Title: A GPS water vapor tomography method based on a genetic algorithm

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Dear Reviewers,

We would like to thank the anonymous reviewers for providing an opportunity to revise the manuscript. The comments and suggestions of the reviewers are all valuable and very helpful. We have studied them carefully and have made revisions to improve the manuscript. Revised portion are marked in **red** in the manuscript and the main corrections and additions are given below with a comment followed by a response (in red color).

Best regards,
Authors

Referee #1:
Comments and Suggestions for Authors
The paper by Yang et al. (2019) introduces new methodological solution to the GNSS tomography ill-conditioned problem. Authors suggest to use of genetic algorithm that is applying optimization principle based on the minimization function of the slant residuals (y-Ax), and stochastic modelling of water vapour field evolution. The concept is sound, methodology quite innovative at least in the tomography community, but comparison with standard method reveals that there is a little or no improvement once the genetic approach is used. Moreover, competitive studies for the same location, shows better performance
p.16 "Xia et al. (2013) obtained a RMS of 1.01 g/m3 by adding the COSMIC profiles, Yao et al. (2016) obtained a RMS of 1.23 g/m3 by maximally using GPS observations and a RMS of 1.60 g/m3 without the operation, Zhao et al. (2017) achieved a RMS of 1.19 g/m3 and 1.61 g/m3 considering the signal rays crossing from the side of the research area and a RMS of 1.79 g/m3 without this consideration, Ding et al. (2017) obtained a RMS of 1.23 g/m3 and 1.45 g/m3 by utilizing the new parametric methods and the traditional methods, Yao et al. (2017) achieved the RMS from 1.48-1.80 g/m3 using different voxel division approaches, etc, the total RMS of 1.43 g/m3 for the two time periods in this paper can be considered as a good agreement with the radiosonde data regardless of the weather conditions".

[Response]: Thank you for the comments.
In the literatures mentioned-above, the RMS achieved by Yao et al. (2016), Zhao et al. (2017) and Ding et al. (2017) are 1.60, 1.79 and 1.45 g/m^3, respectively, when the traditional Least squares method is used. To obtain tomographic results with higher accuracy, Xia et al. (2013) added extra COSMIC data, Yao et al. (2016) and Zhao et al. (2017) added the signals crossed through from the side faces by using Radiosonde and

ECMWF data, respectively. In the articles written by Yao et al. (2017) and Ding et al. (2017), they explored the influence of the voxel division approaches and different parametric methods on the accuracy of tomographic results. It is found that the tomographic results are stable when using the traditional Least squares method with same observation data and same grid model. The normal way to improve the accuracy is to introduce more observations, such as COSMIC data, radiosonde, ECMWF data and extra GNSS data. In our paper, the proposed GA method is conducted based on the common case, i.e. without any other extra observations. The RMS obtained in this paper is 1.43 g/m^3, which is not worse than the results in the literatures (1.60, 1.79 and 1.45 g/m^3) using the traditional Least squares method with similar conditions. Moreover, the difference of the experimental period and the grid division may affect the above comparison.
We also conducted the tomographic experiments using the traditional Least squares method in this paper. The comparison with traditional Least squares method revealed that there is a little improvement once the genetic approach is used. The focus of this paper is to solve the ill-conditioned problem of water vapor tomography using the proposed method, by which to overcome the difficulty of inverting the sparse matrix in Least squares method, the weakening of tomographic technique by a prior information in algebraic reconstruction technique and the restriction of obtaining external data. To significantly improve the accuracy of the tomographic results is not the focus of our research. Similarly, the algebraic reconstruction technique and the Kalman filter approach are also proposed to provide a new solution for water vapor tomography and to solve the shortcomings of the previous methods, rather than focusing on the significant improvement of the tomographic accuracy. In my view, the tomographic accuracy could not be significant different by different methods when the number of water vapor observations and their distribution are the same for each method. Therefore, once the COSMIC data in Xia et al. (2013) or the GPS signals crossed through by the side face in Yao et al. (2016) and Zhao et al. (2017) are introduced to the tomographic model based on the GA, we think the tomographic accuracy will be improved. But this is not the point of this paper and can be validated in the follow-up research.

