “error bars” represent the central 90% quantile.

reaches saturation at a resolution beyond 3 km, except for Nairobi, where it covers a particularly wide range of displacements for 1 km vertical resolution.

3.1.2 MIPAS averaging kernels

The Michelson Interferonmeter To complement the analysis based on idealized averaging kernels, two exemplary case studies have been performed using averaging kernels characterizing temperature retrievals from Michelson Interferometer for Passive Atmospheric Sounding (MIPAS, Fischer et al. 2008) measurements. MIPAS was an infrared limb emission spectrometer operating on the Envisat research satellite. One of its data products was global temperature distributions from the upper troposphere to the mesosphere (von Clarmann et al., 2003b, 2009). We complement the theoretical study presented above with an assessment of how MIPAS temperature averaging kernels affect the lapse rate tropopause determination. For this purpose case

Table 2. Nairobi MIPAS collocations

Date	Time (UTC) 09-06-2010 3 Aug 2010	08:08:53 23 Sep 2010
14-07-2010 Difference Time (min.)	08:08:54 03-08-2010 70	07:40:09 35
23-09-2010 Difference Latitude (°)	07:37:27 28-10-2010 -2.8	19:46:35 -0.1
24-11-2010 Difference Longitude (°)	19:58:09 0.0	1.3
08-12-2010 Tropopause Displacement (km)	19:44:25 -0.35	-1.08

study we use averaging kernels and the a priori of two MIPAS retrievals which are spatiotemporally as close as possible to the radiosonde measurements (Table 2). The MIPAS geolocations deviate from that of the Nairobi radiosonde launch site by less than $\pm 5°$ in latitude and less than $\pm 10°$ in longitude.

Since MIPAS averaging kernels are provided on a 1-km altitude grid, we use the radiosonde profiles resampled on a 1-km vertical grid, using Equation (6) with a $\mathbf{V}$ matrix for linear interpolation. Since the MIPAS averaging kernels ($\mathbf{A}_{\mathrm{MIPAS}}$) are routinely produced on a 1-km grid (see, Fig. 4 for an example), they can then be conveniently applied to these resampled radiosonde profiles. The application of the averaging kernels as a filter function given by

$$\boldsymbol{x}_{\mathrm{degraded}} = \mathbf{A}_{\mathrm{MIPAS}} \boldsymbol{x}_{\mathrm{radiosonde}} \tag{11}$$

yields the radiosonde profile at the vertical resolution of the MIPAS temperatures. It does, however, not include the contribution of the a priori profiles used in the MIPAS retrieval. A more realistic transformation, which provides the radiosonde profiles as MIPAS would see them, involves Eq. (3). Its application to the problem under investigation reads

$$\begin{aligned} \boldsymbol{x}_{\mathrm{degraded}} &= \mathbf{A}_{\mathrm{MIPAS}} \boldsymbol{x}_{\mathrm{radiosonde}} \\ &\quad + (\mathbf{I} - \mathbf{A}_{\mathrm{MIPAS}}) \boldsymbol{x}_{\mathrm{ERA-Interim}}, \end{aligned} \tag{12}$$

where $\boldsymbol{x}_{\mathrm{ERA-Interim}}$ are temperature profiles extracted from ECMWF ERA-Interim analyses (Dee et al., 2011), which were used as a priori information for the MIPAS retrievals. Since actual MIPAS measurement data are not used directly but only for the calculation of the averaging kernels, the and since the goal is to isolate the effect of the averaging kernel, the noise term is not considered here.

Histogram of tropopause altitude offsets for degraded temperature profiles. The top panel shows the tropopause height displacement by resampling the original sonde data on a regular 1-km grid (Eqs. (5) and (6). The lower panel shows the effect of the MIPAS averaging kernels according to Eq. (12).

The resulting profile $\boldsymbol{x}_{\mathrm{degraded}}$ is the radiosonde profile as MIPAS would have seen it, if it had made a noise-free measurement exactly at the place and time of the radiosonde measurements. Again, the effect of the reduced resolution , now along with the effect of the a priori temperature profiles, on the tropopause altitude determination is investigated(Fig. ??). The mean tropopause altitude offset is -640 m; the median is -530 m. The sign of these results are consistent with those obtained for Gaussian kernels. The displacement, however, . Resulting displacements are shown in the last row of Table 2. In the August case the displacement of -0.35 km lies within the standard deviation obtained for a 3-km Gaussian kernel, while

