AMT-2019-107: Retrieval of Temperature from a Multiple Channel Pure Rotational Raman-Scatter Lidar Using an Optimal Estimation Method (Gamage et al.)

Final Response:

- (1) comments from Referees
 - a. Comments from Anonymous Referee #1: pages 2-5
 - b. Comments from Christoph Ritter (Referee): pages 15-16
- (2) author's response (The Referee's comments are in blue, are responses are in black. Red sections have been added to the manuscript)
 - a. Response to Anonymous Referee #1: pages 6-14
 - b. Response to Christoph Ritter (Referee): pages 17-22
- (3) Author's changes in manuscript: Revised sections are in red.

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Interactive comment

Interactive comment on "Retrieval of Temperature From a Multiple Channel Pure Rotational Raman-Scatter Lidar Using an Optimal Estimation Method" by Shayamila Mahagammulla Gamage et al.

Anonymous Referee #1

Received and published: 23 July 2019

The authors present the application of the OEM to pure rotational Raman (PRR) temperature lidar. The OEM has been applied before to other lidar techniques but not to this one. Compared to how PRR temperature lidar data were analysed previously, the OEM shows several advantages, the largest of which is that no calibration with other sensors is needed. Furthermore, the systematic and statistical uncertainties as well as the effect of partial overlap in the near range are also determined.

The authors could highlight these advantages even more clearly. There is one early study on calibrating RRL temperature lidar with the instrumental parameters (Vaughan

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et al.: Atmospheric Temperature Profiles Made by Rotational Raman Scattering. Applied Optics, 1993) but this study concluded that the uncertainties were too high at that time.

The paper is mostly well written (see detailed below). My recommendation is to accept the paper after minor revision.

Specific comments:

Since the proposed technique seems to be universally applicable to temperature rotational Raman lidar systems, more references to the state-of-the-art of this technique and other existing systems would certainly be interesting for the readers and should be included.

You mention two assumptions on page 8, line 25/26. What do you mean with "well known"? How critical are these assumptions? When highlighting which assumptions are NOT needed (see abstract, introduction, conclusions), you should be fair and also mention which are needed. Maybe it would be interesting to explain also in the abstract and conclusions that (how much) the results are independent from the selected a priori temperature profile.

Page 9, line 4: There are always aerosols in the troposphere. How few are acceptable? I think the term "digital measurements" for photon counting signals is odd. Also the analog signals are digitized. Thus, I suggest that you write "photon counting" throughout the text.

Minor comments:

"Lidar constants" and "coupling constants" are terms which are not commonly used. Please define/explain or avoid. E.g., page 1, line 5: "ratio of efficiencies" is better I think since the laser power, transmitter efficiency, telescope area are the same for both channels and thus cancel and become irrelevant (if I understand correctly).

Page 1, line 21: "...2 flights per day". One could add: "only at selected sites world-

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wide".

Page 1, line 22: Please add references to other combined Raman lidar systems. There are several which measure water vapor and temperature.

Page 3, Eq. 1: You assume that all PRR lines of one channel are collected with the same efficiency. This is generally not the case but may be true for RALMO. Please add a comment on this.

Page 3, Eq. 2: This equation is not found in Penney et al. 1974.

Page 5, line 11: Why is the background noise B_RR a function of height? I think the "real background" should be height independent. If the baseline is height dependent, detector non-linearity or electronic cross-talk is present, I guess.

Page 7, table 2: What do you with "transition height"?

Page 8, line 6: beta_par *is* related

Page 9, line 20: I would prefer "model parameters" or "model b parameters"

Page 9, line 21: Please refer to the figure.

Page 10: line 6: What do you mean with "dominant"?

Page 10, line 7: How do you know that the analog signals are linear?

Page 10, line 8: "become saturated". What do you mean with "saturated"?

Page 11, Fig 2: I suggest that you write "four signals". It is *two* channels. How did you determine "measurement noise"? With OEM? Please clarify.

Page 14, line 2: What is coupling constant R_a? How is it defined?

Page 15, Fig 6: How did you obtain the a priori overlap? Why is this important since the ratio of the overlap functions cancels?

Figs 1, 7, 12, 17: I would prefer the same scales. JLa -> JL, JHa -> JH? Should be

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consistent. I think it would be interesting to show the elastic signals for all cases. Why "Eb" for "elastic"?

The language still needs polishing/corrections. Here two examples:

Page 1, line 3ff: "assumption for the form of ..." However, I think the form of the calibration function is not really the point here. Calibration with external sensors (with all the uncertainties related to the accuracy of the reference sensor and to the sampling differences) is usually needed but overcome with OEM.

Page 1, line 13 & 15: "under different sky conditions", "under clear and cloudy conditions"

Interactive comment on Atmos. Meas. Tech. Discuss., doi:10.5194/amt-2019-107, 2019.

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AMT-2019-107: Retrieval of Temperature From a Multiple Channel Pure Rotational Raman-Scatter Lidar Using an Optimal Estimation Method (Gamage et al.) Response to Referee 2 (Anonymous Referee #1) 26 September 2019

The Referee's comments are in blue, are responses are in black. Red sections have been added to the manuscript.

Since the proposed technique seems to be universally applicable to temperature rotational Raman lidar systems, more references to the state-of-the-art of this technique and other existing systems would certainly be interesting for the readers and should be included.

We will add the following paragraphs to the introduction of the paper:

Behrendt (2005) provides a comprehensive overview of the traditional rotational Raman lidar temperature calculation method. Over the years the traditional temperature method has been improved by advancing the instrumentation capabilities and by improving the estimation and calibration techniques. Here are some examples of innovations in this area since the Behrendt (2005) review.

Studies by Radlach et al. (2008) and Weng et al. (2018) introduce changes to the individual Raman lidar systems to improve the temperature measurements. Radlach et al (2008) introduced a new high-resolution rotational Raman lidar system with a receiving system that uses multicavity interference filters in a sequential setup to improve the efficiency of the elastic and rotational Raman signal separation. Together with the filter adjustments they have made noontime temperature measurement with uncertainties less than 1 K up to 1 km for 1 min integration time. Weng et al. (2018) introduced a new PRR lidar system that effectively detects two isolated N_2 molecule PRR line signals and elastic backscatter signals. With this new system, temperatures at any given time can be obtained without calibration.

The accuracy of the traditional Raman temperature estimations highly depends on the estimation of the calibration function and calibration coefficients. Zuev, Vladimir V., et al. has investigated the use of nonlinear calibration functions to improve the accuracy of the traditional Raman temperature estimation and somehow compensate for the heavy assumption of single PRR line. The results showed the 1/T term expressed in a form of a quadratic function of log of the ratio of the PRR measurements is the best for practical use. Another study by He, Jingxi, et al. proposed a new calibration method for PRR lidar temperature profiling based on the different temperature sensitivities of Stokes and anti-Stokes PRR lines. In this paper they reconstruct the expression of

the differential backscatter cross section according to the temperature dependencies of each component and form a temperature factor and a calibration factor in the intensity ratio. This new method has reduced the temperature error by ~50% compared with the commonly-used calibration methods in conditions of low signal-to-noise ratio (SNR).

Temperature profiling from Raman lidar backscatter measurements can also be improved using retrieval schemes based on OEM as we introduce in this paper. A previous study by Yan, Qing, et al. has also proposed an optimized retrieval method for traditional Raman temperature profiling. The proposed method allowed independent alternating solutions to high- and low-quantum-number PRRSs, where high-quantum-number PRR lidar returns are used to solve the channel constant, and low-quantum-number PRR returns with high SNR are used for retrieving temperature profiles. The results showed that the effective temperature retrieval height greatly improved from 17 to 25 km under clear weather conditions and better than 5 K can be obtained up to 25 km.

You mention two assumptions on page 8, line 25/26. What do you mean with "well known"? How critical are these assumptions? When highlighting which assumptions are NOT needed (see abstract, introduction, conclusions), you should be fair and also mention which are needed.

"Well known" was a poor choice of words, we meant the *a priori* extinction profile is based on a backscatter ratio measurement by the lidar, and that the overlap is based on estimates from clear sky measurements compared to the expected overlap (please see our response to your comment on page 15 in the Minor Comments section which explains this in more detail). To estimate an extinction profile requires the assumption of a lidar ratio.

As part of our response to Referee 1 (Christoph Ritter) comments we have added the following paragraph to the paper that explains our approach of the overlap and extinction retrievals.

"The effect of geometrical overlap and particle extinction on the signals are strongly coupled and hence retrieving both parameters simultaneously with the given data channels is not possible unless at least one of the effects is highly constrained. We assume that particle extinction is well known from the backscatter ratio outside clouds, and that overlap is well known above the height of full overlap, i.e. above 6 km (Dinoev et al., 2010). We use this knowledge to define a transition height, 6 km in clear skies or at the cloud base height, whatever is lower. Below this height overlap is retrieved, and above this height particle extinction is retrieved. The *a priori* overlap function is estimated from measurements in clear sky conditions. A 50% standard deviation is used for geometrical overlap below the transition height and a constant standard deviation of 10^{-3} is used above this height, constraining the geometrical overlap to the *a priori* values above the transition height. For particle extinction, a standard deviation of 10^{-6} km⁻¹ is used

below the transition height to constrain the retrieval, then a 50% standard deviation is used above this height, allowing the OEM to retrieve exclusively the particle extinction. The *a priori* covariance matrices for both particle extinction and geometrical overlap are determined using a tent function with a 100m correlation length."

Maybe it would be interesting to explain in the abstract and conclusions that (how much) the results are independent from the selected *a priori* temperature profile.

Similar to Sica and Haefele (2015), we trust our retrievals upto a certain cutoff height where the response function falls below a value of 0.9. Thus, our retrieval depends on maximum of 10% of the *a priori* temperature profile and that dependency matter only in the heights where the signal strength becomes weaker. One can choose different values of response function to define the cutoff height.

We will add the following sentences to the conclusion explaining the dependency of the temperature retrievals from the selected *a priori* temperature profile.

The OEM retrieved temperatures depend nearly to 100% of the measurement. At the cut off height, the measurement contribution falls below 90%. Temperature retrievals above the cutoff height depend to more than 10% on the *a priori* temperature profile and are not considered an independent measurement.

Page 9, line 4: There are always aerosols in the troposphere. How few are acceptable?

In our study we define the clear and cloudy conditions in the atmosphere based on the backscatter ratio profiles we calculate using RALMO elastic and PRR measurements. When there are no clouds or thick aerosol loads found we have observed the backscatter ratio is less than 2. We consider such cases to be cloud free and retrieve the overlap function up to 6 km.

I think the term "digital measurements" for photon counting signals is odd. Also the analog signals are digitized. Thus, I suggest that you write "photon counting" throughout the text.

We will change the term digital measurements to photon counting.

You are correct, this error was also pointed out by Referee 1. We made a mistake in the analog measurement units. Our response here is the same as to Referee 1:

"The RALMO analog raw data are sampled by Licel counters, and then converted to counts (ADC). However, it does not change the unit of the analog signal. Thus, we have made a mistake in the analog signal units in Figures 1, 7,12 and 18. Units for the analog signals are now corrected to the units of mV."

Minor comments:

"Lidar constants" and "coupling constants" are terms which are not commonly used. Please define/explain or avoid. E.g., page 1, line 5: "ratio of efficiencies" is better I think since the laser power, transmitter efficiency, telescope area are the same for both channels and thus cancel and become irrelevant (if I understand correctly).

Lidar constant /lidar system constant (Liu, Z., Voelger, P., & Sugimoto, N. (2000), Tao, Zongming, et al. (2008), Winker, D. M., and M. A. Vaughan, Kovalev(1994), Vladimir A., and H. Moosmüller (1994)) refers to combining instrument and physical constants into a single constant. While individual instrument parameters can often not be determined, such as the optical efficiency of the system, the overall value of the constant can be estimated using the return photocount profiles (e.g. Sica et al. 1995). The term coupling constant, the ratio of the two lidar constants, not a common term. This ratio was called a coupling constant by Sica and Haefele (2016), since it mathematically coupled the lidar equations for physical separate counting channels in their forward model. In this study a coupling constant is used in the forward model to couple the measurements from the two PRR digital/analog channels. We would prefer to define and use the term coupling constant in the manuscript rather than ratio of efficiencies. We will make sure these terms are well defined.

Page 1, line 21: ". . . 2 flights per day". One could add: "only at selected sites worldwide". We will add this.

Page 1, line 22: Please add references to other combined Raman lidar systems. There are several which measure water vapor and temperature.

We have added the following references:

- 1. Mattis, Ina, et al. "Relative-humidity profiling in the troposphere with a Raman lidar." *Applied optics* 41.30 (2002): 6451-6462.
- 2. Wang, Yufeng, et al. "A detection of atmospheric relative humidity profile by UV Raman lidar." *Journal of Quantitative Spectroscopy and Radiative Transfer* 112.2 (2011): 214-219.
- 3. Reichardt, Jens, et al. "RAMSES: German Meteorological Service autonomous Raman lidar for water vapor, temperature, aerosol, and cloud measurements." *Applied optics* 51.34 (2012): 8111-8131.
- 4. Behrendt, Andreas, et al. "Combined Raman lidar for the measurement of atmospheric temperature, water vapor, particle extinction coefficient, and particle backscatter coefficient." *Applied optics* 41.36 (2002): 7657-7666.

