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Interactive comment on "Probabilistic analysis of ambiguities in radar echo direction of arrival from meteors" *by* Daniel Kastinen and Johan Kero

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Dear Referee #2,

Thank you very much for the helpful comments and suggestions! We have below addressed each of the points raised in the review.

Major comments:

1.
To avoid any misunderstanding, what is discussed here is "a constant phase rotation

applied to all elements". In other words, the **same** phase is added to all elements. Thus, such a phase-shift can be extracted from the vector representing radar channels and be written as $\Phi(\mathbf{k})e^{i\theta}$ where θ is the phase-shift. Such an added phase is practically equal to the temporal component of any received signal. A temporal component is a constant phase added over all spatial locations but that changes in time equally everywhere. As a DOA can be determined from a measured signal at any point in time, such a constant phase should NOT affect the results, which is exactly the case for the definition in Eq. 6. This is also why all (to our knowledge) DOA determination algorithms uses phase-differences between antennas rather then the direct phases as measured by the antennas. The phase difference is used to eliminate this "constant phase" added over all antennas. Therefore, invariance to such a phase is not a problem but rather a desirable property. The conventional framework mentioned that uses the inner product uses a beam-forming approach to DOA determination of a measured signal $\mathbf{x}$ where $|\langle \Phi(\mathbf{k}), \mathbf{x} \rangle|$ is maximized. The inner product will be affected by the addition of a constant phase to all elements as $\langle \Phi(\mathbf{k}_1)e^{i\theta}, \Phi(\mathbf{k}_2)e^{i\theta} \rangle = \langle \Phi(\mathbf{k}_1), \Phi(\mathbf{k}_2) \rangle e^{i2\theta}$. I.e, only by a constant change of phase while the magnitude will **not** change. As Eq. 6, as well as the standard framework, depends on the magnitude, both are invariant with respect to a constant phase-shift of all elements. We realize that the section surrounding the definition of Eq. 6 does not discuss these implied dynamics or alternative implementation enough, thereby giving rise the the concern. To address this, we will add a small discussion along these lines on this topic to that section and clarify the choice we made (to minimize rather then maximize).

2-1.
We agree that the relationships are hard to grasp from only text and the suggestion of a illustrative diagram sounds great! We will add this to the manuscript.

2-2.
We did consider the impact of the limited number of MC samples (due to limited

computational resources) but they were not included in the text. These considerations will now be added to the manuscript to support the results further!

The main considerations are as follows: we can consider the discretization as a set of Bernoulli distributions that defines a success as: "the output DOA fall into the inclusion region" and a failure as "it did not". Then, we can measure the probability parameter P_i for region and Bernoulli distribution i trough the fraction of samples inside that region out of all samples, i.e. $\tilde{P}_i$. This estimators variance (not distribution variance, but variance of the estimator itself) $\mathrm{var}(\tilde{P}_i)$ can be approximated by substituting the distribution variance with the measured Bernoulli variance and applying the central limit theorem $\mathrm{var}(\tilde{P}_i) \approx \frac{\tilde{P}_i(1-\tilde{P}_i)}{N_s}$, where N_s is the number of samples. This estimator variance has a upper limit where the parabola is maximum at $\tilde{P}_i = 0.5$. We used 1000 samples, this makes the largest estimator standard error equal to $\sqrt{(\frac{0.5(1-0.5)}{1000})} = 0.01581....$ In other words, a decent approximation of the upper limit of the estimated probabilities standard error is less then 2%. For the special case where no "successful" observations were made (i.e. no points fall into a region), other considerations can be made to approximate an upper limit for a confidence interval of this probability, but this is still guaranteed to be very small at 1000 samples. Thus, we consider this as a reasonable level of accuracy for the large-scale probabilistic dynamics as a function of SNR that is the goal of this study. The models used will probably account for similar or larger errors if/when our approach is applied on real systems.

Yes, this sentence is incorrect as it should account for fringe possibilities and define the range of scenarios where it applies. It is true for the majority of cases where sampling is sufficient and the dynamics of the DOA determination still form clusters. To illustrate, propose we have a radar system where we have measured a large collection of meteor events, all clearly ambiguous where several distinct clusters of DOA determination outputs are formed for each event. One would then probably try to pick a source DOA $\mathbf{k}_0$ from each of these clusters, generate the corresponding

possible ambiguity sets $\Omega(\mathbf{k}_0)$ and see if any set can contain all clusters. If for NONE of the examined meteors, any set calculated fits the measurements: there is probably something wrong with the model used to generate the sets. We will add a clarification like this to the manuscript in place of the previous erroneous sentence!

2-3.
We will add the traditional Bayes theorem form $P(x|D) = \frac{P(D|x)P(x)}{P(D)}$ to the text again. Originally we intended to make this section easier to follow for someone not familiar with Bayesian probability (from personal experience the $P(D)$ and $P(D|x)$ has proven difficult to explain in text). However, the re-writing seems to have made it more confusing instead. We will re-work this section to use standard notation and be more clear.

2-4.
Unfortunately that sentence is slightly confusing as it is not explained how, with respect to measurements, one could acquire a $\mathbf{k}_0$. When dealing with measurements a $\mathbf{k}_0$ would probably be chosen from the largest clustering of output DOA's, where the SNR is sufficient and a range and perhaps Doppler shift has also been determined that is consistent with the range and Doppler of the other measurements of that event. As such, one want to verify that it is actually a meter echo present in the data and that the chosen sample for $\mathbf{k}_0$ comes from that meteor. If we have such a sample $\mathbf{k}_0$ that we assume comes from the meteor: then yes, the statement is true if we assume that the modeling performed to generate Ω is representative of the DOA determination behaviour. The true location must be contained in $\Omega(\mathbf{k}_0)$ because only members of this set as an input to the DOA determination algorithm can generate an output at $\mathbf{k}_0$ with high enough probability by definition. That sentence will be replaced with an short discussion on this topic.

3.
Yes, as mentioned, this was an attempt at introducing the concept quickly to someone

without prior statistical knowledge but we realize that is was counterproductive. We will modify this passage to conform to standard statistical nomenclature and put more effort into a seamless introduction of the concept.

Minor comments:

L. 14: Fixed!
L. 101: Fixed in this and several other locations so that j is consistently used as a channel indicator.
L. 129: Yes, it is a set of initial wave vectors. This has been clarified.
Eq. 9: As we are not considering a difference between models for DOA calculation and SNR calculation this should be Phi not Psi! It has been changed everywhere accordingly.
L. 310: Changed all instances of minimize to minimise.
L. 422: Ooops, the word "instead" should not be there! It has been removed.
Figur 5, ect: An explanation on the scale of the color map has been added to the figure captions (it is a direct consequence of the definition of Eq.6, largest length between two points on a N-dimensional unit sphere).
L. 431: In our "Author Comment #1" to Reviewer #1 this topic was extensively discussed and we will follow that plan to resolve this question.

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