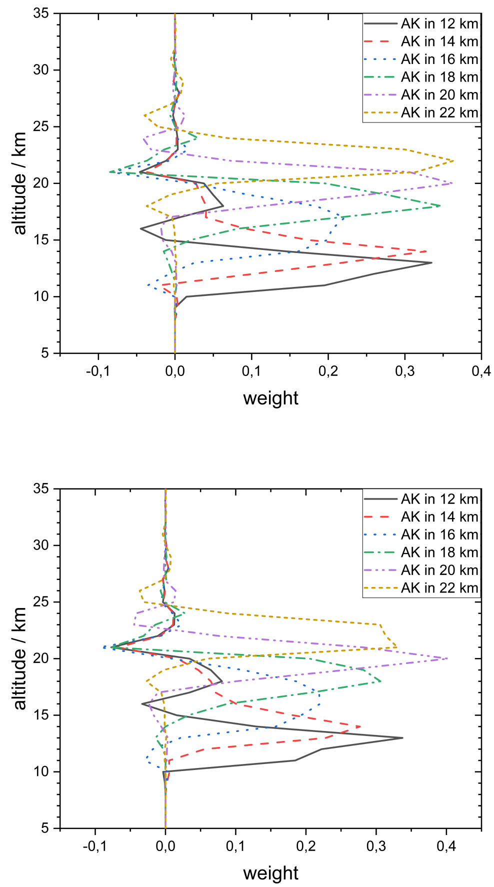

Figure 4. Averaging kernels of a MIPAS ~~measurement~~ measurements near Nairobi on 3 ~~August 2010.~~ Aug 2010 (top) and 23 Sep 2010 (bottom). The MIPAS altitude resolution is altitude dependent and varies between 2.7 and 4.5 km below 22 km altitude, with typical values of around 3.0 km. To avoid ~~an~~ overly busy ~~plot~~plots, only every other averaging kernel is shown.

in the September case it is considerably larger~~for the MIPAS averaging kernels~~, with -1.08 km. The sign of the displacements found in the MIPAS case studies agrees with that found for the application of the Gaussian kernel.

The consideration of the prior information used is important. The reason is roughly this. In remote sensing applications, instead of referring to the resulting profile, the altitude resolution refers to the difference between the resulting profile and the a priori profile. The fine structure of the prior information is propagated into the resulting profile retrieval and only corrections on a larger scale ~~are founded on~~ originate from the MIPAS measurements. We do not expect any systematicity with respect to this effect and we see no way to predict whether the smoothing effect or the fine structure of the a priori profile dominates the tropopause displacement.

~~In Equation (12) errors in MIPAS and radiosonde data and, more important in this context, spatio-temporal~~ For the comparison of the results obtained with the MIPAS averaging kernels and those obtained with the Gaussian averaging kernels, a caveat is adequate. Although coincidences under assessment are quite close, any spatiotemporal mismatch between the MIPAS and the ~~radiosonde measurements are neglected. The latter is particularly problematic. According to Equation (12) the atmosphere as seen by MIPAS contains~~ sonde data can contribute to the displacement. Only MIPAS averaging kernels and a priori information ~~, which~~ are used for the comparison but no MIPAS data. The a priori data is in this case the ERA-Interim temperature profile~~at the MIPAS measurement location~~, which depends on the MIPAS geolocation and measurement time. Any tropopause altitude difference between the ERA-Interim temperature profile at the MIPAS time and measurement location and at the radiosonde time and measurement location will map onto the degraded profile and adds a further component of uncertainty which can hardly be distinguished from the tropopause altitude offset caused by the degraded vertical resolution.

The problem discussed in this Section, that the use of a structured a priori profiles adds additional complication to the assessment of the tropopause displacement obviously is of no concern when a pure smoothing constraint in combination with a flat a priori profile is used for the retrieval.

3.2 The Cold Point Tropopause

In addition to the sensitivity of the lapse rate tropopause altitude to the vertical resolution of the temperature profile, also cold point tropopause altitudes were investigated. Again Gaussian averaging kernels were assessed (Fig. 5).

Here, the degrading with the Gaussian kernel was performed directly ~~to~~ on the radiosonde profiles on their original grids and the cold point tropopauses were determined. ~~The~~ Histograms of related tangent altitude displacements for resolutions from 1 to 5 km are shown in Fig. 6, while the dependence of the cold point tropopause altitude on the vertical resolution is shown in Fig. 7.

~~Also the~~ As for the lapse rate tropopause, also the cold point tropopause altitudes inferred from coarser resolved temperature profiles are lower than those inferred from the original profiles ~~. We assume that the Nairobi profiles are representative of the tropical atmosphere. However, we note that these results might not be applicable for other latitude bands. With -500 m (430 m) the mean (median) tropopause altitude displacement exceeds that of the~~ (Table 3). For almost all stations and resolutions under investigation, the mean tropopause altitude displacement exceeds that of the lapse rate tropopause sizeably. The only exception is the vertical resolution of 1 km. Here the cold point tropopause appears to be less sensitive to the degraded resolution than

Table 3. Statistics for cold point tropopause displacements

Nairobi	
Latitude	1.3°S
Longitude	36.8°E
Sample Size	452
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.16	0.13	-1.30	-0.15	0.43
2.0	-0.35	0.22	-1.65	-0.31	0.09
3.0	-0.51	0.34	-2.68	-0.46	0.59
4.0	-0.66	0.43	-3.20	-0.60	0.51
5.0	-0.75	0.47	-3.54	-0.69	0.29