If we have missed anything else please let us know and we would be glad to add it.

Page 3, Eq. 1: You assume that all PRR lines of one channel are collected with the same efficiency. This is generally not the case but may be true for RALMO. Please add a comment on this.

We do not consider the efficiencies of all the PRR lines are the same. Transmission of the receiver at the wavelength of the PRR line given by $\tau_{RR}(J_i)$ in Eq.1 represents the efficiencies of each detected by the RALMO system. $\tau_{RR}(J_i)$ for RALMO are known (Figure 1, Dinoev, T. S., et al) and used in our forward model.

Page 3, Eq. 2: This equation is not found in Penney et al. 1974.

This equation is from Behrendt A (2005) and the original equations are based on Penney et al. 1974. We will fix this in the manuscript.

Page 5, line 11: Why is the background noise B_RR a function of height? I think the "real background" should be height independent. If the baseline is height dependent, detector non-linearity or electronic cross-talk is present, I guess.

In general background noise B_{RR} is a function of height. For some systems this can be independent of height. On Page 9 line 11 we have explained that for RALMO background noise is independent of height.

Page 7, table 2: What do you with "transition height"?

We have fixed this to:

Geometrical Overlap Function	Estimated using the forward model and measurements	50% below and at transition height
Particle Extinction	Estimated using measurements	10 ⁻⁶ km ⁻¹ below and at transition height

Page 8, line 6: beta_par *is* related

Changed the text.

Page 9, line 20: I would prefer "model parameters" or "model b parameters" Changed the text.

Page 9, line 21: Please refer to the figure.

This sentence is now changed to "The traditional temperature profiles that will be shown later in this section are calculated using count profiles consisting of glued analog and digital measurements which are corrected for non-linearity and background before processing".

Page 10: line 6: What do you mean with "dominant"?

In lower altitudes signal strengths are higher and as the height increases the signal gets weaker. Then the effect from the electrical offset starts to dominate and analog signal in higher altitudes will have values that are not correct.

As we think the word dominant poor choice of words we will change this to be "becomes larger."

Page 10, line 7: How do you know that the analog signals are linear?

RALMO uses Licel transient recorders for data acquisition with Licel's Hammatsu PMTs. The Licel transient recording for analog measurements are designed/tested for linearity by the manufacturer. These systems are widely used in the lidar community, and we are not aware of any evidence that the analog channels are nonlinear.

Page 10, line 8: "become saturated". What do you mean with "saturated"?

What we are implying here is that the photon counting measurements that are above 10MHz are no longer linear. We have made a mistake in the paper by stating the saturation limit is 2 MHz. We will correct this to 10MHz.

Page 11, Fig 2: I suggest that you write "four signals". It is *two* channels. Will change the text.

How did you determine "measurement noise"? With OEM? Please clarify.

The OEM does not determine measurement noise. However, we require measurement noise to evaluate the OEM. As given in page 7 line 5, for photon counting measurements which are linear, the measurement variance is equal to the square root of the photon counting measurement (Poisson statistics). For photon counting measurements that are non-linear and for analog measurements we use the auto-covariance method (refer Lenschow *et al.* (2000).) to estimate the measurement noise.

Page 14, line 2: What is coupling constant R_a? How is it defined?

R_a is the coupling constant for analog channels. It is estimated using Eq.14. This has been stated in the paper as "The coupling constants for analog (Ra) and digital (R) channels are estimated by fitting the ratio of PRR measurements with the ratio of the differential cross section (Eq.(14))."

Page 15, Fig 6: How did you obtain the *a priori* overlap? Why is this important since the ratio of the overlap functions cancels?

The *a priori* overlap function used in the OEM scheme is estimated using the OEM retrievals of temperature during clear sky conditions. A few clear day and nighttime measurements were processed in the OEM using a model overlap function given in Dinoev, T., *et al.* (2013) and corresponding overlap functions were retrieved. Then the average of the retrieved overlap functions from the measurements obtained at clear conditions were used as the *a priori* overlap function in the current PRR temperature OEM scheme.

The temperature retrieval is not sensitive to the choice of the *a priori* overlap function. Both, a sensitivity analysis and the retrieval diagnostics (the overlap averaging kernels) (see below) confirm this. But the overlap function is still important since the lidar signal for each channel is modelled explicitly, as opposed to the traditional method, which works with the signal ratios. Analogously, in the traditional method, the overlap could be determined using the retrieved temperature profile, a forward model and an extinction profile from an ancillary source.

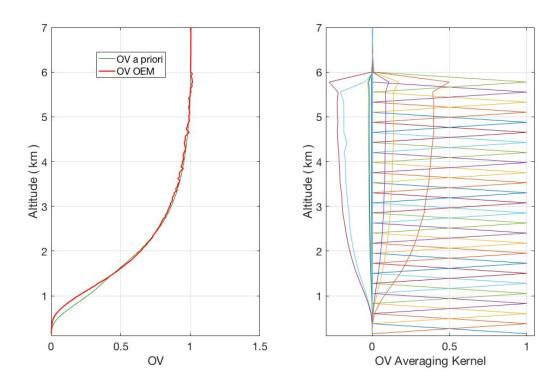


Figure: Left Panel: The OEM retrieved overlap (red curve) and the a priori overlap (green curve) from the clear nighttime RALMO measurements made on 20110909. Right Panel: The overlap averaging kernel.

Figs 1, 7, 12, 17: I would prefer the same scales. JLa -> JL, JHa -> JH? Should be consistent. I think it would be interesting to show the elastic signals for all cases. Why "Eb" for "elastic"?

The reason we are not showing the analog signals in the same height range as digital is that as in altitudes above 15 km analog signals go to background. Thus, the variations in the lower level analog signals are not clearly visible with the extended altitude axis.

We have only showed the elastic signal for Case study 4 as it is a special case. However, we will attach the profiles of elastic signals for the other 3 cases studies as a supplementary document.

Eb is a term that is used in the RALMO system, that refers to the **e**lastic backscatter detected by the **b**ig telescope.

The language still needs polishing/corrections. Here two examples:

Page 1, line 3ff: "assumption for the form of . . ." However, I think the form of the calibration function is not really the point here. Calibration with external sensors (with all the uncertainties related to the accuracy of the reference sensor and to the sampling differences) is usually needed but overcome with OEM.

Thank you. We will go over the text again carefully before submitting. We have fixed the 2 unclear sentences you have mentioned above.

References:

- 1. Behrendt, Andreas. "Temperature measurements with lidar." *Lidar*. Springer, New York, NY, 2005, 273-305
- 2. Penney, C. M., RL St Peters, and M. Lapp. "Absolute rotational Raman cross sections for N 2, O 2, and CO 2." *JOSA* 64.5 (1974): 712-716.
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- 4. Sica, R. J., and A. Haefele. "Retrieval of temperature from a multiple-channel Rayleigh-scatter lidar using an optimal estimation method." *Applied optics* 54.8 (2015): 1872-1889.
- 5. Lenschow, Donald H., Volker Wulfmeyer, and Christoph Senff. "Measuring second-through fourth-order moments in noisy data." *Journal of Atmospheric and Oceanic Technology* 17.10 (2000): 1330-1347.

- 6. Behrendt, A., et al. "Profiles of second-to fourth-order moments of turbulent temperature fluctuations in the convective boundary layer: first measurements with rotational Raman lidar." *Atmospheric Chemistry and Physics* 15.10 (2015): 5485-5500.
- 7. Radlach, Marcus, Andreas Behrendt, and V. Wulfmeyer. "Scanning rotational Raman lidar at 355 nm for the measurement of tropospheric temperature fields." *Atmospheric Chemistry and Physics* 8.2 (2008): 159-169.
- 8. Zuev, Vladimir V., et al. "Tropospheric temperature measurements with the pure rotational Raman lidar technique using nonlinear calibration functions." *Atmospheric Measurement Techniques* 10.1 (2017): 315-332.
- 9. He, Jingxi, et al. "A Novel Calibration Method for Pure Rotational Raman Lidar Temperature Profiling." *Journal of Geophysical Research: Atmospheres* 123.19 (2018): 10-925.
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- 12. Tao, Zongming, et al. "Measurements of cirrus cloud backscatter color ratio with a two-wavelength lidar." *Applied optics* 47.10 (2008): 1478-1485.
- 13. Winker, D. M., and M. A. Vaughan. "Vertical distribution of clouds over Hampton, Virginia observed by lidar under the ECLIPS and FIRE ETO programs." *Atmospheric research* 34.1-4 (1994): 117-133.
- 14. Kovalev, Vladimir A., and H. Moosmüller. "Distortion of particulate extinction profiles measured with lidar in a two-component atmosphere." *Applied optics* 33.27 (1994): 6499-6507.
- 15. Sica, R. J., et al. "Lidar measurements taken with a large-aperture liquid mirror. 1. Rayleigh-scatter system." *Applied optics* 34.30 (1995): 6925-6936.
- 16. Reichardt, Jens, et al. "RAMSES: German Meteorological Service autonomous Raman lidar for water vapor, temperature, aerosol, and cloud measurements." *Applied optics* 51.34 (2012): 8111-8131.
- 17. Behrendt, Andreas, et al. "Combined Raman lidar for the measurement of atmospheric temperature, water vapor, particle extinction coefficient, and particle backscatter coefficient." *Applied optics* 41.36 (2002): 7657-7666.
- 18. Povey, Adam C., et al. "Estimation of a lidar's overlap function and its calibration by nonlinear regression." *Applied optics*51.21 (2012): 5130-5143.
- 19. Dinoev, T. S., et al. "Monitoring of Eyjafjallajökull ash layer evolution over payerne Switzerland with a Raman lidar." *Proceedings of the TECO* (2010).

Review of "Retrieval of Temperature from ... by optimal estimation method" by Mahagammulla Gamage, et al.

The paper describes the temperature retrieval by lidar using the optimal estimation method. Four different cases (each day / night and clear / cloudy) from Payerne station are used to demonstrate the feasibility of the method. While the OEM method for temperature retrieval is not entirely new and the first and third bullet point of the conclusions may be found elsewhere, the paper as a whole contents enough information for publishing. It is generally well written and well structured. The 4 selected cases are new and demonstrate the usage of the still not too far spread OEM technique.

I have a short list of questions and remarks to the author which should be clarified prior to publishing:

You show a complete error analysis for the OEM technique, which I found convincing. However, to better judge these results it would be very interesting to have a similar error estimation for the traditional Raman technique. Especially in your case 1 fig. 4 it seems as if the Raman result is only plotted up to 12 km although in the manuscript a change of vertical resolution in this altitude is mentioned (page 11). Can you comment on this?

Do you assume the same overlap for both channels? Why do you need the overlap? Do you really think to be able to retrieve the overlap with the required precision to obtain aerosol extinction information? In this case an error analysis would be required, otherwise revise your wording. I think for this paper such an effort is not necessary – as the particle extinction is the same for the high and low channel, hence the temperature from the OEM should not depend on the extinction?

Page 11 and Fig. 5: your error analysis is nice and one of the strong selling points of this paper. However, in the current form I cannot reproduce the values. A bit more information is required, how the values were obtained.

Minor points:

Page 1 quote Mahagammulla Gamage: I think in the introduction it is not necessary to quote a paper which is still under preparation. You may choose another quote here

Page 3: (and elsewhere) is d sigma / d Omega really the ATTENUATED cross section? I am not sure, as you have Gamma^2 as extinction term in your eq. 1.

Page 4: explain x_a in eq. 7.

Page 5: minus sign in eq. 8 is missing

Page 9: 2 times "from" in line 14

Page 10, fig. 1: is the units of your analog signal MHz, how did you convert it?

Page 11: line 13: reduced

Page 13: line 4: agrees, line 4: deviates

Page 19: a lidar ratio of 5sr is already quite small, can you estimate an error for the lidar ratio?

Page 20: line 8: "by" missing ... that by the OEM method; line 9: I don't understand the "temperature range" – I thought the OEM only depends on K and K_a?

Page 21: line 11: what do you mean be "uncorrected" PRR measurements?

AMT-2019-107: Retrieval of Temperature From a Multiple Channel Pure Rotational Raman-Scatter Lidar Using an Optimal Estimation Method (Gamage et al.) Response to Referee 1 (**Christoph Ritter**) 04 July 2019

The Referee's comments are in blue, responses are in black, and the texts added to the manuscripts are in red.

You show a complete error analysis for the OEM technique which I found convincing. However to better judge these results it would be very interesting to have a similar error estimation for the traditional Raman technique. Especially in your case 1 fig4 it seems as if the Raman result is only plotted up to 12 km although in the manuscript a change of vertical resolution in this altitude is Mentioned page 11. Can you comment on this? We strongly agree that the community of users of the traditional temperature method would benefit from a detailed investigation of that technique in the spirit of the 3 papers by the group of NDACC lidar scientists who have done comprehensive studies of the traditional ozone and Rayleigh scatter temperature determinations, as well as quantifying the vertical resolution of the determinations [see the references at the end of this reply], and we would be glad to participate in that effort. It is, however, tangential to the purpose of this manuscript which is to show a new way to retrieve temperature, which offers some advantages (and disadvantages) relative to the traditional method. For instance in Jalali et al. 2018 and Farhani et al. 2019 the OEM uncertainty budgets were shown to compare well with previous studies for Rayleigh scatter temperature profiles and DIAL ozone profiles (including the NDACC work). These comparisons give us confidence in our uncertainty budgets for lidar retrievals using OEM.