Therefore, two questions should be asked: are there any information left in the slants observations that can be utilized by the tomography framework, if positive, one might ask whether approach with introducing new algorithm to old parametrization will aid in the development of tomography processing. I suggest to address these two major questions in the revision process.
[Response]: Thank you for the comments.
We think that there is no information left in the slant observations that can be utilized by the tomography framework. Since the slant observations can be used in the tomographic model are those crossed through from the top boundary. The common tomographic experiments, including our research, are modeled by this part of the slant observations. In the articles of Yao et al. (2016) and Zhao et al. (2017), they added the slant observations crossed through from the side face to the tomographic model by using the radiosonde and ECMWF data. But the accuracy of this part of the slant

observations outside the tomographic region remains to be further tested. The tomographic results in their articles showed that the accuracy is improved, it is still not the common method for water vapor tomography. The current tomographic researches are based on the slant observations with high accuracy passing through from the top boundary. Ding et al. (2017) proposed a new parametric method which use the vertex value of the voxel to represent the water vapor density of the voxel. In the common method, the value of the central point in the voxel is considered as the water vapor density of the voxel. The properties of the tomographic observation equation in the above two method are still the same. The Ding's method is not a commonly used method in water vapor tomography.

In this paper, we adopted the common method of tomographic research, which only used the slant observations crossed through from the top boundary and considered the value of the central point in the voxel as the water vapor density of the voxel, to conduct the tomography based on GA. We believe that the research based on the above method is universal and reasonable. In the follow-up research, the studies can be done by adding the slant observations passing through from the side face and using the Ding's parametric method. Actually, many similar methods were proposed to explore the improvement in tomographic accuracy, but they are not commonly used. To study the application of the genetic algorithm in water vapor tomography, it is reasonable to construct the tomographic equation by using the common methods.

Overall, the manuscript presentation quality is high, however few points need to be addressed (in addition to two major questions, stated above):

1. The genetic algorithm should be clearly explained and compared to the classic Least Square, Kalman Filter or Algebraic Reconstruction Technique solutions, reader need to understand the principles of approach and its application to the tomography problem. This comment is related to: The Introduction section where Authors only briefly p.3 l. 1-10 discuss differences between new method and standard methods, 2 Methodology where Authors should add one subsection discussing classic Least Squares applied in next section. 3. Experiment and Analysis, where reads would expect how Table 1 and steps discussed on pages 5 and 6 links to real data, it should be clear how choices of parameters from Table 1 translates into algorithm performance in more detailed, step-wise manner.

[Response]: Thank you for the suggestions.

To make this paper better understood by readers, we have fully considered the three comments and carefully revised the relevant parts of the article. According to the comments, we added appropriate information in the chapters of Introduction, Methodology, Experiment and Analysis. Below is the added content and you can see them in the revised manuscript.

"The ART techniques are iterative algorithms that proceed observation by observation. Only two vector y, x and a data structure containing the slant subpaths in each voxel are required to solve the observation equations. The algorithms consist two loops. The inner loop processes SWV by SWV and applies an adequate correction to each voxel. After all SWVs have been executed the next iteration is started in the outer loop (Bender et al., 2011). It is not necessary to perform the matrix inversion and therefore avoids the ill-conditioned problem. But it only updates the results of the voxels traveled through by signal rays and the tomographic results heavily depend on the exact initial field, the data quality and relaxation parameter (Wang et al., 2014)."

"It assumes that the water vapor density in each voxel meet the Gauss-Markov random walk behavior for a certain period of time, and establishes the corresponding state equation of Kalman Filter. The observation vector is utilized based on the mathematical model to perform the best estimation of the state vector, which is a process of continuous prediction and correction."