Hilo	
Latitude	19.4°N
Longitude	155.4°W
Sample Size	534
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.19	0.31	-2.38	-0.16	2.05
2.0	-0.43	0.45	-3.16	-0.36	2.19
3.0	-0.62	0.54	-3.02	-0.52	1.29
4.0	-0.77	0.68	-4.72	-0.63	1.18
5.0	-0.89	0.76	-4.56	-0.71	2.47

Munich	
Latitude	47.8°N
Longitude	10.9°E
Sample Size	297
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.15	0.18	-0.86	-0.14	2.24
2.0	-0.35	0.32	-3.40	-0.30	1.59
3.0	-0.53	0.56	-5.29	-0.37	0.67
4.0	-0.64	0.73	-4.80	-0.42	0.38
5.0	-0.71	0.97	-6.57	-0.39	2.01

Greifswald	
Latitude	54.1°N
Longitude	13.4°E
Sample Size	277
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (km)	Standard Deviation (km)	Minimum Deviation (km)	Median Deviation (km)	Maximum Deviation (km)
1.0	-0.17	0.28	-3.81	-0.14	0.10
2.0	-0.30	0.41	-4.92	-0.23	0.20
3.0	-0.41	0.56	-5.00	-0.28	0.21
4.0	-0.47	0.71	-6.95	-0.28	0.22
5.0	-0.48	0.81	-6.52	-0.25	0.98

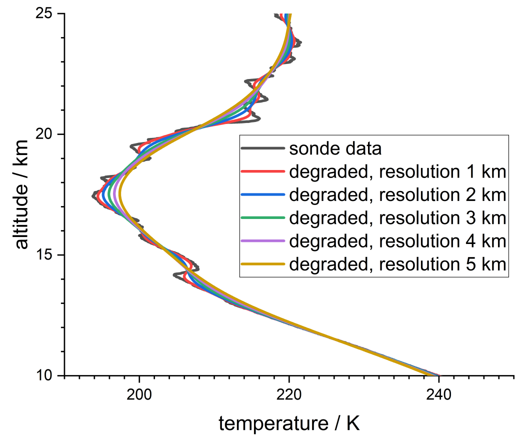

Figure 5. An example of a radiosonde profile, Nairobi, ~~1.21~~1.27°S 36.8°E, 7 July 2010, at original vertical resolution along with degraded profiles at vertical resolutions of 1 to 5 km. Contrary to the example shown in Fig. 1, the smoothed profile is represented on the native radiosonde grid, not on the 1-km regular grid.

the lapse rate tropopause. For the median tropopause displacement the situation is not so clear. Here the cold point tropopause altitude is less sensitive to coarser resolutions than the lapse rate tropopause ~~by far~~altitude for the midlatitudinal stations Munich and Greifswald.

MIPAS data are sampled on a 1-km grid, and data of satellite instruments of similar vertical resolution are typically sampled on even coarser grids. Determination of the cold point tropopause on such a grid would thus by far be dominated by sampling effects. Tropopause shifts of a magnitude as determined with the Gaussian kernel thus cannot be safely resolved.

4 Implications for Water Vapour Content

The cold point temperature largely determines the water vapour content of air entering the stratosphere. Thus, temperature profiles with finite vertical resolution affect the estimated saturation mixing ration of water vapour. For the tropical stations the related error in the saturation mixing ration seems to be fairly proportional to the vertical resolution. Reasonably good agreement between the mean and the median errors is found (Tab. 4 and Fig. 8). In the tropics, which are the entry region of air into the stratosphere, the error in the saturation mixing ratio in units of ppmv is about one or two times the resolution in km. For the midlatitudinal stations the error is much larger, with approximately seven ppmv per km of vertical resolution. These large errors, however, are of little concern because the midlatitudinal tropopause is not the preferred pathway of tropospheric air into the stratosphere.