As far as to our choice of plotting the traditional method up to 19km (Fig4: traditional temperatures goes up to 19km not 12km), the traditional temperature estimates are derived from the MeteoSwiss routine temperature analysis, which works as follows.

The vertical resolution is not constant; it starts at 30m and it increases to 400m.

- The change in vertical resolution is based on the calculated uncertainty of the temperature profile at each height. The maximum height resolution allowed is 400m, until the temperature statistical uncertainty becomes smaller than a threshold value of 1K.
- The calculated profile is cut off based again on the uncertainty. When the algorithm can't calculate the next range gate within the threshold value of 1K it stops the profile at the last range gate.

In Fig. 4 the traditional retrievals are shown up to \sim 19km. For that specific case (Case 1), the traditional temperature vertical resolution is 30m up to 12.5km, then changes to 400 m above this height. The temperatures are cutoff at the height where the uncertainty reaches 1K (19km).

Above information on the traditional method is given in Section 5 of the manuscript.

• Do you assume the same overlap for both channels? Why do you need the overlap? Do you really think to be able to retrieve the overlap with the required precision to obtain aerosol extinction information? In this case an error analysis would be required otherwise revise your wording. I think for this paper such an effort is not necessary – as the particle extinction is the same for the high and low channel, hence the temperature from the OEM should not depend on the extinction?

Yes, we assume the same overlap for both channels based on the work of Dinoev et al. 2012. We need the overlap since in OEM we forward model the raw backscatter profile of each channel and hence, have to specify an overlap function. At a given altitude, it is not possible in our retrieval to determine both overlap and extinction. Therefore, our approach is to retrieve whatever quantity we know less well. Generally this is overlap below 6km and extinction above 6km, since above 6km full overlap can be safely assumed. In case of clouds with a ceiling below 6km, we retrieve overlap up to the ceiling and extinction above. Furthermore, the results for extinction and lidar ratio look reasonable compared to other studies. One benefit of OEM is the ability to estimate the effect of a model parameter on the retrieved quantity, so given we have made reasonable choices for the model parameter uncertainties their impact on the retrieved temperature is well characterized.

Moreover, we trust the extinction retrieval and its uncertainty above 6km but not below and we changed the text in Section 4.3 of the manuscript as shown below to clarify this.

"The effect of geometrical overlap and particle extinction on the signals are strongly coupled and hence retrieving both parameters simultaneously with the given data channels is not possible unless at least one of the effects is highly constrained. We assume that particle extinction is well known from the backscatter ratio outside clouds, and that overlap is well known above the height of full overlap, i.e. above 6 km (Dinoev et al., 2010). We use this knowledge to define a transition height, 6 km in clear skies or at the cloud base height, whatever is lower. Below this height overlap is retrieved, and above this height particle extinction is retrieved. The a priori overlap function is estimated from measurements in clear sky conditions. A 50% standard

deviation is used for geometrical overlap below the transition height and a constant standard deviation of 10⁻³ is used above this height, constraining the geometrical overlap to the a priori values above the transition height. For particle extinction, a standard deviation of 10⁻⁶km⁻¹ is used below the transition height to constrain the retrieval, then a 50% standard deviation is used above this height, allowing the OEM to retrieve exclusively the particle extinction. The a priori covariance matrices for both particle extinction and geometrical overlap are determined using a tent function with a 100m correlation length."

Figure 1, shows the full uncertainty budgets of the overlap and particle extinction retrievals for the Case 3: Nighttime with cirrus cloud given in the manuscript. Overlap uncertainties are shown up to 6km and the particle extinction uncertainties are shown above 6 km.

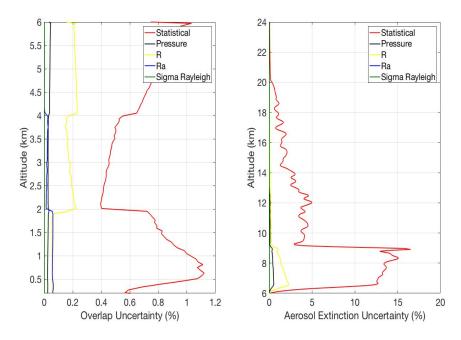


Figure 1: Left Panel: Full uncertainty budget of the overlap retrievals from the nighttime RALMO measurements on 05 July 2011 with a cirrus cloud present at 6km height. Uncertainty due to Rayleigh cross section, pressure, and coupling constants are in the orders of 10⁻⁴, 10⁻¹, and 10⁻¹ respectively. Right Panel: Full uncertainty budget of the particle extinction. Uncertainty due to Rayleigh cross section 10⁻³ and due to analog coupling constant is 10⁻². Uncertainty due to pressure and digital coupling constants are about 1% and 2% respectively.

• Page 11 and Fig. 5 your error analysis is nice and one of the strong selling points of this paper. However, in the current form I cannot reproduce the values. A bit more information is required, how the values were obtained.

Forward model uncertainties are calculated based on the theory presented in the Rodgers textbook Section 3.2.2. We have also added the following equations that we used to compute the errors into the revised manuscript.

The uncertainty budget is determined from the measurement and model parameter covariance matrices (Rodgers, 2000). The total covariance S_{total} is:

$$S_{total} = S_m + S_F$$

where S_m is the retrieval covariance due to measurement noise and S_F is the retrieval covariance due to the forward model parameter uncertainty. The retrieval covariance due to measurement noise S_m is

$$S_{\rm m} = GS_{\rm v}G^{\rm T}$$

where G is the gain matrix that indicates the sensitivity of the retrieval to the measurements. The retrieval covariance due to the forward model parameters S_F is $S_F = GK_bS_bK^T_bG^T$

where K_b and S_b are the forward model parameter Jacobian and covariance matrices respectively. The model parameter Jacobians K_b , can be estimated analytically or numerically for each model parameter. To construct S_b we require the uncertainties of the model parameters. We recommend Rodger's textbook (Rodgers, 2000) for more details of the OEM.

Furthermore, in our work presented in the manuscript we have used the uncertainties used in the Table 2 in the manuscript to construct construct S_b .

- Page 1 quote Mahagammulla Gamage: I think in the introduction it is not necessary to quote a paper which is still under preparation. You may choose another quote here: agree will remove this.
- Page 3: and elsewhere is d sigma /d Omega really the ATTENUATED cross section? I am not sure, as you have Gamma^2 as extinction term in your eq. 1?

Yes. this term is attenuated cross section. We define attenuated differential cross section term as the convolution of the instrument function with each line in the spectrum. This is shown in EQ 2, where attenuated cross section contains the terms $\tau^+(J_i^-)$ and $\tau^-(J_i^-)$ that are the transmission of the receiver at the wavelength of each RR line. However, this term is not a standard one and it is easy to see why it could be confused with attenuation due to atmospheric transmission. The atmospheric transmission is defined in EQ 1 in the usual manner.

• Page 10..fig. 1: is the units of your analog signal MHz.how did you convert it?:

However, in the OEM it doesn't matter what unit we use for signal as long as the forward model generated signal and the real measurements are in the same unit. Another advantage of OEM is that multiple detector channels, both analog and digital with differing vertical resolutions and units can be easily used. Thus, these changes have no effect on the results in the manuscript.

• Page19:a lidar ratio of 5sr is already quite small. Can you estimate an error for the lidar ratio?

We can estimate a statistical uncertainty using standard error propagation. The lidar ratio is not directly retrieved in our OEM. Our estimate of LR is based on the retrieved particle extinction and ASR profiles calculated from elastic and PRR lidar measurements. Even though it is small mathematically it is possible to estimate an error and that is what we have done. We will add a phrase to the manuscript explaining how the uncertainty of lidar ratio was calculated. We have also updated Figure 17 in the manuscript, indicating the estimated LR errors.

• Page20: line9: I don't understand the "temperature range"—I thought the OEM only depends on K and K a?

The calibration coefficients of the calibration function used in the traditional temperature retrievals needs to be estimated over a wide range of possible temperatures. However, our OEM calibration constants (coupling constants) can be estimated over a narrow range of temperatures, or even one single point, without introducing extrapolation errors in the retrieval. The OEM depends on the two coupling constants R and R_a and estimation of those require temperatures, but does not require the assumption of a functional form relating these constants to temperature (refer to Eq.10 and 11 in the manuscript); the requirement of a functional form depending on multiple parameters rather than a constant is why the traditional method must use a wide range of temperatures.

• Page21:line11:what do you mean be"uncorrected "PRR measurements?: This was meant to be raw PRR measurements, that is a Level 0 product not corrected for saturation or background. We will change this word from "uncorrected" to "raw".

Thank you for noticing the following typos. We will fix these mistakes in the revised paper

- Page 4: explain x_a in eq. 7.: x_a is the a priori.
- Page 5: minus sign in eq. 8 is missing: Fixed.
- Page 9: 2 times "from" in line 14: Fixed
- Page 11 : line 13 : reduced : Fixed
- Page13: line 4: agrees line 4:deviates: Fixed
- Page20:line8:"by"missing...that by the OEM method^a: Fixed

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Retrieval of Temperature From a Multiple Channel Pure Rotational Raman-Scatter Lidar Using an Optimal Estimation Method

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Abstract. We present a new method for retrieving temperature from Pure Rotational Raman (PRR) lidar measurements. Our Optimal Estimation Method (OEM) used in this study uses the full physics of PRR scattering and does not require any assumption of the form for a calibration function nor does it require fitting of calibration factors over a large range of temperatures. The only calibration required is the estimation of the ratio of the lidar constants of the two PRR channels (coupling constant) that can be evaluated at a single or multiple height bins using a simple analytic expression. The uncertainty budget of our OEM retrieval includes both statistical and systematic uncertainties, including the uncertainty in the determination of the coupling constant on the temperature. We show that the error due to calibration can be reduced significantly using our method, in particular in the upper troposphere when calibration is only possible over a limited temperature range. Some other advantages of our OEM over the traditional Raman lidar temperature retrieval algorithm include not requiring correction or gluing to the raw lidar measurements, providing a cutoff height for the temperature retrievals that specifies the height to which the retrieved profile is independent of the *a priori* temperature profile, and the retrieval's vertical resolution as a function of height. The new method is tested on PRR temperature measurements from the MeteoSwiss Raman Lidar for Meteorological Observations system in clear and cloudy sky conditions, compared to temperature calculated using the traditional PRR calibration formulas, and validated with coincident radiosonde temperature measurements in clear and cloudy conditions during both day and night time.

Copyright statement. TEXT

1 Introduction

High time and space resolution measurements of atmospheric temperature are necessary to improve our understanding of many atmospheric processes, both dynamical and chemical. Radiosounding is the most widely used method for temperature profiling in the troposphere and lower stratosphere, and has the advantage of operation in most weather conditions, but is typically limited to 2 flights per day only at selected sites worldwide. Pure rotational Raman (PRR) lidars have excellent vertical and temporal resolution, and can be combined with vibrational Raman channels to determine relative humidity (Mattis et al., 2002;

Wang et al., 2011; Behrendt et al., 2002; Reichardt et al., 2012). Lidar temperature measurements can be assimilated with atmospheric models to improve weather forecasts, as recently demonstrated by Adam et al. (2016).

The traditional Raman lidar temperature retrieval method, introduced by Cooney (1972), uses the ratio of two PRR signals from the Stokes branch which have been corrected for saturation, background and other instrumental effects as required. The PRR spectrum contains two branches: Stokes and anti-Stokes. Both branches have approximately the same intensity and they are positioned symmetrically in wavelength on either side of the excitation line. The traditional Raman lidar temperature retrieval algorithm requires the assumption of an analytic form of a lidar calibration function whose coefficients are usually determined with external measurements, such as radiosondes (Behrendt, 2005). The calibration function is an approximation of the relationship of the signal ratio and temperature and depends on two or more coefficients. Calibration errors exceeding 0.5 K can arise if the calibration data do not cover a sufficient temperature range (Behrendt, 2005).

Of primary importance is in the calibration of the lidar returns to allow absolute temperature measurements. In the traditional method, the ratio of the corrected photo-counts is fit to a set of corresponding temperature data points usually obtained from radiosondes. Behrendt (2005) gives a comprehensive overview of the traditional rotational Raman lidar temperature calculation method. Over the years the traditional temperature method has been improved by advancements in the instrumentation capabilities and improvements to the estimation and calibration techniques. Below some examples of innovations in this area since the (Behrendt, 2005) review.

Radlach et al. (2008) and Weng et al. (2018) introduce changes to the their Raman lidar systems to improve the temperature measurements. Radlach et al. (2008) introduced a new high-resolution rotational Raman lidar system with a receiving system that uses multicavity interference filters in a sequential setup to improve the efficiency of the elastic and rotational Raman signal separation. Together with the filter adjustments they have made noontime temperature measurement with uncertainties less than 1 K up to 1 km for 1 min integration time. Weng et al. (2018) introduced a new PRR lidar system that effectively detects 2 isolated nitrogen molecule PRR line signals and elastic backscatter signals. With this new system, temperatures at any given time can be obtained without a calibration.

