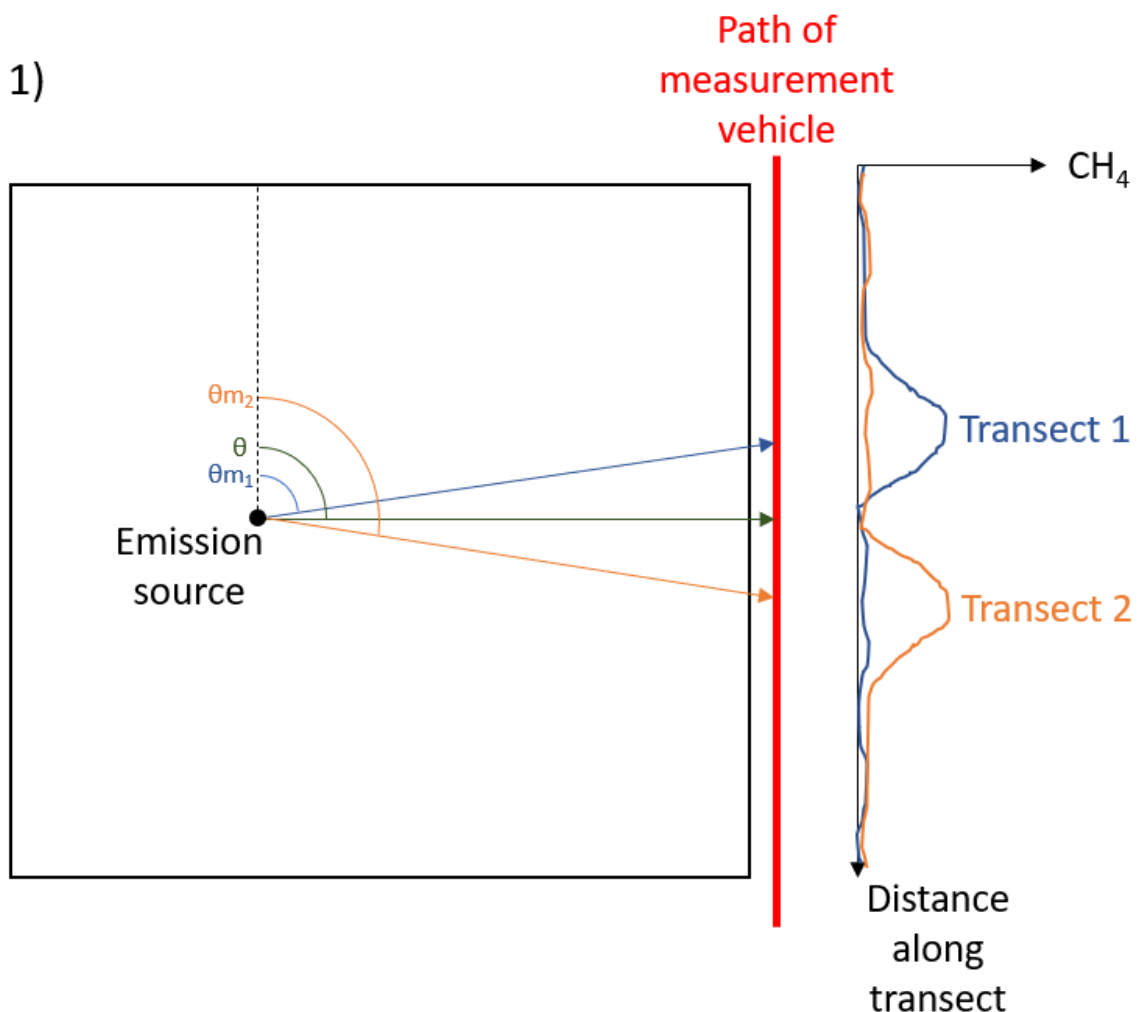