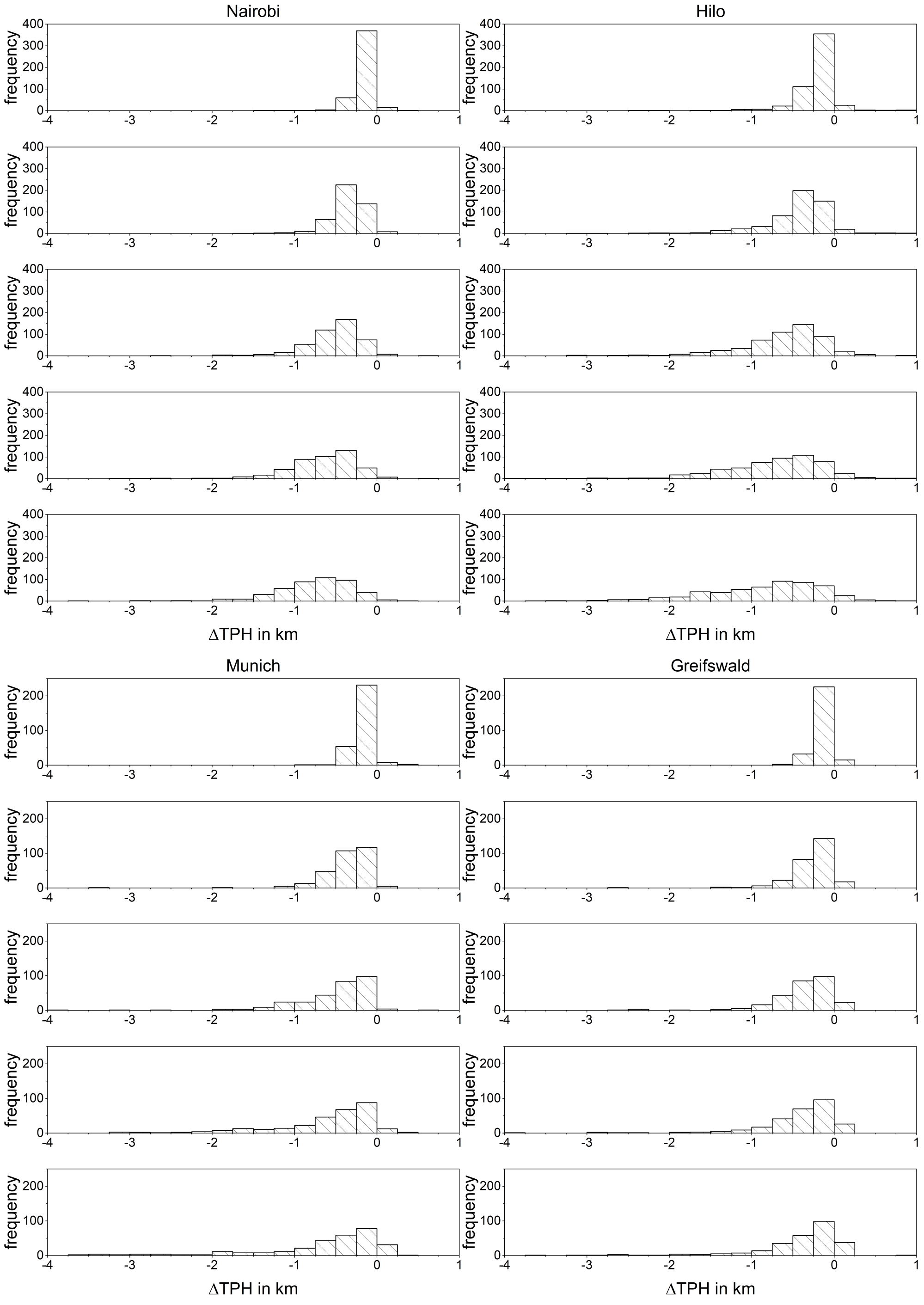

Figure 6. Histograms of tropopause height (TPH) offsets of degraded temperature profiles (Gaussian kernel) with resolutions of 1 to 5 km in steps of 1 km (from top to bottom) for Nairobi (top left), Hilo (top right), Munich (bottom left) and Greifswald (bottom right).

Table 4. Statistics for cold point water vapour saturation mixing ratio errors

Nairobi	
Latitude	1.3°S
Longitude	36.8°E
Sample Size	452
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (ppmv)	Standard Deviation (ppmv)	Minimum Deviation (ppmv)	Median Deviation (ppmv)	Maximum Deviation (ppmv)
1.0	1.10	0.70	-0.12	0.93	4.53
2.0	2.20	1.19	0.14	1.92	8.16
3.0	3.31	1.59	0.72	2.93	12.1
4.0	4.48	1.92	1.55	4.04	15.5
5.0	5.75	2.25	2.39	5.22	18.7

Hilo	
Latitude	19.4°N
Longitude	155.4°W
Sample Size	534
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (ppmv)	Standard Deviation (ppmv)	Minimum Deviation (ppmv)	Median Deviation (ppmv)	Maximum Deviation (ppmv)
1.0	2.38	6.70	-37.0	1.53	46.3
2.0	3.85	7.14	-35.8	2.82	49.1
3.0	5.19	7.49	-34.6	4.07	52.7
4.0	6.62	7.79	-33.2	5.39	55.7
5.0	8.16	8.06	-31.5	6.73	58.2

Munich	
Latitude	47.8°N
Longitude	10.9°E
Sample Size	297
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (ppmv)	Standard Deviation (ppmv)	Minimum Deviation (ppmv)	Median Deviation (ppmv)	Maximum Deviation (ppmv)
1.0	7.03	4.69	0.33	5.72	37.3
2.0	13.1	7.35	3.53	11.4	51.0
3.0	18.8	9.65	5.04	16.7	64.5
4.0	24.5	12.1	7.20	21.9	81.8
5.0	30.6	14.7	7.98	27.7	99.0

Greifswald	
Latitude	54.1°N
Longitude	13.4°E
Sample Size	277
Years	2007-2018

Altitude Resolution (km)	Mean Deviation (ppmv)	Standard Deviation (ppmv)	Minimum Deviation (ppmv)	Median Deviation (ppmv)	Maximum Deviation (ppmv)
1.0	7.42	6.45	-58.9	6.49	34.2
2.0	14.8	9.40	-54.2	13.4	58.6
3.0	22.5	12.9	-51.5	19.8	83.6
4.0	30.6	16.8	-47.7	27.4	110.
5.0	39.2	20.9	-43.23	35.0	136.