The accuracy of the traditional Raman temperature estimations is highly sensitive to the estimation of the calibration function and calibration coefficients. Zuev et al. (2017) has investigated the use of nonlinear calibration functions to improve the accuracy of the traditional Raman temperature estimation and relaxing the assumption of sampling two single PRR lines. The results showed the ¹/T term expressed in a form of a quadratic function of log ratio of the PRR measurements as best for practical use. He et al. (2018) proposed a new calibration method for PRR lidar temperature profiling based on the different temperature sensitivities of Stokes and anti-Stokes PRR lines. They reconstruct the expression of the differential backscatter cross section according to the temperature dependencies of each component and form both a temperature factor and a calibration factor in the intensity ratio. This new method reduces the temperature error by 50% compared with commonly-used calibration methods in conditions of low signal-to-noise ratio (SNR).

Yan et al. (2019) has proposed an iterative method for determining Raman temperature. Their method allows independent alternating solutions to the high and low quantum number PRRSs separately, where high quantum number PRR lidar returns are used to solve for the channel constant, while low quantum number PRR returns with higher SNR are used for retrieving

temperature profiles in an iterative fashion. Their results showed that the effective temperature retrieval height for their system improved from 17 to 25 km.

The application of the Optimal Estimation Method (OEM) for temperature retrievals using Pure Rotational Raman (PRR) lidar measurements has several advantages over the traditional method, including the ability to find temperature without assuming an analytic form of the temperature/count ratio relation. Our OEM retrieval does not use the ratio of the counts. Rather we use a forward model which includes complete physics to describe the raw count profiles. For calibration the ratio of the lidar constants, here from referred to as coupling constant, needs to be determined. The coupling constant can in principle be estimated at a single point, such as a nearby flux tower or surface measurement. Our OEM retrieval has other important benefits over the traditional method, as it can directly retrieve ancillary parameters to temperature, such as geometrical overlap, particle extinction, and the lidar constant. Our OEM is also capable of providing a full uncertainty budget, including both random and systematic uncertainties on a profile-by-profile basis, including the systematic uncertainty introduced in the retrieved temperature by the estimation of the coupling constant. The OEM is an inverse method, and is a standard tool in the retrieval of geophysical parameters from passive atmospheric remote sensing instruments. Recent studies including (Povey et al., 2014; Sica and Haefele, 2015, 2016; Farhani et al., 2019) have shown that OEM can be used to retrieve atmospheric aerosol, water vapor mixing ratio, middle and upper atmospheric temperature and ozone using lidar measurements.

In Section 2 a brief description of the instrument and the measurements used in this study is presented. Section 3 presents the development of the PRR lidar equation.

The development of a forward model for application of the OEM to PRR temperature retrieval is given in Section 4. The OEM-retrieved temperature results from the PRR measurements for different atmospheric conditions are shown in Section 5. A discussion of these results is presented in Section 6 followed by conclusions.

2 The Raman Lidar for Meteorological Observations

PRR measurements from the RAman Lidar for Meteorological Observations (RALMO), located in Payerne, Switzerland (46°48′ N, 6°56′ E) are used for the OEM temperature profiling. RALMO is a fully automated lidar built at the École Polytechnique Fédérale de Lausanne and operated by MeteoSwiss (Dinoev et al., 2013). It is dedicated to operational meteorology, validating models and satellite measurements, and climate studies. RALMO has been operating nearly continuously since 2008, with an average data availability of 50%. Data gaps are due to rain and low clouds (approximately 30% of the time), maintenance (1-2 days per month) and other occasional technical problems. RALMO consists of a frequency tripled, Q-switched Nd:YAG laser of 354.7 nm producing up to 400 mJ emission energy at 30 Hz repetition rate. The pulse duration is 8 ns. The laser is operated at 300 mJ energy per pulse to extend the lifetime of the flash lamps from 20 to approximately 60 million shots. The RALMO receiver uses four parabolic mirrors each with 1 m focal length and 30 cm diameter, and it is fiber coupled to a two stage grating polychromator. The data acquisition system consists of photo-multipliers and analog/digital transient recorders from Licel. The system obtains a measurement by adding together 1800 laser shots (every minute) at a vertical resolution of

3.75 m. For a detailed description of the lidar and a detailed validation of the temperature measurements the reader is referred to Dinoev et al. (2013).

The returns of the Raman-shifted backscatter arising from rotational energy state transitions of nitrogen and oxygen molecules due to the excitation at the laser wavelength at 354.7 nm are detected in analog and photon counting mode. The high quantum number channel (JH) of RALMO is assigned to the backscattered signals from the energy exchange that occurs in the high quantum states for both the Stokes (355.77-356.37 nm) and anti-Stokes (353.07-353.67 nm) branches. The low quantum number channel (JL) is assigned to the signals from the energy exchange occurring in the low quantum states in the Stokes (355.17-355.76 nm) and anti-Stokes (353.67-354.25 nm) branches.

3 The PRR lidar equation

10 The backscattered PRR signal is given by the Raman lidar equation:

$$N_{RR,t}(z) = \frac{C_{RR}}{z^2} O(z) n(z) \Gamma_{atm}^2(z) \left(\sum_{i=O_2, N_2} \sum_{J_i} \eta_i \left(\frac{d\sigma}{d\Omega} \right)_{\pi}^i (J_i) \right) + B_{RR}(z)$$
 (1)

where the true backscattered PRR signal $N_{RR,t}$, is a function of height z, C_{RR} is the lidar constant, n(z) is the number density of the air molecules, O(z) is the geometrical overlap, η_i is the volume mixing ratio of nitrogen and oxygen, $\Gamma_{atm}(z)$ is the atmospheric transmission, $\left(\frac{d\sigma}{d\Omega}\right)^i_{\pi}(J_i)$ is the attenuated differential backscatter cross section for each RR line J_i and $B_{RR}(z)$ is the background of the measured signal. For different lidar systems the background can either be a constant or vary with height. Since air below 80 km is a constant mixture of oxygen and nitrogen, η_i is a constant. The lidar constant C_{RR} depends on the number of transmitted photons, detector efficiency, and the area of the telescope. The attenuated differential backscatter cross section for Stokes and anti-Stokes line pairs of equal quantum number of the PRR spectrum is expressed as (Penney et al., 1974):

$$\begin{pmatrix}
\frac{d\sigma}{d\Omega}
\end{pmatrix}_{\pi}^{i}(J_{i}) = \frac{112}{15} \cdot \frac{\pi^{4}g_{i}(J_{i})hcB_{0,i}(v_{o} + \Delta v_{i}(J_{i}))^{4}\zeta_{i}^{2}}{(2I_{i} + 1)^{2}kT} \times (X^{+}(J_{i})\tau^{+}(J_{i}) + X^{-}(J_{i})\tau^{-}(J_{i})) \exp\left(\frac{-E_{rot,i}(J_{i})}{kT}\right) \tag{2}$$

where for the Stokes branch

$$X^{+}(J) = \frac{(J+1)(J+2)}{2J+3}$$
 for $J = 0, 1, 2, ...$ (3)

and for the anti-Stokes branch

$$X^{-}(J) = \frac{J(J-1)}{2J-1}$$
 for $J = 2, 3, 4, \dots$ and $X^{-}(J) = 0$ for $J = 0, 1$. (4)

 $\tau^+(J_i)$ and $\tau^-(J_i)$ are the transmissions of the receiver for the Stokes and anti-Stokes lines J_i , respectively. $g_i(J)$ is the statistical weighting factor, which depends on the nuclear spin I_i for each atmospheric constituent, h is Planck's constant, c is

the velocity of light, k is Boltzmann's constant, $B_{0,i}$ is the ground-state rotational constant, v_0 is the frequency of the incident light, and ζ_i is the anisotropy of the molecular polarizability. The rotational energy $E_{rot,i}(J)$ for each Stokes and anti-Stokes branch is estimated based on the assumption of a homonuclear diatomic molecule in the quantum state J for nitrogen and oxygen molecules with no electronic momentum coupled to the scattering (Behrendt, 2005).

The response of photomultiplier tubes used as detectors in lidar systems and operated in the digital mode can become nonlinear at high count rates. In the case of RALMO, the true and observed counts are related by the equation for non-paralyzed systems where γ is the counting system dead time:

$$N_{observed} = \frac{N_{true}}{1 + N_{true}\gamma}. (5)$$

The dead time of the counting system is the time taken after measuring a photon, before the detector is able to accurately measure another incident photon.

4 Application of the OEM for PRR temperature retrieval

4.1 Brief review of the optimal estimation method

The OEM is an inverse method that uses the measurements y to estimate the state (retrieval) variables x of a system via a forward model. The forward model F contains all the atmospheric and instrumental physics that describe the measurements. The forward model can include model parameters b, which are assumed and not retrieved, and their effect on the retrieved quantity uncertainties can be calculated.

The measurements are related to the forward model by:

25

$$y = F(x, b) + \epsilon \tag{6}$$

where ϵ represents measurement noise. Under the assumption that all parameters have Gaussian probability density functions 20 Bayes' theorem can be applied to determine the cost function,

$$cost = [y - F(x,b)]^{T} S_{y}^{-1} [y - F(x,b)] + [x - x_{a}]^{T} S_{a}^{-1} [x - x_{a}],$$
(7)

where x_a is the *a priori* of the retrieval parameters, S_y is the measurement covariance, which describes the random measurement uncertainty and S_a is the *a priori* covariance. The cost function measures the goodness of fit for a solution, and for good models the cost is on the order of unity. For non-linear forward models, the Marquardt-Levenberg method can be used iteratively to minimize the cost of the retrieval (see Section 5.7 in Rodgers (2000) for details).

The uncertainty budget is determined from the measurement and model parameter covariance matrices (Rodgers, 2000). The total error covariance, S_{total} , is:

$$S_{total} = S_m + S_F \tag{8}$$

where S_m is the retrieval covariance due to measurement noise and S_F is the retrieval covariance due to the forward model parameter uncertainty. The retrieval covariance due to measurement noise, S_m , is

$$S_m = GS_uG^T \tag{9}$$

where G is the gain matrix which is the sensitivity of the retrieval to the measurements. The retrieval covariance due to the forward model parameters, S_F , is

$$S_F = GK_b S_b K_b^T G^T, (10)$$

where K_b and S_b are the forward model parameter Jacobian and covariance matrices, respectively. The model parameter Jacobians K_b , can be estimated analytically or numerically for each model parameter. To construct S_b we require the uncertainties of the model parameters. We recommend Rodgers (2000) for more details of the OEM.

10 4.2 The forward model for a PRR lidar

The forward model describes the measurement as a function of both the state of the atmosphere and instrumental parameters. The core of our forward model is the lidar equation as presented in Section 3. It is called 4 times to generate the measurements corresponding to high and low quantum numbers, i.e. JH and JL, with digital and analog detection. Analog detection is assumed to be linear and hence the saturation equation (Equation 5) is not applied.

The pressure, P(z), and temperature, T(z), can be taken from either a radiosonde measurement or an atmospheric model. The background noise, B_{RR} , is in general a function of height, z, but is constant with height for RALMO. Unlike all the other existing forward models for lidar except Povey et al. (2012) (which was designed specifically to determine overlap) we retrieve O(z) the geometrical overlap function in addition to temperature. The overlap functions of JH and JL channels were assumed to be the same (Dinoev et al., 2010).

The atmospheric transmission, $\Gamma_{atm}(z)$ in Eq.(1), includes both molecular and particle scattering.

$$\Gamma_{atm}(z) = \exp\left[-\int_{0}^{z} (\alpha_{mol} + \alpha_{par})dz\right]$$
(11)

where α_{mol} is the molecular extinction coefficient and α_{par} is the particle extinction coefficient. The molecular extinction can be expressed using the Rayleigh cross section σ_{Ray} and air density n_{air} as:

$$\alpha_{mol} = \sigma_{Ray}.n_{air} \tag{12}$$

where σ_{Ray} is calculated using the expressions given by Nicolet (1984).

For each channel the subscript RR is replaced by JL and JH, the high and low quantum number PRR channels. Then C_{RR} , B_{RR} and J_i then have different values.

RALMO detects multiple Stokes and anti-Stokes lines from both nitrogen and oxygen PRR spectrum. Therefore, to determine the attenuated backscatter cross-section in the forward model we require knowledge of the exact number of quantum

states detected by each the RALMO PRR channel. From the JH and JL channel characteristics we can calculate the range of frequency shifts for each channel relative to the elastic wavelength 354.7 nm. Then using the equations given by Herzberg (2013) we can determine the quantum numbers J_i for both nitrogen and oxygen molecules. This calculation process is repeated to determine the J_i numbers for the JL channel of the RALMO. The summary of the findings are given in Table 1.

Table 1. The respective quantum lines from nitrogen and oxygen PRR spectrum detected by the RALMO temperature polychromator. Note the given quantum lines are valid for both the Stokes and anti-Stokes branches.

Channel		Nitrogen	Oxygen
JL	Quantum lines	3,4,5,7,8	5,7,9,11
JH	Quantum lines	10,11,12,13,14,15	15,17,19,21

In order to establish absolute calibration, we define the coupling constant R as the ratio of the lidar constants C_{JL} and C_{JH}

5

$$R = \frac{C_{JH}}{C_{JL}} \tag{13}$$

and use the substitution $C_{JH} = RC_{JL}$. The coupling constant is height-independent and can be determined with no assumptions at, if desired, a single altitude using the following equation derived from Eq.(1).