"2.2 Water vapor tomography based on Least squares method

After obtaining the observation equation (Eq. (2)), three kinds of constraints are usually added:

$$0=\mathrm{H}\cdot x \qquad (1)$$

$$0=V\cdot x \qquad (2)$$

$$0=T\cdot x \qquad (3)$$

Equations (6)-(8) are the vertical constraints, horizontal constraints and top constraints. For the horizontal constraint equation, it assumes that the distribution of water vapor density is relatively stable in the horizontal direction within a small region. Thus, the water vapor density within a certain voxel can be represented by the weighted average of its neighbors in the same layers. For the vertical constraint equation, it is a relationship established for the voxels between two adjacent layers basing on the analysis of meteorological data for many years. The top constraint is to set the water vapor density of the top boundary to a small constant. Based on the principle of Least square, the tomographic results can be achieved by the following formula:

$$x=\left(A^T A+H^T H+V^T V+T^T T\right)^{-1}\times\left(A^T y\right) \qquad (4)$$

To obtain the inverse matrix in Eq. (9), the singular value decomposition is required and its detail instruction can be seen in the relevant literature (Flores et al. 2000)."

"According to the flowchart 1, the above GPS observation data were processed to construct the tomographic equation and further convert it into the fitness function for the optimization algorithm. The population size is chosen based on the total number of unknown parameters (water vapor density). The value of 200 is the default option of the algorithm when the number of unknows exceeds a certain amount. The elite count is chosen to be 10 to specifies the number of individuals that are guaranteed to survive to the next generation, since it is based on the population size (0.05 * population size). The other parameters are selected as Table 1, which are the default settings of the algorithm for the common use.
"

2. Comparisons with radiosondes fig 9, and with ECMWF fig 12 are corner stones of this manuscript. Therefore, it is difficult to understand why LS and Genetic algorithms were only compared to ECMWF but no to RS, as in fig 9. It should be done only for overlapping voxels. Why not to add to fig 9 two extra lines one for tomography LS and one for ECMWF, this will clearly indicate the quality of retrieval in time

[Response]: Thank you for the suggestion.

In the revised manuscript, we compared the GA and Least squares method using the radiosonde and ECMWF data as reference data and listed the statistical results. In the new figure (Fig. 14), we added two extra lines, one for tomography LS and one for ECMWF, to show the comparison of profiles of GA, Least squares method, radiosonde and ECMWF data during the rainless days. Fig. 9 belongs to section 3.4, the focus of which is to demonstrate the good consistency of GA tomographic results and radiosonde data.

3. The choice of research area to be one of the well-studied Hong-Kong cases has to be evaluated positively. However, division into rainy and rainless days is not supported by any meteorological analysis such as air mass origin, rain type, rain intensity, other associated phenomena. This is important as not all-weather types associated with rain will produce increase of SIWV. Moreover, there is limited evidence that the differences between so called "rainy" and "rainless" days are significant.

[Response]: Thank you for pointing it out.

We reviewed the meteorological data and provided more relevant weather information in the revised manuscript. The daily rainfall and relative humidity in different period are presented in detail. Moreover, we counted the SWV produced in the selected stations and the results in different days are listed in the table below.

Table 1. The value of SWV produced in the selected stations (unit: mm)

DOY	163	164	165	166	167	168	169	Average
SWV(mm)	69.9	68.6	69.4	92.9	87.9	85.1	79.7	79.1
DOY	225	226	227	228	229	230	231	Average
SWV(mm)	108.5	109.4	107.8	108.2	115.3	123.1	118.4	112.9

The above listed data can show that Hong Kong experienced different weather conditions during these two periods, one with continuous rainfall and the other without rainfall. The value of SWV used for the water vapor tomography are different in the two period of time. Similar to the literature (Zhao et al. 2017, Guo et al. 2016, Yao et al, 2019), this paper is focused to prove that the water vapor tomography can achieve good results in rainy and rainless weather condition, not to show the differences between rain and rainless days are significant. However, the comparisons showed that the tomographic results in rainless day is better than those of the rainy days, which are consistent with the previous articles (Zhao et al. 2017, Guo et al. 2016, Yao et al. 2019). We tried to explain the reasons for the different tomographic results in different weather conditions at the end of the article. The research about the effects of rain type, rain intensity and other phenomena on tomographic results is relative rare and is not the focus of current tomographic study. In the follow-up research, we would pay more attention on this issue.