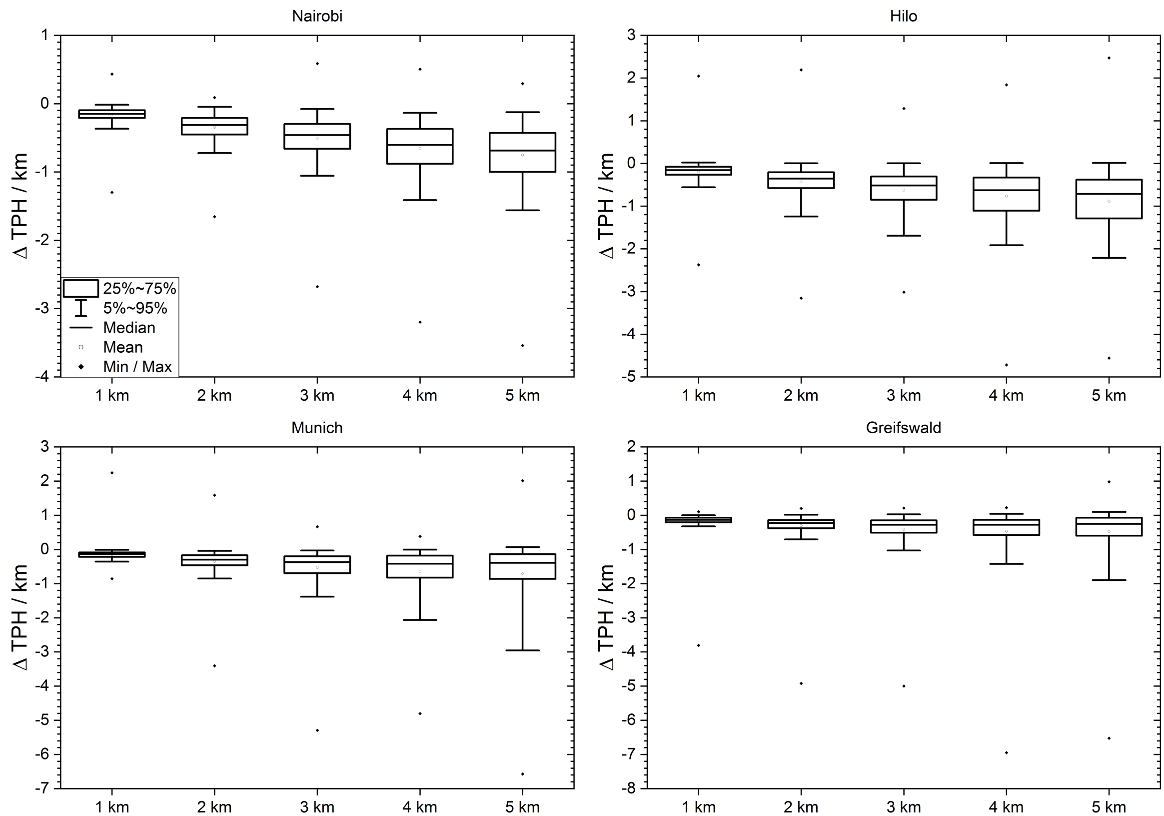

Figure 7. The cold point tropopause displacement displacements for four geolocations as a function of the vertical resolution for Gaussian kernels. For details, see Fig. 3.

5 Feasibility of Correction Schemes

Since degraded, i.e., less resolved, temperature profiles are thought to exhibit less steep gradients, it suggests itself to adjust the lapse rate threshold in the WMO definition of the tropopause to compensate the smoothing effect. Doing this, one might expect to find the tropopause at the correct altitude even from degraded profiles. Obviously, the threshold, if any useful, must be a function of the vertical resolution of the temperature profile used.

We have searched for a temperature gradient that, when applied to the coearse coarse resolution profiles, reproduces the same tropopause altitude as the -2 K/km gradient applied to the original profiles. Fig. 9 shows histograms of the obtained adjusted lapse rate criteria for vertical resolutions of 1 to 5 km for all four stations under investigation. There is a tendency that lapse rates between -1 and -2 K/km are more adequate for applications to coarsely resolved profilesbut the spread .

For the tropical stations Nairobi and Hilo, there is a clear tendency that for coarser vertical resolutions a smaller absolute value of the lapse rate would be more adequate to determine the tropopause. The spread, however, is very large . Therefore, (Table 5). The standard deviations of the optimal lapse rates (1.28–1.84 K/km) are approximately the same as the absolute