10
$$R = \left(\frac{N_{t,JH} - B_{JH}}{N_{t,JL} - B_{JL}}\right) / \left(\frac{\sigma_{JH}(T,z)}{\sigma_{JL}(T,z)}\right). \tag{14}$$

The differential cross section terms are determined by applying temperature from a coincident reference temperature, typically from a radiosonde. For a well designed lidar system the coupling constant should be stable over weeks. Unlike the fitting of an analytic calibration function to the data as in the traditional method, R can be estimated at a specific height or range of heights, which can be over a narrow range of temperature without introducing extrapolation errors. We extensively tested this assertion using both synthetic and real measurements. The results show that the estimation of R is indeed height independent. The value of R is only affected by the measurement noise. Hence, we recommend using a range of heights or a specific height where the photocounts have a high signal-to-noise ratio.

Using R in the forward model allows us to retrieve only one lidar constant, while constraining the two channels to vary so as to satisfy Eq.(13). We will see in the next section that any variations or uncertainty in the determination of R introduces an uncertainty on the order of $0.2 \, \text{K}$ to the retrieved temperature profile.

The retrieval parameters (Table 2) in our OEM algorithm are temperature, background signals (including photo multiplier shot noise, sky background, and offset for analog channels), the lidar constants, dead times of the digital photon counting systems, geometrical overlap, and particle extinction as a function of height. In OEM we can retrieve parameters on a height

grid where the resolution can be same or different than the vertical resolution of the height grid that the measurements obtained. If the retrieval grid is coarser than the measurement grip we use linear interpolation on retrieved quantities when they are required in the forward model.

Table 2. Values and associated uncertainties for the OEM retrieval and forward model parameters.

Parameter	Value	Standard Deviation	
Measurements			
Photon Counting	Measured	Poisson Statistics	
Analog	Measured	Auto Covariance Method	
Retrieval Parameters (a priori)			
Temperature	US Standard Model	35 K	
Geometrical Overlap Function	Estimated using the forward model and measurements	50% below and at transition height	
		10^{-3} above transition height	
Particle Extinction	Estimated using measurements	$10^{-6} {\rm km}^{-1}$ below and at transition height	
		50% above transition height	
Lidar Constants (analog/digital)	Estimated using the forward model	100%	
Digital Background Noise	Mean above 50 km	Standard Deviation above 50 km	
Analog Background Noise	Mean above 50 km	nighttime- Standard Deviation above 50 km	
		daytime- normalized standard deviation above 50 km	
Dead Time	Empirical fitting	10%	
Forward Model Parameters			
Pressure	Radiosonde	30 Pa	
Coupling Constants (analog/digital)	Estimated measurements and sonde temperature	Standard deviation of the coupling constants over a height range	
Air density	Radiosonde	1%	

4.3 Implementation of the RR temperature retrieval

5 The OEM solver in the Qpack software package is used for our retrieval (Eriksson et al., 2005). The solver requires the following as inputs: the measurements from each lidar channel and their covariances, *a priori* values for the retrieval parameters and their covariances, model (b) parameters and their covariances, and the Jacobians of the forward model.

The lidar measurements from the digital channels follow Poisson counting statistics in the range where the counts are linear. Thus, the measurement variance S_y is equal to the number of photons in each height bin, assuming no correlation between the height bins (that is, the off-diagonal terms in the S_y matrix are zero). The auto-correlation function method of Lenschow et al. (2000) is used to estimate the measurement covariance of the analog and digital measurements in the non-linear region. For both analog and digital channels, the *a priori* backgrounds and their variances are taken as the mean and the variance of the measurements above 50 km height.

The U. S. Standard Atmosphere (NASA, 1976) model temperature profile is normalized to the surface temperature from the coincident sonde temperature, and then used as the *a priori* temperature profile in our retrievals. Due to the high variability of the temperature, a standard deviation of 35 K for all heights is used to specify the covariance matrix for *a priori*. Other choices of *a priori* temperature profile are possible, but as an operational, fully automated lidar system RALMO retrievals must be

performed automatically every 30 min, so the choice of the US Standard Model with this covariance simplifies this procedure. As discussed in Eriksson et al. (2005), the elements of the retrieval and model parameters are often correlated, and some of the covariance matrices should have off-diagonal elements. Off-diagonal elements are parametrized with the correlation length and an appropriate analytical function describing the decay of the correlation. For this study, we used a tent function with a 1 km correlation length for temperature retrievals (Eriksson et al., 2005).

Molecular and particle extinction terms occur in the atmospheric transmission term of Eq.(11). An *a priori* particle extinction profile is estimated based on the following expression:

$$\alpha_{par} = LR \cdot \beta_{par} = LR \cdot \beta_{mol} \cdot (\Re_{\beta} - 1) \tag{15}$$

where LR is the lidar ratio, β_{par} is the particle backscatter coefficient. β_{par} is related to the backscatter ratio \Re_{β} as Whiteman (2003):

$$\Re_{\beta} = \frac{(\beta_{mol} + \beta_{par})}{\beta_{mol}} \tag{16}$$

The backscatter ratio \Re_{β} is estimated using the RALMO PRR and elastic measurements. Bucholtz (1995) gives a method for calculating β_{mol} using pressure, temperature and Rayleigh cross sections. The Rayleigh extinction cross sections required for β_{mol} estimation are computed using the formula of Nicolet (1984). Calculated Rayleigh extinction cross sections are also used to estimate the air density profile used as a b parameter in the forward model, assuming an uncertainty of 1% for the standard deviation.

The lidar ratio LR is chosen based on the \Re_{β} values for the given measurements. Typically \Re_{β} values inside clouds are greater than 2. Thus, for this study the height at which \Re_{β} is first equal to 2 is considered as the height of the cloud base or the height of an aerosol layer (cloud/aerosol layer base height). The cloud/aerosol layer base height is later used to determine the transition height that constrains the range of the geometrical overlap and the particle extinction retrievals. In cloud free sky conditions that we will refer as clear sky conditions from here on (that is if \Re_{β} does not exceed 2), LR is assumed to be 80 sr inside the boundary layer and 50 sr elsewhere. In cloudy conditions, LR is assumed to be 20 sr within the clouds present below 6 km. If the cloud is above 6 km, LR is assumed to be 15 sr within the cloud. These choices for lidar ratios are taken from Ansmann et al. (1992a) and Pappalardo et al. (2004). Accurate LR is not crucial, as it is used to estimate an *a priori* particle extinction profile. However, we can calculate a LR profile using the OEM retrieved α_{par} and compare it with the initially chosen LR values to evaluate how good a choice of the initial value is.

The effect of geometrical overlap and particle extinction on the signals are coupled and hence retrieving both parameters simultaneously is not possible unless at least one of the effects is highly constrained. The particle extinction is indefinitely measured using the backscatter ratio outside clouds, and overlap is well known above the height of full overlap, i.e. above 6 km (Dinoev et al., 2010). We use this knowledge to define a transition height, in clear skies 6 km or at the cloud base height, whatever is lower. Below this height overlap is retrieved, and above this height particle extinction is retrieved. The *a priori* overlap function is estimated from measurements in clear sky conditions. A 50% standard deviation is used for geometrical overlap below the transition height and a constant standard deviation of 10^{-3} is used above this height, constraining the

geometrical overlap to the *a priori* values above the transition height. For particle extinction, a standard deviation of 10^{-6} km⁻¹ is used below the transition height to constrain the retrieval, then a 50% standard deviation is used above this height, allowing the OEM to retrieve exclusively the particle extinction. The *a priori* covariance matrices for both particle extinction and geometrical overlap are determined using a tent function with a 100 m correlation length.

The *a priori* lidar constants for the JL analog and JL digital channels are estimated by fitting the measurements generated using the sonde temperature and pressure used in the forward model to the PRR measurements. For analog measurements, the fitting range is between 1.5 to 2 km height. For digital measurements with clear conditions or cloud/aerosol presence above 8 km, 6 to 8 km is used as the fitting range, to insure the photocounts are linear. If the digital measurements contain a cloud or aerosols in the geometrical overlap region, a fitting range below this is used, typically 3.5 - 4 km height. The fitting uncertainty 0 for each analog and digital lidar constants is used as the variance of the *a priori* lidar constant.

The *a priori* dead times for the two digital photon counting systems are considered to be 3.8 ns, consistent with estimations from previous studies for RALMO and with values specified by the manufacturer (Sica and Haefele, 2016, 2015; Dinoev et al., 2010). The uncertainty in the dead time is taken as 10%. Coincident radiosonde pressure profile are used assuming a 10% standard deviation. The coupling constants for analog (R_a) and digital (R) channels are estimated by fitting the ratio of PRR measurements with the ratio of the differential cross section (Eq.(14)). The coupling constants are estimated using the same fitting range as the lidar constants. Table 2 gives a summary of the parameters and associated uncertainties of the retrieval and model parameters used in the forward model.

5 Results from the temperature retrieval

We present four different measurement situations which demonstrate the robust nature of our OEM temperature retrieval. Details of each case study are given in Table 3. The RALMO measurements used in the retrievals are added in time over 30 min and to 15 m in height. Analog measurements are used from the surface to 6 km height, while digital measurements are used from 4 to 28 km. The retrieval grid has a vertical resolution of 60 m at all heights. For all the cases given in Table 3 we used radiosonde measurements that coincide within 1 hour of the lidar measurements for comparison purposes, to estimate the required *a priori* information, and to determine the forward model *b* parameters (Table 2).

Table 3. Details of the 4 cases in clear and cloudy sky conditions we present to demonstrate the flexibility of our OEM temperature retrieval.

Case	Date	Time (UT)	Sonde Launch (UT)	Sky Condition
1	20110909	2300-2330	2200	Clear-nighttime
2	20110910	1100-1130	1100	Clear-daytime
3	20110705	2300-2330	2300	Cirrus Cloud (~ 6 km)-nighttime
4	20110621	2300-2330	2300	Lower Cloud (4km)-nighttime

The traditional temperature profiles that will be shown later in this section are calculated using count profiles consisting of glued analog and digital measurements which are corrected for non-linearity and background before processing. The vertical resolution of the traditional temperature profiles is $30 \, \text{m}$ at the lowest heights, increasing to $400 \, \text{m}$ by the upper heights to decrease the magnitude of the statistical uncertainty. A calibration function linear in 1/T is used and the two coefficients have been determined with radiosonde data using the altitude range from surface to $10 \, \text{km}$. Hence, for this comparison the temperature profile is smoother than the OEM retrieved temperature profile. The change in vertical resolution is based on the random uncertainty of the temperature profile at each height. The vertical resolution is decreased until the temperature uncertainty becomes less than a threshold value, set here as $1 \, \text{K}$.

5.1 Case 1: Nighttime with clear conditions

10 Fig. 1 shows the RALMO 30 min coadded count measurements in the four PRR channels for case 1 given in Table 3. Analog signals suffer day and night from a high electrical offset which becomes larger above 5 km. The analog signals stay linear over the entire encountered range. On the other hand, the digital signals have a very low system background but become saturated above 10 MHz.

Fig. 2 shows the residuals, which are defined as the difference between the forward model and the measurements. For a good retrieval with cost on the order of unity, the residuals (blue curve) should be on the order of the standard deviation of the measurement noise (red curve), and indeed this is the case, hence the forward model is not over-fitting the measurements (e.g. cost much less than unity).

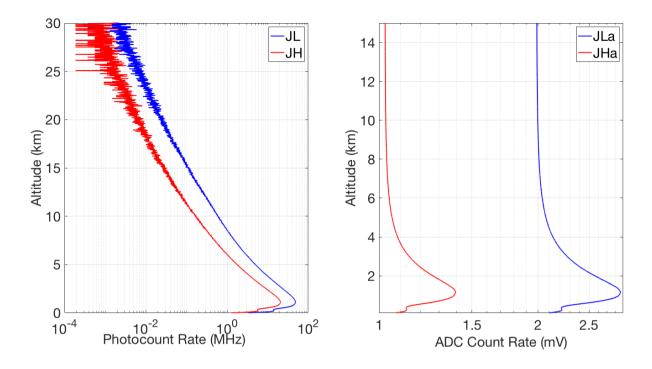


Figure 1. Count rate for 30 min of coadded RALMO measurements from 2300 UT on 09 September 2011, a clear night. Left panel: digital channels (blue curve, JL; red curve, JH). Right panel: analog channels.

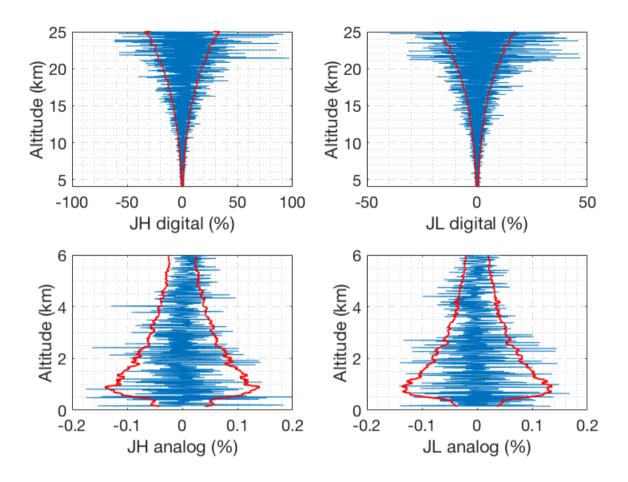


Figure 2. Difference between the forward model and clear nighttime RALMO measurements on 09 September 2011 for the four signals (blue). The red curves show the standard deviation of the measurements noise.