Referee #2:

General Comments

This paper proposes a new approach to solving GPS tomographic inversion to retrieve 3-D atmospheric water vapor distribution above a network of GPS receiving stations. It should circumvent the strong constraints of classical techniques that have to deal with the inversion of an often very sparse matrix. Hence, this approach could be of great interest to the community. However, to demonstrate the good performances of their technique, the authors set to provide an ensemble of statistical indicators, some with respect to GPS slant delays, some with respect to radio-sounding profiles, some with respect to "classical techniques", etc. but usually considering only global RMS and MAE scores. Although statistical estimates are of interest, they do provide the physical/meteorological understanding of the actual objective (and challenge) of tomography inversion: it is the 3-D distribution of water vapor and more particularly its vertical variability. Hence, it would have been, in my view, much more informative and useful to the community to have comparisons of profiles of RS + ERA + GA + Least-Square + Ref GPS, possibly with the corresponding statistical metrics in order to perceive the actual capacity of the new technique to resolve the water vapor distribution, rather that producing a dispersed set of statistics which makes it difficult how the technique compares globally. This article is globally well written and easily readable, nonetheless, the English phrasing is at times a bit awkward which might lead to some misunderstandings.

[Response]: Thank you for the comments.

In the community of water vapor tomography, the comparison with SWV, radiosonde and ECMWF data are commonly used to validate the tomographic results. The global RMS and MAE scores computed from the reference data (Gamit-estimated SWV, radiosonde, ECMWF data), which is adopted in almost all relevant tomographic articles, is an effective way to evaluate the performance of tomographic method. As you said, comparisons of profiles of radiosonde, ECMWF data, GA, Least squares method could be much more informative and useful to the community. In the revised manuscript, we conducted the corresponding comparison to make up for this deficiency. We collected all the radiosonde data during the period of tomographic experiment and plotted the profiles of tomographic results and reference data. Specifically, the new figure (Fig. 14) shows the profiles of GA and Least squares method during the rainless days and utilized the radiosonde and ECMWF data as reference data.

Specific comments:

4. P.5, L.8: after criteria, do you mean a "," or a ":"? i.e., are there 3 (",") or 2 (":")

conditions for termination. In any case, a precise description of the termination criteria and how they are defined should be clearly stated here.

[Response]: Thank you for pointing it out.

We rewritten this part in the revised version.

“The search terminates when a group of approximates meets the requirements of the fitness value. Generally, we set the stopping criteria for generation or calculation time.”

5. P.9, L.15: for clarity one could add something like "The change of tomography computed VS GAMIT-estimated slant water vapor residuals". . . Likewise, if this tells us that the GA method compares reasonably well with the original data, it would have been very interesting (and useful to evaluate the method) to know how would have fared a "classical" inversion technique.

[Response]: Thank you for the suggestion.

We added the information in the revised manuscript.

6. P.9, L.17: "It is clear . . . residuals decreased with . . . elevation angle". Readers can read a graph. Hence, if that is stating the obvious, then the sentence can be deleted. . . otherwise, if that is a point of interest, than it should be discussed. . . Likewise with the following sentence: "The right . . . angles".

[Response]: Thank you for the suggestion.

We deleted the corresponding part in the revised manuscript.

7. P.10, L.11-13: Is there some altitude difference between the 2 stations? I would guess that if that is the case, hkmw is higher than hkpc. Actually, at this point, one could also discuss the reason why in fig 5 all zenith residuals are positive!

[Response]: Thank you for the comment.

The altitude of hkmw is a little higher than hkpc, their specific values are 194.95m and 18.13m, respectively. Considering that the vertical height of each layer of the tomographic model is 800m, both stations are located at the bottom of the first layer of the voxels. Thus, we think it still needs further research to discuss whether the height is the case. In fig 5, the MAE means mean absolute error, which is always a positive value. The zenith residuals of each station were calculated as the MAE that used in fig 5.

8. P.13, L.15-17: I guess this sentence relates to the green box plot of Fig. 8. . . but are you sure that the range [-7.08, 4.47] is a sign of good water vapor restitution for the new method??? That is, I guess, what should be appreciated rather than a good statistical distribution!

[Response]: Thank you for pointing it out.

We corrected the corresponding part in the revised manuscript.