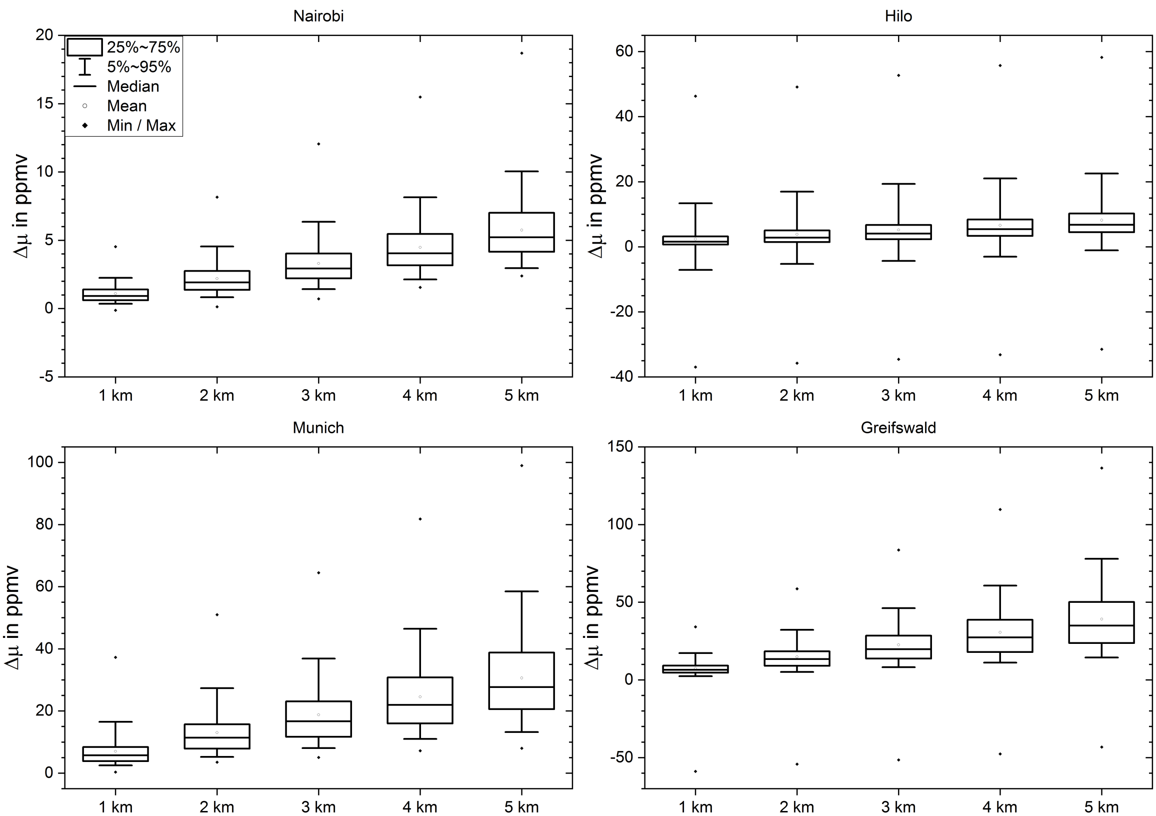

Figure 8. The effect of the vertical resolution and the related displacement of the cold point tropopause altitude on the water vapour saturation mixing ratio for four geolocations as a function of the vertical resolution for Gaussian kernels. For details, see Fig. 3.

values of the optimal lapses rates themselve (1.26–2.28 K/km), and even exceed them in some cases (Nairobi for 4 and 5 km resolution and Hilo for 5 km resolution). Often the standard deviation even exceeds the difference of between the nominal lapse rate of 2 K/km and the optimal lapse rate by a factor of ten or more. E.g. this difference is 0.11 K/km for Nairobi profiles at 2 km vertical resolution while the standard deviation is 1.63 K/km. Thus we cannot recommend ~~a general correction scheme.~~ to use adjusted lapse rates to infer tropical tropopause altitudes from coarsely resolved temperature profiles.

The situation is different for midlatitudinal temperature profiles. The optimal lapse rate criterion turns out to be -1.3 K/km (Table 5). This value was found to be representative for both Munich and Greifswald. For Munich profiles the spread (0.62–0.85 K/km) is little more than half of the absolute mean optimal value, and for Greifswald it is even less than a third (0.39–0.41 K/km). For Munich the spread is of similar size as the difference between the nominal and the mean optimal lapse rates, and for Greifswald it is only little larger than half this difference. Astonishingly enough the lapse rate was found to have a very weak dependence on the resolution on the vertical resolution, suggesting that the optimal lapse rate is not a continuous function of the vertical resolution but that this is a kind of threshold problem where the discontinuity is located at vertical resolutions even better than 1 km. We are confident that for midlatitudinal temperature profiles the inductive inference of adapted lapse rate criteria can indeed improve tropopause altitude determination from coarsely resolved profiles.

6 Discussion

6 Discussion and Conclusion

In the tropical region the determination of both, the lapse rate and the cold point tropopause altitude from temperature profiles of degraded altitude resolution, typically leads to an underestimation of the tropopause height. The mean magnitudes of this effect range from 0 to about 500 m for altitude resolutions of 1 to 5 km. Often considerably larger effects are found for the cold point tropopause. For midlatitudinal temperature profiles larger tropopause altitude displacements were found, and, broadly speaking, the cold point tropopause turns out to be less sensitive to vertical resolution issues. This suggests that, while in the tropics the cold point tropopause is commonly used, in the case of coarsely resolved profiles, the lapse rate tropopause appears to be more robust, and *vice versa* for midlatitudinal atmospheres. However, adaptive, resolution dependent, lapse rates can improve the tropopause determination in mid-latitudes. In contrast, the variability of the tropopause dislocation is fairly large in the tropics such that the recommendation of an inductively generalized correction scheme for tropical tropopause heights seems audacious and even inappropriate to us.