The averaging kernels of temperature and the vertical resolution of the retrievals are shown in Fig. 3. The area of the averaging kernels is defined as the response function of the retrievals and is close to 1.0 below 24 km, meaning that contribution of the *a priori* temperature profile to the retrieved temperature is negligible (Rodgers, 2000). With decreasing signal-to-noise ratio the measurement response quickly decreases above about 27 km (Fig. 3). The full-width at half-maximum of the averaging kernels is the retrieval's vertical resolution (Fig. 3, right panel). The vertical resolution of the retrieval is smaller than the resolution of the retrieval grid above about 10 km. We consider the height at which response function decreases to 0.9 as the cutoff height for the OEM retrievals as at this level the contribution of the *a priori* temperature profile is about 10%.

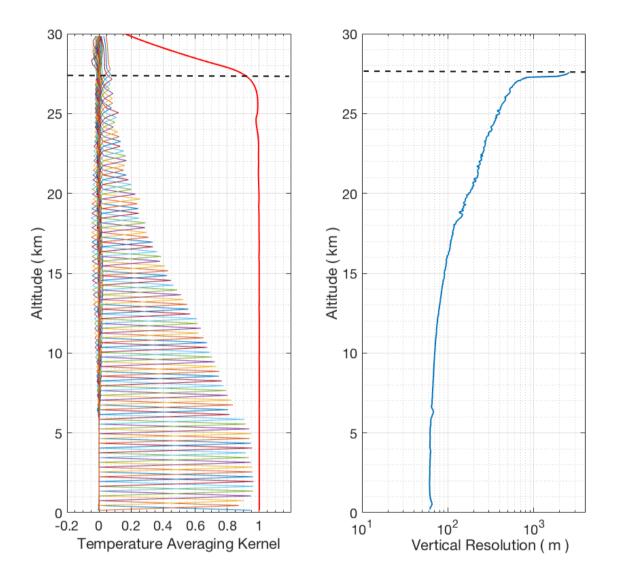


Figure 3. Averaging kernels (left) and vertical resolution (right) for temperature retrievals from the clear nighttime RALMO measurements on 09 September 2011. The area of the averaging kernels at each height, the response function, is the red solid line. The horizontal dashed line shows the height below which the retrieval is due primarily to the measurement and not the *a priori* temperature profile. For clarity averaging kernels for every fifth height bin of the retrieval grid are shown.

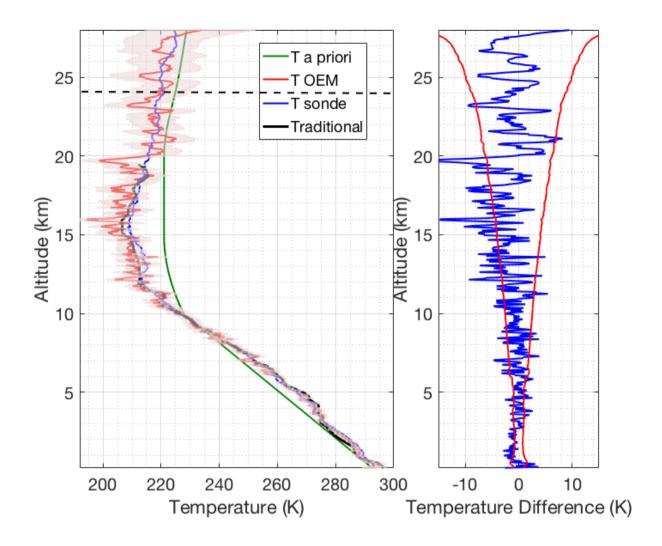


Figure 4. Left panel: Retrieved temperature profile and the statistical uncertainty (red curve and shaded area) using the OEM from the clear nighttime RALMO measurements on 09 September 2011. The blue curve is the radiosonde measurement. The sonde was launched at 2200 UT. The green curve is the *a priori* temperature profile used by the OEM. The black curve shows the temperature retrieved using the traditional analysis method from glued analog and digital signals. The horizontal dashed line shows the height below which the retrieval is due primarily to the measurement and not the *a priori* temperature profile. Right panel: The blue curve shows the temperature difference between OEM-retrieved temperature and the sonde temperature. The red curves in the figure show the statistical uncertainty of the OEM temperature.

Figure 4 shows a comparison of the OEM-retrieved temperature profile with coincident sonde temperature and temperature obtained using the traditional method. The traditional method profile from the RALMO glued (digital and analog) measure-

ments provided by MeteoSwiss has a vertical resolution of 30 m, below 12.5 km and 400 m above this height, and is interpolated to the same retrieval grid as the OEM and shown in black. The change in vertical resolution and the cutoff height of the traditional temperature retrieval are based on temperature uncertainty thresholds. As shown in the right panel of Fig. 4, the temperature difference between OEM-retrieved and sonde temperature (blue curve) fits inside the statistical uncertainty of the OEM-retrieved temperature. Temperature from the traditional method follow the same trend as the sonde and the OEM-retrieved temperature.

The OEM provides a complete uncertainty budget of both random and systematic uncertainties (Figure 5). Uncertainties due to the Rayleigh cross section is in the order of $0.01 \,\mathrm{mK}$. Pressure accounts for up to $0.1 \,\mathrm{K}$ below $10 \,\mathrm{km}$ and up to $0.7 \,\mathrm{K}$ up to $25 \,\mathrm{km}$ and is a non-negligible source of uncertainty in the stratospheric part of the retrieval. Note that this error could be reduced by choosing a better pressure profile. The uncertainty due to the analog coupling constant R_a is in the order of $0.07 \,\mathrm{K}$ up to $4 \,\mathrm{km}$ and the uncertainty due to digital coupling constant R is $0.15 \,\mathrm{K}$ in 4- $7 \,\mathrm{km}$ height range and less than $0.1 \,\mathrm{K}$ everywhere else. The largest contribution to the temperature uncertainty is from measurement noise, which increases with height.

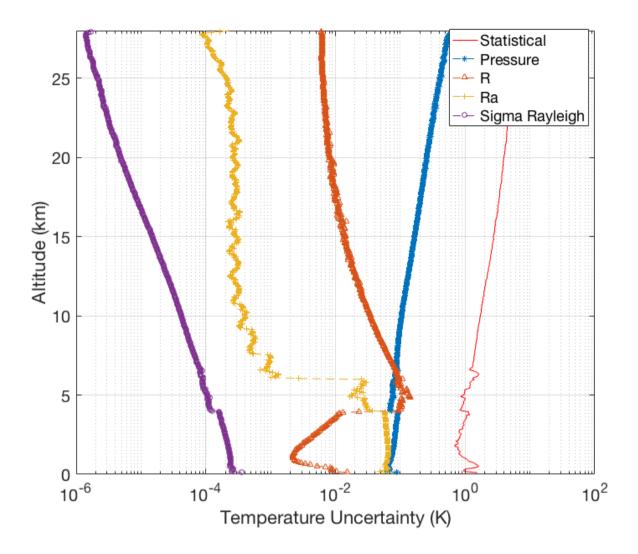


Figure 5. Random uncertainties (red curve) and systematic uncertainties due to the forward model parameters for the temperature retrievals from the clear nighttime RALMO measurements on 09 September 2011. The systematic uncertainties included are Rayleigh-scatter cross section (purple dot-dash curve), R digital coupling constant (orange triangle-dash curve), R_{α} analog coupling constant (yellow cross-dash curve), and pressure (blue dot-dash curve).

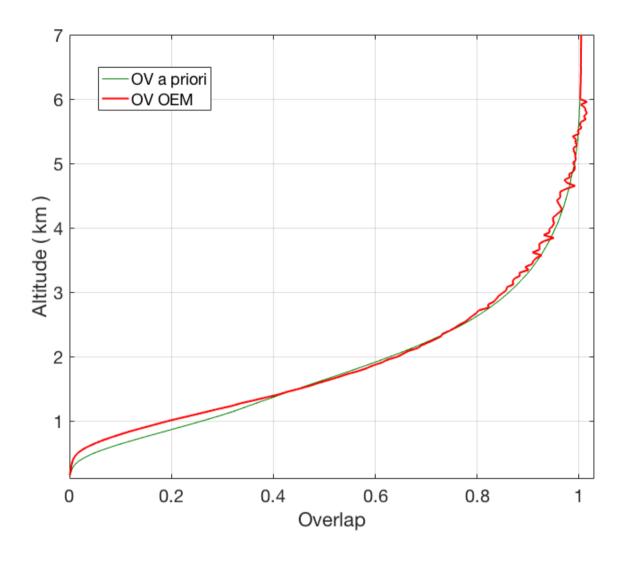


Figure 6. Retrieved geometrical overlap function (red curve) from the clear nighttime RALMO measurements on 09 September 2011 compared to the *a priori* geometrical overlap function (green curve).

Figure 6 shows the OEM-retrieved geometrical overlap function for the RALMO PRR channels. It illustrates that the overlap retrieval is constrained to be equal to 1 above the transition height (6 km), above which the extinction coefficient is retrieved (see Sec. 4.3).

5.2 Case 2: Daytime with clear conditions

The retrieval setup for second case study, which is a daytime measurement (Table 3), is identical to the one used for nighttime. The major difference between daytime and nighttime retrievals is the large solar background in the measurements, which is evident in the measurements of the digital PRR channels (Fig. 7).

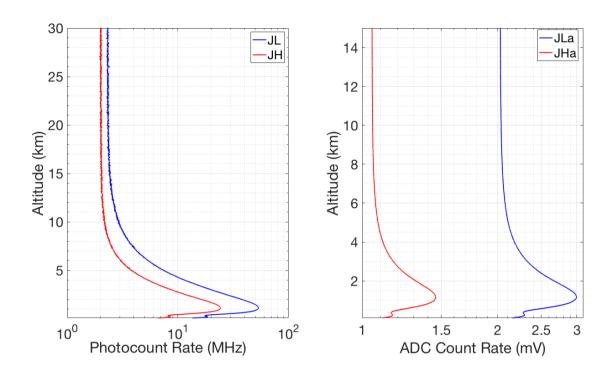


Figure 7. Count rate for 30 min of clear coadded RALMO measurements from 1100 UT on 10 September 2011. Left panel: digital channels (blue curve, JL; red curve, JH). Right panel: analog channels.

The residuals are unbiased and fall within the limits of the measurement standard deviation (Fig. 8). This result confirms the capability of our forward model in daytime conditions. As shown in Fig. 9, the vertical resolution (right panel) is the same as the retrieval grid below 13 km where response function (left panel) is equal to 0.9. The vertical resolution starts to deviate from the retrieval grid as the response function decreases and the background starts to dominate the digital channels.

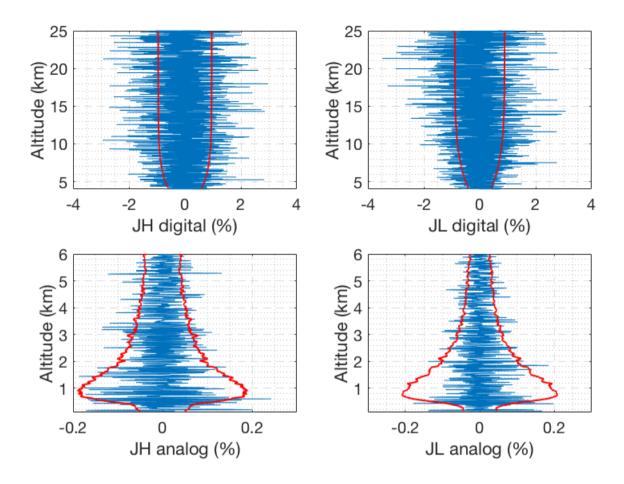


Figure 8. Difference between the forward model and the clear daytime RALMO measurements on 10 September 2011 for the four signals (blue). The red curves show the standard deviation of the measurements.

Similar to the clear nighttime case, the OEM-retrieved temperature agrees with the sonde temperature within the statistical uncertainty (Fig. 10). It is also evident for this specific case study that the temperature from the traditional method deviates from the sonde temperature more than the OEM retrieved temperature. This difference compared to the traditional method is due to the fact that the OEM adapts the vertical resolution in an optimal way as a function of height while the traditional method is, constrained by the filter employed to a specific signal-to-noise ratio of the measurements from both channels.

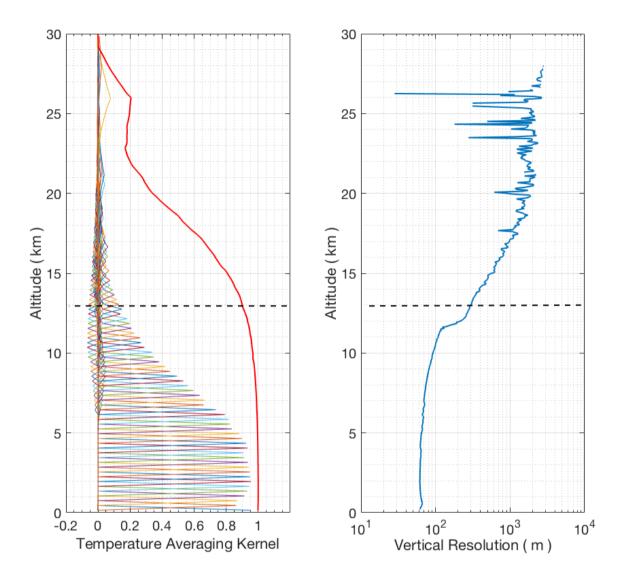


Figure 9. Same as Fig. 3 but for 10 September 2011.