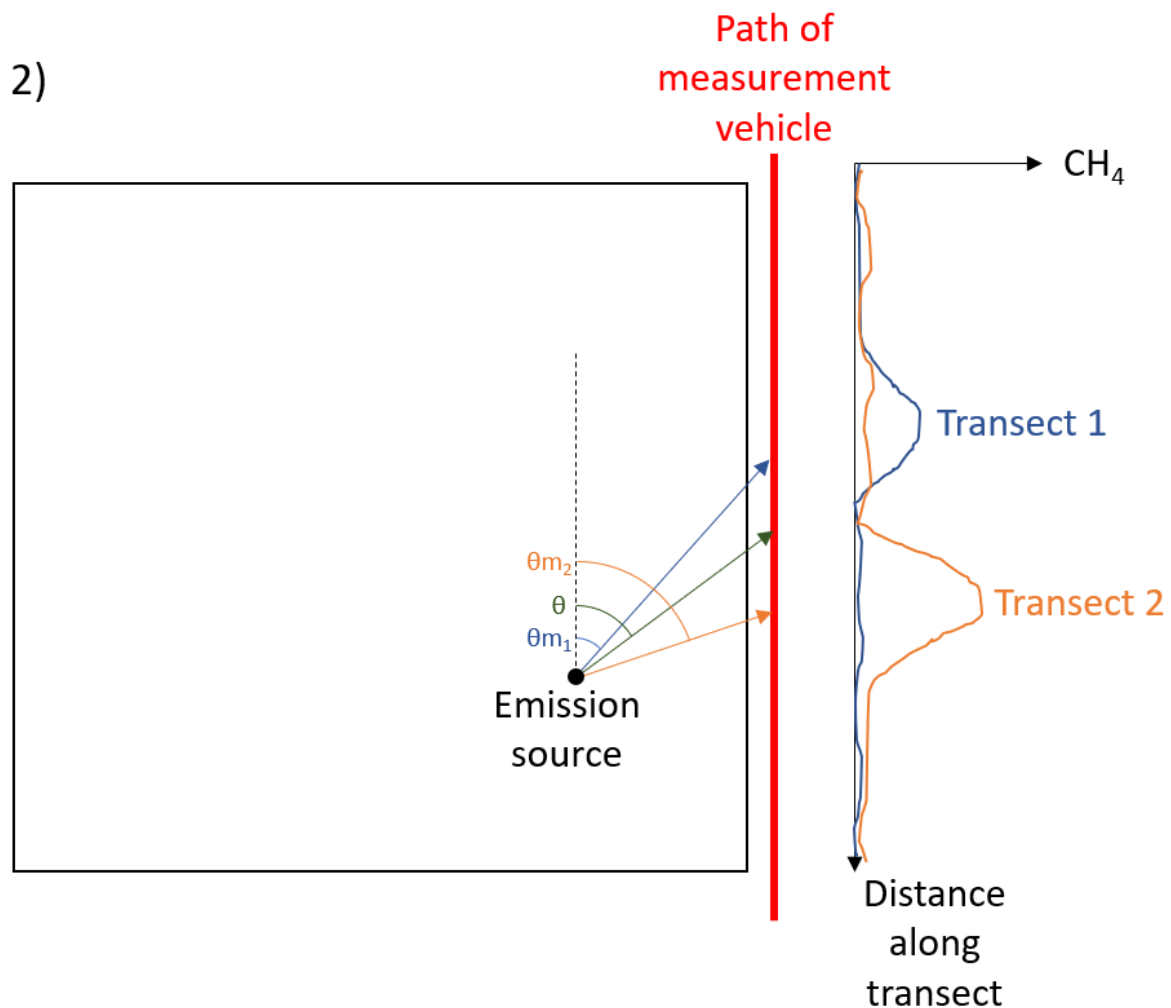