Given the variety of retrieval schemes used to infer temperature profiles from satellite measurements, the following caveats with respects to the generality of our results need to be discussed.

Often satellite retrievals use a retrieval scheme similar to Eq. (1) along with a highly structured a priori profile $\boldsymbol{x}_\mathrm{a}$. A retrieval with a vertical resolution which that is significantly coarser than that of $\boldsymbol{x}_\mathrm{a}$ will modify only the coarse structure of the temperature profile, while the fine structure of the $\boldsymbol{x}_\mathrm{a}$ will survive the retrieval process. This is because the resolution of the retrieval as defined by the averaging kernel refers, rigorously speaking, not to the resulting profile, but to the difference between the resulting profile and the a priori profile. Related effects on the tropopause displacement are then complicated to predict., because it depends largely on the surviving fine structure. Tropopause determination procedures do not distinguish between the a priori contribution and the measurement contribution to the final temperature profile. The large displacements along with the large scatter found in the analysis of MIPAS averaging kernels (Section 3.1.2) are attributed to this effect. In consequence, it seems to be more appropriate to use smooth a priori profiles if retrieved temperature profiles are intended to be used for tropopause altitude determination.

Another issue of concern is the retrieval grid of the temperature retrieval. It is the exception rather than the rule that limb sounders use a retrieval grid which is about a factor of three finer than the vertical resolution, as the MIPAS dataset used here. More often the vertical grid is close to the vertical resolution of the retrieval. In these cases, the tropopause altitude determination is limited directly by the sampling of the retrieval and not by its resolution.

We focused our investigation on the tropical region. Temperature profiles in other latitude bands can deviate significantly in shape and magnitude, particularly near the tropopause. For midlatitudinal temperature profiles with weak or non-existent gradients above the tropopause even a coldpoint displacement of opposite sign than found here could be possible.

Table 5. Statistics for lapse rate adjustments

Nairobi	
Latitude	1.3°S
Longitude	36.8°E
Sample Size	452
Years	2007-2018

Altitude Resolution (km)	Mean optimal lapse rate (K/km)	Standard Deviation (K/km)	Minimum (K/km)	Median (K/km)	Maximum (K/km)
1.0	-2.28	1.84	-7.30	-2.46	2.35
2.0	-1.89	1.63	-5.89	-2.03	2.70
3.0	-1.55	1.51	-4.97	-1.66	2.57
4.0	-1.35	1.41	-4.85	-1.41	2.49
5.0	-1.26	1.32	-4.76	-1.31	2.51

Hilo	
Latitude	19.4°N
Longitude	155.4°W
Sample Size	534
Years	2007-2018

Altitude Resolution (km)	Mean optimal lapse rate (K/km)	Standard Deviation (K/km)	Minimum (K/km)	Median (K/km)	Maximum (K/km)
1.0	-1.93	1.62	-7.02	-1.73	3.15
2.0	-1.70	1.46	-6.39	-1.55	2.25
3.0	-1.50	1.38	-5.90	-1.38	2.09
4.0	-1.36	1.33	-5.57	-1.22	2.11
5.0	-1.27	1.28	-5.31	-1.15	2.07

Munich	
Latitude	47.8°N
Longitude	10.9°E
Sample Size	297
Years	2007-2018

Altitude Resolution (km)	Mean optimal lapse rate (K/km)	Standard Deviation (K/km)	Minimum (K/km)	Median (K/km)	Maximum (K/km)
1.0	-1.30	0.85	-2.78	-1.52	2.34
2.0	-1.29	0.78	-2.71	-1.49	2.19
3.0	-1.30	0.70	-2.67	-1.47	1.40
4.0	-1.30	0.65	-2.64	-1.45	1.15
5.0	-1.29	0.62	-2.60	-1.42	0.97

Greifswald	
Latitude	54.1°N
Longitude	13.4°E
Sample Size	277
Years	2007-2018

Altitude Resolution (km)	Mean optimal lapse rate (K/km)	Standard Deviation (K/km)	Minimum (K/km)	Median (K/km)	Maximum (K/km)
1.0	-1.34	0.41	-2.91	-1.30	0.34
2.0	-1.32	0.39	-2.84	-1.29	0.00
3.0	-1.30	0.39	-2.78	-1.28	0.00
4.0	-1.29	0.39	-2.75	-1.26	0.00
5.0	-1.29	0.40	-2.79	-1.25	0.00