The analog coupling constant R_a uncertainty of the temperature from the daytime measurements are also in the order of 0.07 K and the uncertainty due to digital coupling constant R is less than 0.2 K for all heights.

The retrieved geometrical overlap function for the clear daytime case (not shown) agrees with the geometrical overlap retrieved for the nighttime case within 10% statistical uncertainty.

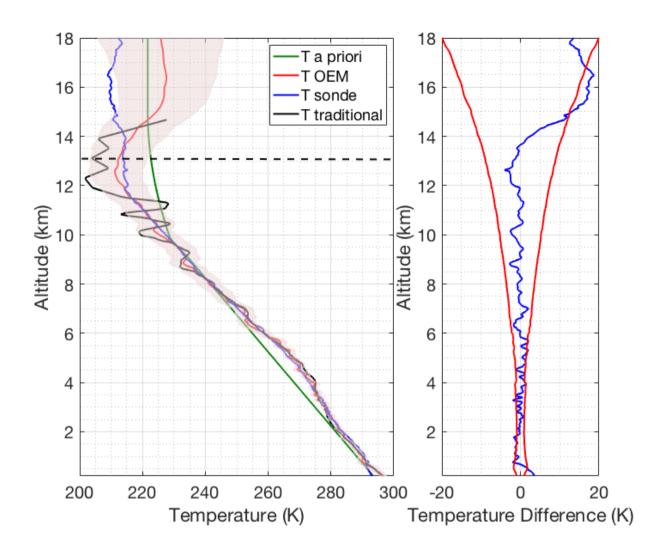


Figure 10. Same as Fig. 4 but for 10 September 2011.

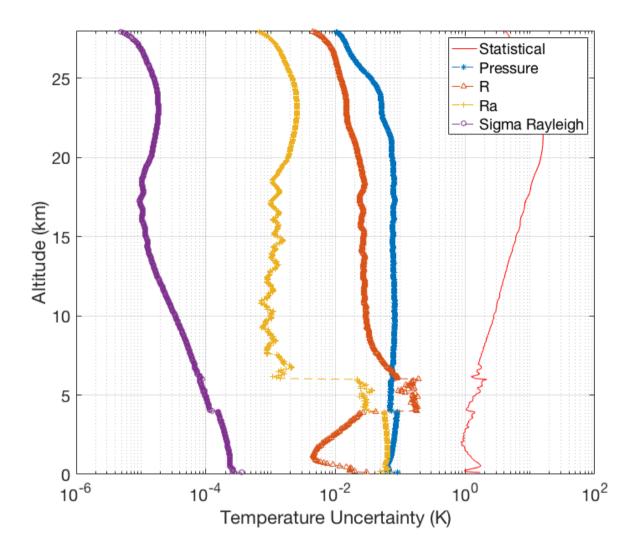


Figure 11. Same as Fig. 5 but for 10 September 2011.

5.3 Case 3: Nighttime with cirrus cloud

The third case (details are given in Table 3) features a cirrus cloud at 6 km height (Fig. 12).

The retrieval setup is identical to the previous cases, as the cloud base is above the height of full geometrical overlap of the transmitter and receiver. The *a priori* profile of the particle extinction coefficient is derived from the backscatter ratio assuming a lidar ratio of 15 sr.

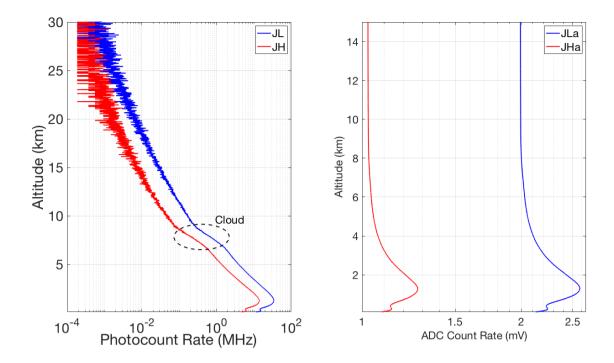


Figure 12. Count rate for 30 min of coadded RALMO measurements from 2300 UT on 05 July 2011. Left panel: digital channels (blue curve, JL; red curve, JH). Right panel: analog channels.

The residuals (Fig. 13) are unbiased and fall within the square root of the measurement variance. This is also true for the altitude range of the cirrus cloud demonstrating that the particle extinction coefficient was determined correctly. The response function (left panel, Fig. 14) decreases to the 0.9 cutoff value at about 23.5 km, clearly lower than the clear-sky nighttime case because of the attenuation of the cirrus cloud. Similar to the two previous cases, the OEM-retrieved temperature agrees with the sonde temperature within the statistical uncertainty of the OEM retrieved temperature (Fig. 15).

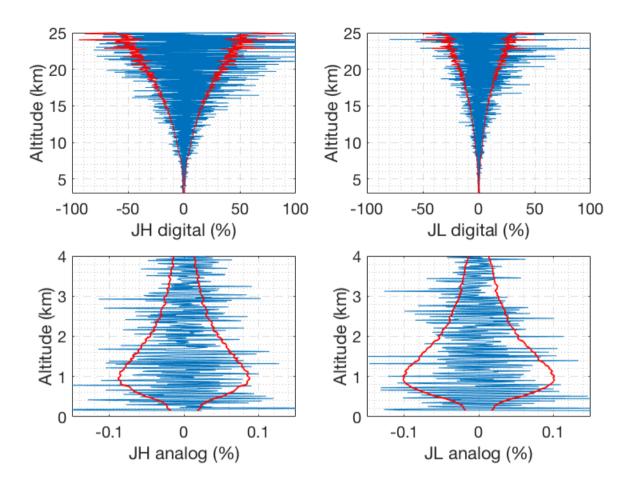


Figure 13. Difference between the forward model and the nighttime RALMO measurements on 05 July 2011 with the presence of a cirrus cloud for the four signals (blue). The red curves show the standard deviation of the measurements.

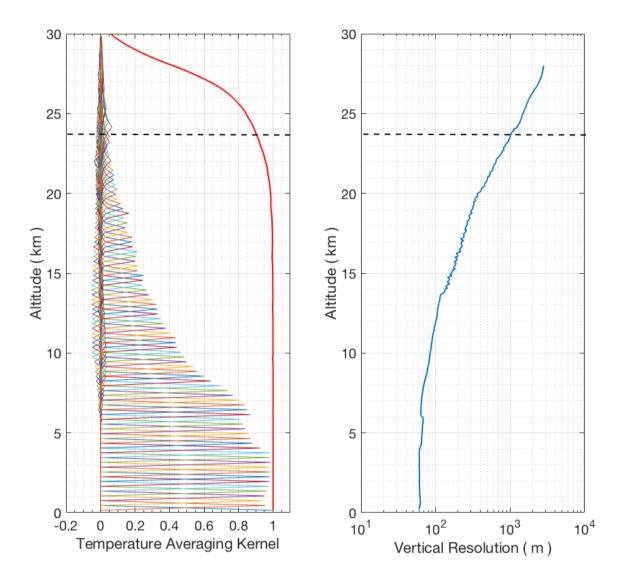


Figure 14. Same as Fig. 3 but for 05 July 2011 with a cirrus cloud at 6 km height.

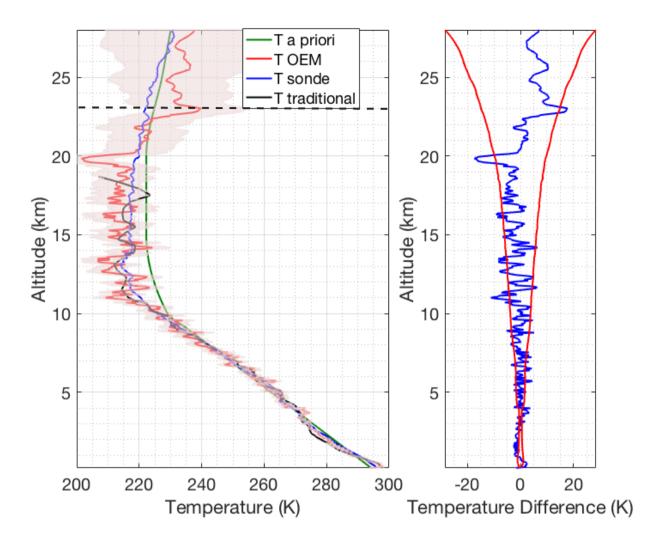


Figure 15. Same as Fig. 4 but for 05 July 2011 with a cirrus cloud at 6 km height, using the OEM.

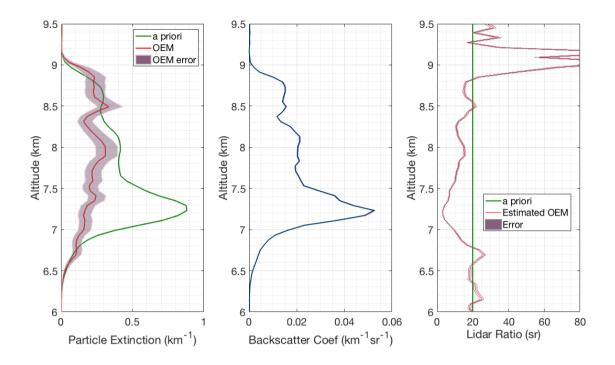


Figure 16. Left: Retrieved particle extinction (red) and the *a priori* particle extinction used in the OEM (green). Center: Backscatter coefficient calculated from the nighttime RALMO measurements on 05 July 2011 with of a cirrus cloud present at 6 km height. Right: Lidar ratio used to determine *a priori* particle extinction (green) and the estimated lidar ratio using the OEM-retrieved particle extinction (red).

The retrieved geometrical overlap function from the measurement with the cirrus cloud (not shown) agrees within 10% uncertainty with the geometrical overlap functions retrieved from the measurement with clear sky conditions, as the cloud is above the region of complete geometrical overlap. The red curve in the first plot in Fig. 16 shows the OEM-retrieved particle extinction and the green curve is the *a priori* particle extinction estimated using the RALMO PRR and elastic measurements, assuming a lidar ratio for cirrus clouds in order to estimate an *a priori* extinction (Section 4.4.3). Above 6.75 km, the OEM-retrieved particle extinction is around 0.25km⁻¹ and approximately two times smaller than the *a priori* yielding a lidar ratio of 5-15 sr while the initial guess was 20 sr (Fig. 16. Ansmann et al. (1992b), using independent measurements of particle extinction and backscatter profiles in cirrus clouds, show similar extinction values (0 - 0.5km⁻¹) and also similar values for the lidar ratio inside the cloud 0 - 10 sr. Thus, the OEM-retrieved extinctions for this cirrus cloud appears to be reasonable.Below 6.75 km, the lidar ratio is around 20 sr which could be an indication that this part of the cloud is super-cooled liquid.

The uncertainty budget for this case (not shown) is similar to the previous 2 cases shown; the cloud has little effect on the uncertainty values. As before, the statistical uncertainty makes the largest contribution to the full uncertainty. We have also calculated the statistical uncertainties for the estimated lidar ratio profile by standard uncertainty propagation of the OEM particle extinction and backscatter coefficient statistical uncertainties.

5.4 Case 4: Nighttime with lower level cloud

A cloud at about 4 km is present in measurements used for the last case study (Table 3). In this situation we use our OEM-retrieved geometrical overlap during clear conditions as our *a priori* geometrical overlap profile. We then retrieve geometrical overlap to the cloud base (4 km height) and particle extinction above 4 km. In this case the retrieved geometrical overlap up to 4 km agrees within 10% uncertainty with the OEM-retrieved geometrical overlap for clear conditions.

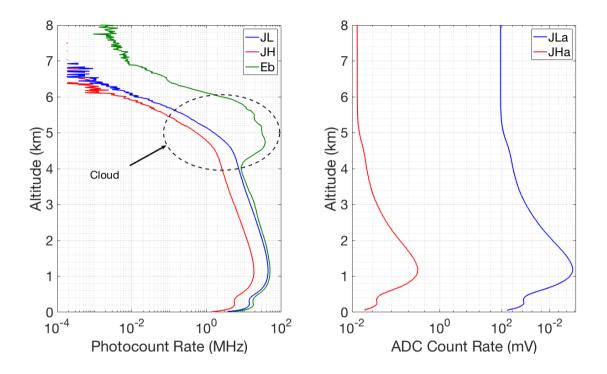


Figure 17. Count rate for 30 min of coadded RALMO measurements from 2300 UT on 21 June 2011, which has a cloud base at an height about 4 km. Left panel: digital channels (blue curve, JL; red curve, JH; green, Elastic). Right panel: analog channels.

