# Comments to the revised version.

In the revised edit, the paper has been well refined and now it looks pretty concise. However, the problem of the ambiguity measure defined by Eq. 6 has not been resolved yet. In my first review comments, I tried to make a brief explanation about the problem mentioning the simplest "constant phase rotation" case among those arise by accepting the definition of Eq. 6. The authors answered in part from L. 125 in the paper text, but it does not address the concerns.

In the reply letter, it is mentioned that

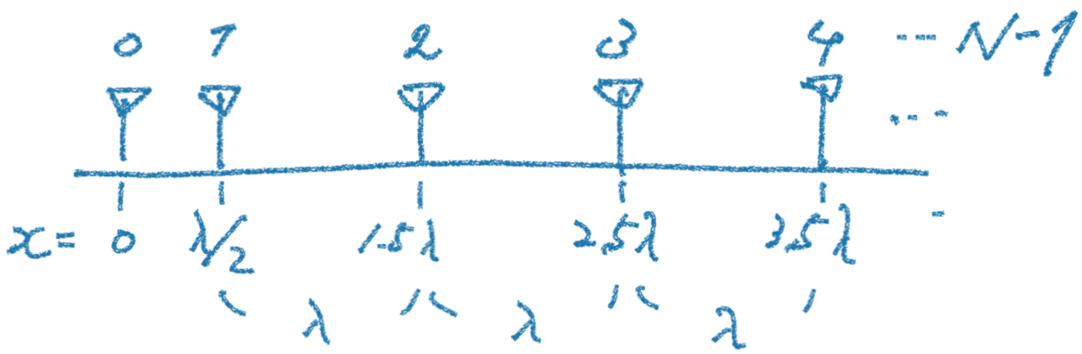
> As a DOA can be determined from a measured signal at any point in time, such a constant phase should NOT affect the results, <u>which is exactly the case for the definition in Eq. 6.</u>

I do agree with the former half, but I do not understand what you mean by the last part underlined. Applying a constant phase shift to Eq. 6, the resulting distance will look like

$$d = \left| \hat{\Phi}_0 - \hat{\Phi} \cdot e^{j\theta} \right|, \qquad (R1)$$

and this is **not invariant** to the phase shift $\theta$.

I try to give another explanation in the following. Think about the following array consisting of $N$ antennas. In this case, antenna-0 is put to the phase reference to which the phase is fixed to 0 degree.




The array responses with respect to the vertical (V) and horizontal (H) directions i.e.,

$$\Phi(k_V) = [e^{j0}, e^{j0}, e^{j0}, e^{j0}, e^{j0}, e^{j0}, \cdots] = [1,1,1,1,1,\cdots], \qquad (R2)$$

$$\Phi(k_H) = [e^{j0}, e^{j\pi}, e^{j\pi}, e^{j\pi}, e^{j\pi}, e^{j\pi}, \cdots] = [1, -1, -1, -1, -1,\cdots]. \qquad (R3)$$

When $N$ is large, the MUSIC (and most other inner product-based DOA estimation algorithms) spectrum exhibits almost the **maximum** response towards H given V. On the other hand, the "distance" shows the **maximum** (not **minimum** as it is supposed to be in the authors thought) value that is

$$d = \frac{|\Phi(k_V) - \Phi(k_H)|}{N} \sim 2. \qquad (R4)$$

This means that the distance measure is not capable of evaluating the ambiguity associated to MUSIC.

As the authors newly mentioned from L. 125, the measure of "ambiguity" can be an authors' choice from numbers of possible definitions. At least, however, the choice must be reasonably compatible with the choice of DOA estimation algorithm (=MUSIC). Simply speaking, the choices of "distance" and MUSIC algorithm are conflicting with each other. In the simulations presented, such conflicts (as in a case depicted above) have been avoided, presumably unintentionally.

In my opinion, this paper is well written having precise descriptions of the methodology and simulation contained. There is the only one contradiction in terms, however; the proposed technique is based on *distance*, while the DOA estimation is based on *inner product*. If my understanding is correct, this problem is too serious to overlook for publishing this paper in this form. Then, there are a few options for the authors can take from.

The most straightforward and seemingly the best option to take is simply to replace the *distance* with *inner product*. Another possible option that I can think of is to add local minimization to replace the *distance* with *the minimum distance by applying an arbitrary but the same phase constant* to all antennas. In an equation form, this can be like,

$$d = \min_{\theta} \left| \hat{\Phi}_0 - \hat{\Phi} \cdot e^{j\theta} \right| . \qquad (R5)$$

So far, however, I am not yet pretty sure whether this idea works or not.

Yet another work around is to add discussions in the text about the drawbacks and the limitations arising via the choice of the distance, including; what in principle and how much the two measures (*distance* and *inner product*) are different in a **quantitative way** in association with **use of MUSIC**; in what cases and how much the *distance* can be a good measure as **ambiguity observed with MUSIC**; and, how the authors did and the readers can avoid failures in the proposed technique nevertheless of the two mismatched choices.