We thank Joseph Pitt for his series of reasoning on our inversion approach and results. His new suggestions have helped, again, to refine our analysis and discussions.

Thanks to the authors for further explaining their approach. I can now see that with multiple transects this method can in theory constrain the location as well as the strength of the emission. I had missed the important point that the X and Y axis in Eq. 1 are defined according to θ_m rather than θ . Consequently, while J_w will always be minimised by moving the source further away, in cases where the plume amplitude differs significantly between transects J_p could have lower values for release locations closer to the transect. So I agree with the conclusions of the authors: in theory there could be enough information to constrain the location and emission rate using this approach, but in practice it has not worked in this case. I think that conclusion is a useful one, so I suggest that this paper should be published in AMT, but I have some additional comments that I think it would be good to address. I now see how the relative amplitude of plumes on different transects could in theory provide a constraint on the location of the source. However, if I have understood this correctly, this constraint only exists when the wind direction has a significant component parallel to the transect. To demonstrate what I mean, consider the two diagrams below. In the first the wind is perpendicular to the transect, while in the second there is non-negligible parallel wind component. Both diagrams represent a topdown view of the area, with the path of the measurement vehicle in red and the measured CH_4 enhancements along the transect shown right.





In diagram 2), where there is a significant component of the wind parallel to the transect, the plume is measured closer to the source on transect 2 than on transect 1. Consequently the plume amplitude is larger on transect 2, and the relative amplitude of the plume on transect 2 vs transect 1 can (in theory) help to constrain the emission source location.

However, in diagram 1), where the wind is perpendicular to the transect, the plume amplitudes are the same on transect 1 and transect 2. In this case there is insufficient information to constrain the location of the source, even in theory, because the plume amplitudes could be equally well simulated by various combinations of emission rate and source location. Because J_w will always be minimized by increasing the distance between the source and the measurements, the estimated emission location will be pushed further away from the transects.

So it would seem that to give this method the best chance of working, one would want to conduct sampling in the wind conditions shown in diagram 2) and avoid the perpendicular wind direction shown in diagram 1). If this is the case then I think this is a conclusion worth highlighting for future studies.

Two processes drive the amplitude of plume transects perpendicular to the wind at fixed height above the ground as a function of the distance from the source (for given wind speed and direction): the plume amplitude is smaller at larger distance due to (i) the loss of larger tails of the plume when integrating between the "edges of the observed peak" because of its wider extent and smoother shape (ii) the larger vertical mixing decreasing

the concentrations close to the ground. The (iii) change of angle between the wind direction and the plume transect adds to the variations of the amplitude of the plume transects.

Of note is that the modeling framework is driven by the effective wind direction corresponding to the direction from the source to the observed plume transect (see J. Pitt's own statement above: "the X and Y axis in Eq. 1 are defined according to θ_m rather than θ "), so that a plume transect perpendicular to the measured wind is hardly perpendicular to such effective wind directions.

We are not sure about how to interpret diagrams 1) and 2). In 1) transects 1 and 2 are represented at the same distance from the source with the same angles between the corresponding effective wind direction and the line of measurement. This would lead to the same plume amplitude, but the reason would not be that the measured wind is perpendicular to the line of measurements. If shifting transects 1 and 2 in a dissymmetric way along the line of measurements, the three processes (i-iii) described above would lead to different plume amplitudes as for the two transects in 2).

Therefore, we do not agree with this general reasoning. We add that plume transects perpendicular to the effective wind are actually preferable since providing clearer limits for the plume, and since being less prone to large errors due to uncertainties in the wind direction.

We insert in the manuscript part of the clarifications given in this answer.

From table 2 of the manuscript it seems that sampling took place under a variety of wind directions, allowing this hypothesis to be investigated. I have two suggestions for this investigation:

We do not agree with the previous reasoning but we still consider the following suggestions since they can help characterize the behavior of the inversion.

1. Include plots of the form of Figure 3 for all releases in the SI. This will demonstrate whether the relative plume amplitude on different transects does sometimes constrain the source location (using the current definition of J), or whether the estimated source location is pushed to the edge of the box in all cases

The figures are now included in the SI (Figures S2-S17). They reveal that J_p is very smooth, and, if ignoring the bounds of the ATEX area, it would have its minimum outside this area. Consequently, minimizing J_p leads to locate the source on a border of the ATEX zone too (most of the time on a border different from that where the minimization of J_w pushes the source). One explanation is the lack of plume transects for constraining the computation. For example, if having two plume transects only, an infinity of solution can lead to the respective amplitude of these transects. Even when having more plume transects, J_p may tend to be driven by a subset of transects, which is another reason for having tried to work with J^{log} . The idea of having a cost function J combining J_w and J_p was partly to overcome such a limitation.

Again, we expand the explanations in the manuscript to better clarify these considerations.

2. Test the impact of varying the plume amplitude uncertainty in J_p . Currently a 100% uncertainty in modelled plume amplitude is assumed – as mentioned in the discussion, lower values for this uncertainty could help to constrain the source location. It would be useful to test various choices of this parameter (e.g. 80%, 60%, 40%, 20%, 10%). In cases where there is a component of the wind parallel to the transect, there may be a value below which this constraint kicks in and the location estimate is no longer forced to the furthest distance from the transects.