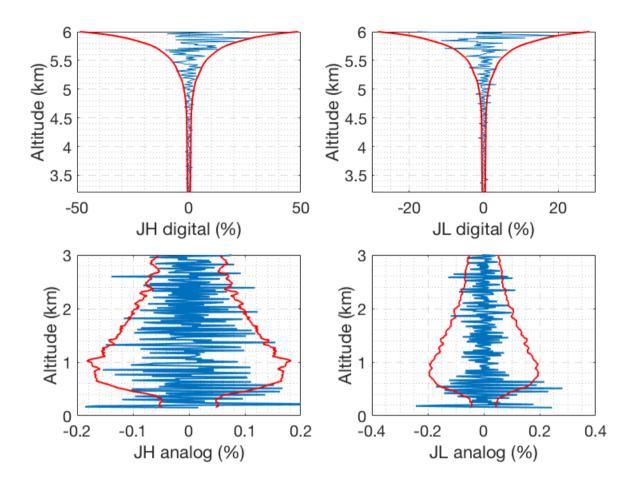


Figure 18. Difference between the forward model and the nighttime RALMO measurements on 21 June 2011 with the presence of a lower level cloud for the four signals (blue). The red curves show the standard deviation of the measurements.

Figure 17 shows the measurements in the four PRR channels and the elastic channel measurement (left panel, green curve). It can be seen in the elastic signal that a cloud base is at around 4 km height. The Raman measurements drop above 4 km and are fully attenuated at 7 km.

We use these measurements obtained at a cloudy condition as input to our OEM, and obtain unbiased residuals which fall within the standard deviation of the measurements, meaning the forward model accurately retrieve temperature in the presence of the cloud.

The response function (left panel, Fig. 19) is 0.9 at 6 km, which is considered the maximum height where the OEM-retrieved temperature are valid. At this height the vertical resolution (right panel, Fig. 19) rapidly increases as the cloud thickens. As shown in Fig. 20 (right panel), up to 6 km, the temperature from the sonde launched at 2300 UT from Payerne and OEM temperature agree with each other within the statistical uncertainty of the OEM temperature. Temperature retrieved using

the traditional method are similar to the OEM and sonde measurements up to 3.5 km, while inside the cloud the traditional temperature starts to deviate.

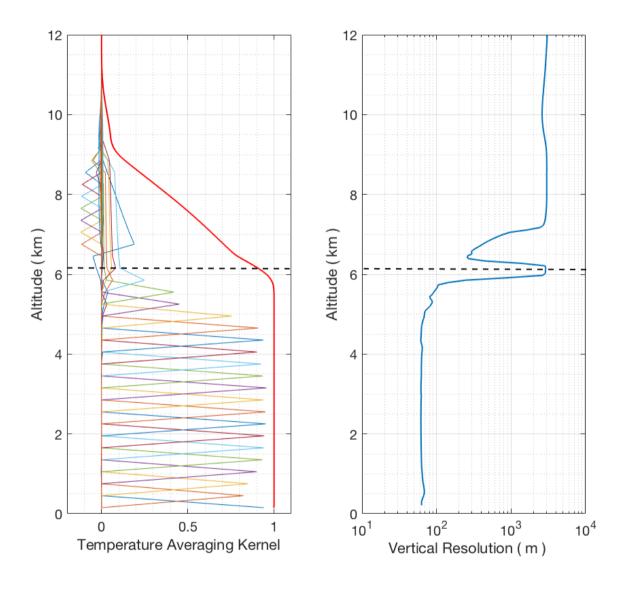


Figure 19. Same as Fig. 3 but for 21 June 2011 with the presence of lower level cloud.

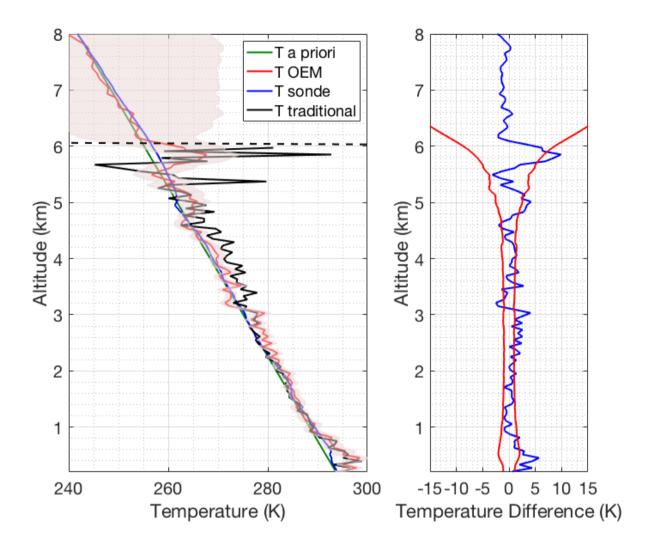


Figure 20. Same as Fig. 3 but for 21 June 2011 with the presence of a lower level cloud, using the OEM.

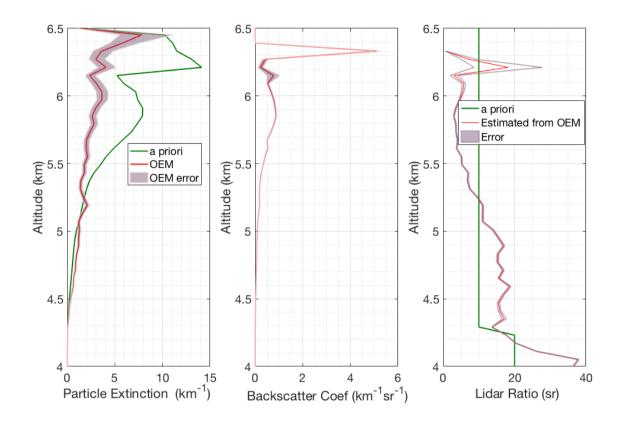


Figure 21. Same as Fig. 16 but for 21 June 2011 with the presence of a lower level cloud.

Below 5.25 km the retrieved particle extinction coefficient agrees well with the *a priori* values and the corresponding lidar ratio is between 15 and 20 sr indicating a liquid cloud. Above 5.25 km the retrieved particle extinction is smaller than the first guess yielding again a lidar ratio around 5 sr (left panel in Fig. 21). This could be an indication that the cloud became an ice cloud above 5.25 km.

6 Discussion

The four retrievals discussed in the previous section demonstrate that the OEM provides robust and accurate retrievals of temperature, geometrical overlap, and particle extinction coefficients, during both clear and cloudy day and night conditions. Unlike the traditional Raman lidar temperature analysis method (Cooney, 1972; Arshinov et al., 1983; Di Girolamo et al., 2004; Behrendt, 2005; Zuev et al., 2017), the OEM does not require an analytic form of a calibration function; rather a single calibration coefficient has to be estimated using a reference temperature profile and were shown to have a small effect on the retrieved temperature.

The calibration function plays a key role in the traditional temperature retrieval algorithm from the PRR backscattered signals, in particular if calibration is not done over a the entire observed temperature range. Typically, a calibration function linear in 1/T is used for systems that detect only one or multiple RR lines (Behrendt, 2005), although other forms of calibration functions have been employed. Recently Zuev et al. (2017) showed closer agreement at times with temperature calculation used a higher order polynomial for the calibration function. All calibration coefficient estimation methods require multiple reference data points which span ideally the entire range of temperatures to avoid extrapolation errors.

The only calibration required in our OEM scheme is the determination of the two coupling constants, R and R_a . The coupling constants can be estimated at a specific height (that is over a narrow range of temperature) without introducing extrapolation errors. Using the OEM we can show that the contribution of the coupling constant to the temperature uncertainty is in the order of 0.07 K in the height below 4 km and about 0.2 K or less above 4 km for a wide variety of sky conditions.

The OEM temperature retrievals of 4 very different sky conditions have been compared against coincident radiosonde temperature measurements. cases presented is the US Standard model normalized to the surface temperature from the coincident sonde temperature. We successfully used other *a priori* temperature profiles, such as the smoothed sonde temperature measurements and temperature from the Mass Spectrometer and Incoherent Scatter radar (MSIS) model to retrieve temperature using our OEM algorithm. All the retrieved temperature profiles using each *a priori* profile for heights where the response function is 0.9 or greater are identical within the statistical uncertainty.

In our study, we have successfully retrieved a geometrical overlap function for the RALMO system using the PRR measurements simultaneously with the temperature retrieval. Ray-tracking studies have concluded that the RALMO system reaches its full geometrical overlap by 5.0-5.5 km in height. These calculations are consistent with our geometrical overlap retrievals in both clear daytime and nighttime conditions. Measuring the geometrical overlap function and its uncertainty allows a more accurate estimation of the particle extinction coefficient when clouds or aerosols are present. The particle extinction profiles we retrieved in the two cloudy condition cases are consistent with measurements collected by Ansmann and Müller (2005) for cirrus clouds and with O'Connor et al. (2004) for liquid clouds.

The particle extinction coefficient is retrieved in the full geometrical overlap region, i.e. above 6 km or above the cloud base. The extinction values and lidar ratios we obtained for high and mid-level clouds are in agreement with other publications. The two case studies featuring clouds suggest, that both clouds consisted of a liquid and a ice part with lidar ratios at 18 and 5 sr.

For all the case studies we presented, the lidar constants for the lower quantum channels (analog and digital), the dead times for each digital channel (JL and JH), and background for all four signals are also retrieved. The retrieved lidar constants for each channel agreed within 20% uncertainty for all four cases. The retrieved dead times are about 3.8 ns and consistent with the dead times specified by the manufacturer and other independent estimates.

The uncertainty budget provided by the OEM contains both random and systematic uncertainties. Estimation of the uncertainty budget requires assignment of appropriate covariances to the model parameters. Using the standard deviations given in Table 2, the uncertainty budgets for all the case studies are estimated. The largest contribution towards the temperature uncertainty originates from the statistical uncertainty due to the measurement noise. Overall contribution from the coupling constants to the temperature uncertainty are less than 0.2 K for all heights. Given the fact that the measurement noise can be

35

reduced with longer integration times this result suggests that by the OEM method very precise temperature measurements are possible even if calibration is only possible over a small temperature range.

The systematic uncertainties of pressure and air density are on the order of 0.1 K to 0.1 mK respectively. Understanding the full uncertainty budget of temperature is of particular importance for trend analysis and process studies. The observational basis for super-saturation studies in the upper troposphere is still unsatisfactory and the OEM framework allows to combine different data sources to provide a high quality data set including profile-by-profile uncertainty budgets.

7 Conclusions

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We have demonstrated the ability of the OEM to retrieve multiple geophysical and instrumental parameters from PRR lidar measurements. The first-principle forward model adequately represents the raw PRR measurement and allows us to retrieve temperature, geometrical overlap, particle extinction, lidar constants, background counts, and dead time using multiple analog and digital channels. The retrievals discussed for four different cases that represent different (and typical) sky conditions. We found the following results from our OEM temperature retrievals from PRR measurements:

- The forward model presented, based on the lidar equation, contains the essential physics to reproduce the analog and digital measurements, leading to unbiased residuals and robust estimates of temperature.
- Our OEM retrieval does not require a calibration function as used in the traditional temperature retrieval method. It only requires determination of the two coupling constants, R and Ra, using a reference temperature profile that can be estimated at a specific height bin (or over a range). Retrieved temperature profiles from both day and night raw PRR measurements in clear and cloudy conditions agree well with coincident radiosonde measurements.
 - The OEM provides a cutoff height for the temperature retrievals that specify up to which height the retrieved profile is primarily due to the measurements and not the *a priori* temperature profile.
 - Vertical resolution is determined at each height, and is automatically adapted in the retrieval in response to increasing measurement noise with height.
 - The OEM provides a complete uncertainty budget, including random and systematic uncertainties due to model parameters, including the assumed pressure, air density and the coupling constants.
- Simultaneous analog, which are linear, and digital measurements allow the dead time to be retrieved.
 - The OEM-retrieved geometrical overlap function for the RALMO using the measurements in clear conditions is determined and shown to be consistent with, but not the same, as that calculated by Dinoev et al. (2013). Hence, retrievals of the particle extinction coefficient are possible using the OEM from the measurements in cloudy conditions or when aerosol layers are present.

 The OEM is a computationally fast and practical for routine temperature retrievals from lidar measurements as required for operational lidar systems.

We have demonstrated that the OEM allows retrieval of temperature from Pure Rotational Raman lidar measurements that are consistent with the coincident sonde temperature. The OEM retrieved temperatures depend nearly to 100% of the measurement.

5 At the cut off height, the measurement contribution falls below 90%. Temperature retrievals above the cutoff height depend to more than 10% on the *a priori* temperature profile and are not considered an independent measurement. We discussed the advantages of the OEM over the traditional temperature retrieval algorithm. We can use the OEM-retrieved temperature to study temperature trends with the benefit of a full uncertainty budget provided by our OEM. Our OEM temperature retrieval can also be used for routine measurements in a wide variety of observing conditions, and is applicable to any similar PRR lidar system.

We are in the process of implementing the OEM for routine temperature measurements from the RALMO system. We are also combining the OEM PRR temperature retrieval with the OEM water vapor mixing ratio retrieval of Sica and Haefele (2016) to directly retrieve relative humidity from the RALMO measurements, both for its importance to operational forecasting and to allow the study of ice super-saturation events. We are also assimilating ERA5 hourly reanalysis data into OEM relative humidity and temperature algorithm to improve the accuracy of the OEM relative humidity retrievals.

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