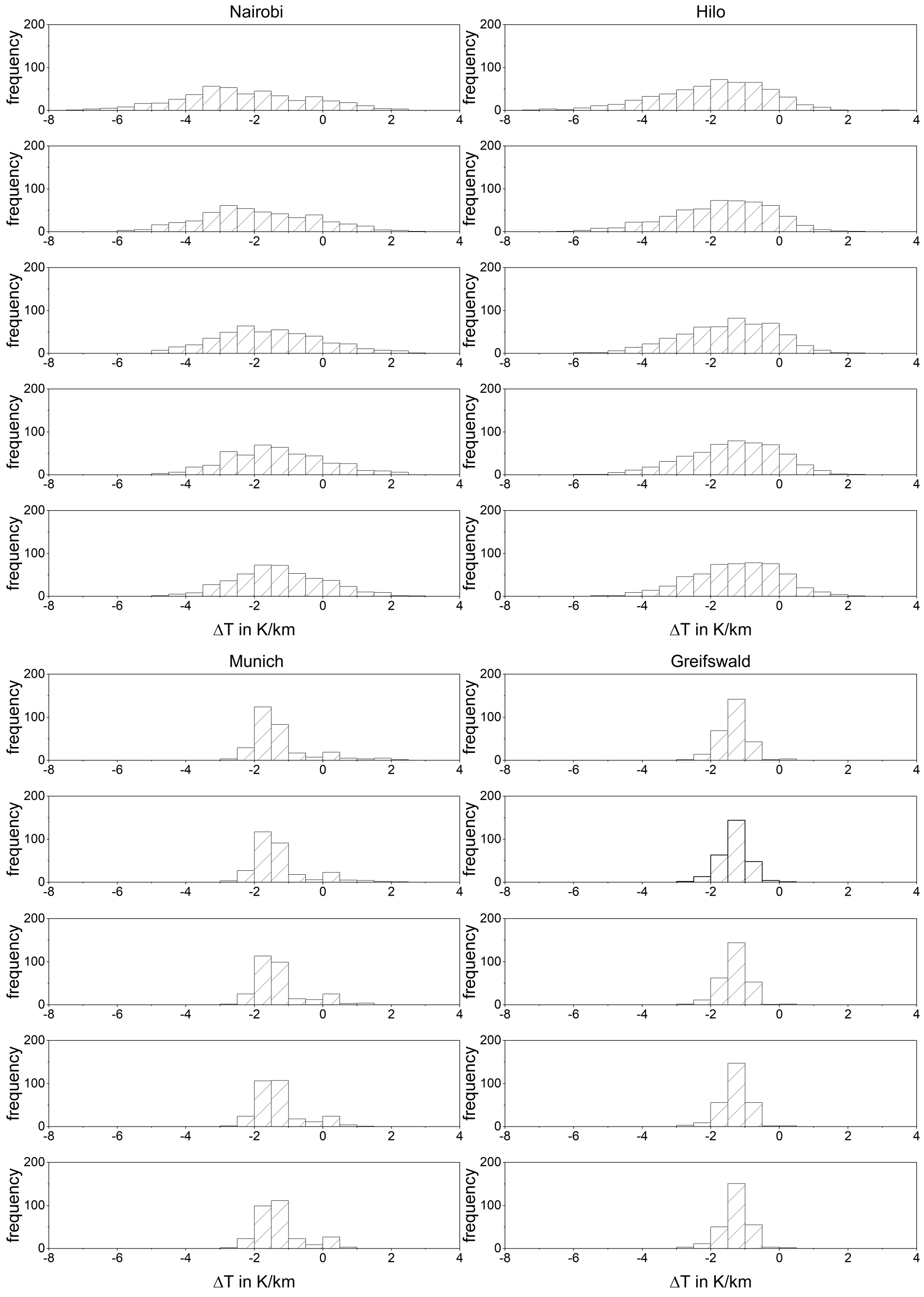

Figure 9. Histograms of optimal lapse rates for tropopause determination from coarsely resolved temperature profiles ~~, along with fitted Gaussian distributions~~for four stations. Vertical resolutions vary from 1 km (top) to 5 km (bottom).

In the tropical region the determination of both, the lapse rate and the cold point tropopause altitude from Our results may have implications for other than remotely sensed temperature profiles of degraded altitude resolution, typically leads to the an underestimation of the tropopause height. The order of magnitude of this effect is in the range of -100 to -500 m for altitude resolutionsof 1 to 5 km. This effect is approximately twice as large for the cold point tropopause than for the lapse rate tropopause. This suggests that, while in the tropics the cold point tropopause is commonly used, in the case of coarsely resolved profiles, the lapse rate tropopause appears to be more robust. The variability of this dislocation is fairly large such that recommendation of an inductively generalized correction scheme seems audacious and even inappropriate to uslimited vertical resolutions, such as model or analysis data. While our methods seem apt also for related investigations, such problems are beyond the scope of this paper.

Data availability. Radiosonde data were obtained from the SHADOZ website: https://tropo.gsfc.nasa.gov/shadoz/. MIPAS data are available via https://www.imk-asf.kit.edu/english/308.php. German radiosonde data were obtained from German Weather Servive via ftp://ftp-cdc.dwd.de/.

Competing interests. TvC is associate editor of AMT but has not been involved in the evaluation of this paper.

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