We have conducted these tests and the summary of the results is provided in SI (Figures S18 & S19). J_w has been reweighted by a factor λ in J and J^{log} which is equivalent (via the division of the resulting J by λ) to consider a relative model error of $\sqrt{\lambda}$ when modeling the plume area A in J_p or when modeling $\log(1+A)$ in J_p^{log} . Surprisingly, on average, the smallest location error is generally found for $\lambda=1$ or 0 , so that the location error is hardly smaller than that with our default inversion configuration. But for CO_2 release inversions when minimizing J and J^{log} , the optimal average location error can be found for $\lambda=1.6\%$ (i.e. a relative model error of $\sim 13\%$) and $\lambda=0.4\%$ (i.e. a relative model error of $\sim 6\%$), respectively. Furthermore, the curves of average location errors as a function of λ for CH_4 releases when minimizing J or J^{log} have local minima with values close to the optimal one obtained for $\lambda=1$ (for $\lambda=0.016$, i.e. a relative model error of $\sim 13\%$, for J and 0.08 , i.e. a relative model error of $\sim 9\%$, for J^{log}). With such values, some of the releases are located well inside the ATEX zone (see SI Figures S20 and S21 for release no. 2). However, most of the release locations keep on being pushed against the border of this area since the resulting J and J^{log} functions keep on being quite smooth. We assume, again, that the lack of plume transects coupled to the model error explain it and the fact that the system misses the actual release location which should correspond to a local minimum of the cost functions.

We now discuss these results in section 5.3.

Clearly this would not mean that a lower uncertainty is justified, but it would give us a sense of the model accuracy that would be required for the successful application of this method.

We are ready to agree with the idea that it might give us "a sense" of the required model accuracy but the situation seems too complex for us to get robust insights on it and to discuss it correctly in one or two sentences. J_p should balance the misfits and the model error, and normally, the two terms should be strongly correlated. Furthermore, here the requirements strongly depend on the formulation of J_w and, strictly speaking, the optimal values of λ for the error location is 1 for the CH_4 releases (which corresponds to our standard set-up). We thus prefer to avoid discussing this specific idea in the manuscript but we add few sentences on the topic. Of note is that the results from the analysis when reweighting J_w lead us better identify the potential need to derive model errors that are specific to the different plume transects.

This threshold accuracy would presumably be a function of wind direction (relative to the transect). If conducting this analysis for all releases is not feasible, it would at least be good to see these results for a selection of releases covering different wind conditions.

We have the results for all releases, but the problem appears to be too complex to derive such a general understanding of the behavior as a function of the wind conditions. Also, see our answer to the general comment above.

I appreciate that my suggestion 2 has some overlap with the current analysis, where J^{\log} is used in place of J . But I think it would be useful for future studies to include an estimate of the required model accuracy, even if it is only strictly applicable to the conditions encountered during these controlled release experiments.

We hope that our answers above clarify the reason why we prefer to avoid discussing the "required model accuracy".

Finally, if in this study the cost function was always minimised by placing the source location at the furthest point from the transect, then I think the comparison of estimated emission rates to other studies should be done using the fixed-location results. As the authors point out, the tendency to overestimate the distance to the source partially counterbalances the tendency of the inversion to underestimate emissions. Presumably if you increased the size of the grid then the location error would be larger and the emission rate error smaller, but this would not mean that the accuracy of the method at estimating the emission rate was inherently improved. I think it is useful to separate the two issues (location and emission rate) in the discussion section; first discuss what improvements would enable the method to estimate the location of the emission with some skill, then discuss the accuracy of the fixed- location emission rate estimates relative to other studies.

We agree with this point: we now better highlight the results from the experiments in which the release is fixed to its actual location in sections 5 and 6 and by revising the ranges summarizing the precision for the release rate estimates.

In addition to my general suggestions above, I have a couple of specific comments:

- Why was θ_m set to $\theta \pm 2\sigma_\theta$ when it was outside this range? Surely if the angle between a potential release location and the observed plume is very different to the measured wind direction then that location is unlikely to be the source of the release? Therefore it seems reasonable that J_w will be very large for such a location, and it is not clear to me why it needs to be limited in this way.

Following this comment, we re-assessed the relevance of setting θ_m to $\theta \pm 2\sigma_\theta$ maximum, and we decided to remove this strong constraint. All the results have been updated. In a general way, they look very similar after this revision and this does not impact our analysis.

- It would be useful to include the results of the fixed-location experiments (using both J and J^{\log}), either in tables 3 and 4 or in the SI.

We agree: we have included them in both tables 3 and 4.