9. P.14, Fig.9: it is a pity that the equivalent graphs for the no rain days are not provided. Indeed, the major limitation of GPS tomographic inversion is its ability to retrieve the vertical variability and that can only be assessed by profiles comparisons with

RS, not by global statistics.

[Response]: Thank you for the suggestion.

We added the corresponding graphs for the no rain days in the revised manuscript. In the figure (Fig. 14), the tomographic results at UTC 0:00 and 12:00 from DOY 225 to 231 derived from GA and Least squares method are compared and the radiosonde and ECMWF data are used as reference data.

10. P.15, L.3-5: At this point, one could think in terms of relative error rather than absolute error.

[Response]: Thank you for the comment.

We added the information about the relative error and rewritten the corresponding part in the revised manuscript.

"The WVD profiles reconstructed by the GA tomographic solutions are in conformity with those derived from the radiosonde data, especially in the upper troposphere from the perspective of absolute error. With respect to the relative error, the values of the voxels upper than 5km and lower than 5km are 31% and 15%, respectively. The reason for this phenomenon is that the value of water vapor in the upper layers is relatively low, even a small difference between the radiosonde and tomographic result can also lead to a large relative error, while water vapor content accounts for more than 90% below 5km near the Earth's surface."

11. P.18, L.7-8: If there are cases when GA performs better than lest square methods and others when it is the opposite, the authors should at least try to sort out if there are some "signature" to those contrasted behaviors (like the presence or amplitude of rain, the type of weather regimes, or more technical reasons such as GPS constellation configurations, . . . etc) in order to provide informative comments to the reader. Indeed, it is important to know how reliable the GA method is compared to the established least square ones: if it performs globally as well and is more computing effective, or performs better, than it is a real progress. If it under performs compared to others, than it has less interest.

[Response]: Thank you for the comment.

The focus of this paper is to solve the ill-conditioned problem of water vapor tomography using the proposed GA method, by which to overcome the difficulty of inverting the sparse matrix in Least squares method, the weakening of tomographic technique by a prior information in algebraic reconstruction technique and the restriction of obtaining external data. To significantly improve the accuracy of the tomographic results is not the focus of our research. Similarly, the algebraic reconstruction technique and the Kalman filter approach are also proposed to provide a new solution for water vapor tomography and to solve the shortcomings of the previous methods, rather than focusing on the significant improvement of the tomographic accuracy. In my view, the tomographic accuracy could not be significant different by different methods when the number of water vapor observations and their distribution are the same for each method.

In this paper, the comparison with tomographic results of the Least squares method is to prove that the results of the GA are appreciated. Table 3 listed the numerical results including RMS and MAE during the whole experimental period and showed that the result is a little better than that of the least squares method when the ECMWF data is regarded as the true value. The solutions that least squares method yields better results than the GA does only accounts a small part. This is similar to the situation that least squares method can obtain results with different accuracy in different time period. We are concerned with the comparisons during the entire experiment, which show that the accuracy of GA is comparable to, or even a little higher than, the least squares method. The very few different solutions do not affect this conclusion. We think that it is reasonable to have these few different results. Since the proposed GA is not designed as the method to significantly improve the accuracy of the least squares method. Moreover, it does not show obvious relationship with the presence or amplitude of rain, the type of weather regimes, or GPS constellation configurations. More research is needed in the follow-up study to find the reasons. We corrected the corresponding part in the revised manuscript.

12. P.19, L.10: The statement that GA can achieve good tomographic results is certainly true, but it should be discussed in light of the comment regarding the comparison with other methods (see comment above).

[Response]: Thank you for the comment.

In the revised manuscript, a more detailed comparison between GA and Least squares method is conducted using the voxels above the radiosonde station. The changes of water vapor density derived from GA and Least squares method with altitudes in different days (rainless days) are shown in the new figure (Fig. 14), in which the radiosonde data and ECMWF data are considered as reference data. Moreover, the statistical values are computed and listed to better show the comparison of GA and Least squares method.

13. P.20, L.9-10: "more water vapor information exists in rainy weather"!!! That needs to be explained (or stated in an understandable way). In my view, weather conditions do not modify the amount of information but the value of such!

[Response]: Thank you for pointing it out.

We stated it in an understandable way in the revised version.

14. P.20, L.10-11: the sentence "Moreover . . . experiments" is unclear or seem unachieved. . . the reader expects something like "and . . ." to know what is the consequence of making measurements during experiments!

[Response]: Thank you for the suggestion.

We rewritten the corresponding part in the revised manuscript.

"Moreover, all the water vapor density along the radiosonde path were collected during the experiments and their changes with altitude were shown in Fig. 15, in which the rainy and rainless weather were represented by blue and red dots."

15. P.20, L.18 and following: Indeed, neglecting water vapor above 8km in near tropical

conditions is far from ideal as it much below the tropopause. Hence a significant part of the water vapor distribution (and dynamics) is not considered. That actually questions the adequacy of Rain / No Rain comparisons throughout the paper at this stage as it could be explained solely by the vertical development of cloud systems.

[Response]: Thank you for the comment.

As you said, the tropopause is different in different region. It is important to determine the top boundary of the tomographic model. Chen and Liu (2014) said that atmospheric regions above 8.5 km should not be considered in the tomography model for Hong Kong. Otherwise, extra unknows will unnecessarily be introduced into the tomography model. Using 8.5km as the top boundary of tomographic modeling can save 43.3% of unknowns compared with using 15km. In addition, since in tomographic reconstruction only those rays entering from the top boundary of the voxel are considered, a higher top boundary implies that more rays will be rejected. Moreover, a more detailed comparison of the top boundary for the tomographic model is described in Yao and Zhao (2017). Two different height were selected as the top boundary in the paper, one is 10.4 km (Scheme 1) and the other one is 8km (Scheme 2). The results of experiment conducted once per hour show that the average utilization of signals increased by 7.51% from 51.51% (Scheme 1) to 59.02% (Scheme 2) and the percentage of voxels crossed by signals increased by 2.73%. The results of experiment conducted once per day also show a similar improvement. The average vertical water vapor profile and STD for 40 years (1974-2014) derived from a radiosonde station (45004) were collected and analyzed, which also shows that 8km is a reasonable choice of top boundary for Hong Kong tomographic model. In addition, the article entitled “Maximally using GPS observation for water vapor tomography” also discussed the choice of the top boundary for the Hong Kong tomographic model, which indicated that 8km is a good choice for Hong Kong region in term of utilization of signal rays and percentage of voxels crossed by signals. In other articles about the Hong Kong water vapor tomography (Chen and Liu, 2016; Chen and Liu, 2017; Zhao and Yao, 2018), 8km or 8.5km was selected as the top boundary, and good tomographic results were achieved.

Therefore, we think that 8km is a good choice for the top boundary in Hong Kong tomographic model considering the change of water vapor density with altitude in a long period, the utilization of signal rays and the percentage of voxels crossed by signals. It was selected and demonstrated by previous articles.

Technical corrections:

1. P.1, L.16 (and throughout the text): I think one should use the term "a priori" rather than "priori" information or data.

[Response]: Thank you for pointing it out.

We corrected it in the revised manuscript.

2. P.1, L.28: "and are . . ." this sentence is not grammatically correct and one wonders to what this part relates to.

[Response]: Thank you for pointing it out.

We rewritten it in the revised version.

3. P.2, L.3: I guess you meant "to improve the restitution of the spatio-temporal variations".

[Response]: Thank you for pointing it out.
We corrected it in the revised manuscript.

4. P.8, Fig.3: Isn't there a graph issue: why is the coloring not matching the grid, for example, in the last part of the figure, there are voxels with some black and some white in it while I understand from the text that it should be either black or white only.

[Response]: Thank you for the comment.
The lower panel of each graph ((a) and (b)) is to show the distribution of voxel with (black) and without (white) sufficient signal. In our experiment, 1.79% of total SWV is taken as a criteria to distinct whether the voxel is crossed by sufficient signal or not. If the number is greater than the threshold, the color of the voxel is black, otherwise the color of the voxel is white. For example, if the number of signal rays crossing the voxel is greater than 88, the voxel is painted black in the last part of the figure (the lower panel of (b)). Thus, the there is no graph issue.