

Radiative transfer acceleration based on the Principal Component Analysis and Look-Up Table of corrections: Optimization and application to UV ozone profile retrievals

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Abstract

In this work, we apply a principal component analysis (PCA)-based approach combined with look-up tables (LUTs) of corrections to accelerate the VLIDORT radiative transfer (RT) model used in the retrieval of ozone profiles from backscattered ultraviolet (UV) measurements by the Ozone Monitoring Instrument (OMI). The spectral binning scheme, which determines the accuracy and efficiency of the PCA-RT performance, is thoroughly optimized over the spectral range 265 to 360 nm with the assumption of a Rayleigh-scattering atmosphere above a Lambertian surface. The high level of accuracy (~ 0.03 %) is achieved from fast-PCA calculations of full radiances. In this approach, computationally expensive full multiple scattering (MS) calculations are limited to a small set of PCA-derived optical states, while fast single scattering and 2-stream MS calculations are performed, for every spectral point. The number of calls to the full MS model is only 51 in the application to OMI ozone profile retrievals with the fitting window of 270-330 nm where the RT model should be called at fine intervals (~0.03 nm with ~ 2000 wavelengths) to simulate OMI measurements (spectral resolution: 0.4-0.6 nm). LUT corrections are implemented in order to accelerate on-line RT modeling, by first reducing the number of streams (discrete ordinates) from 8 to 4, and improving the accuracy to the same level attained from simulations using a vector model with 12 streams and 72 layers. Overall, we speed up our OMI retrieval by a factor of 3.3 over the previous version, which has already been significantly sped up over line-by-line calculations due to various RT approximations. Improved treatments for RT approximation errors using LUT corrections improve spectral fitting (2-5 %) and hence retrieval errors, especially for

36 tropospheric ozone by up to ~10%; the remaining errors due to the forward model errors are within 5 %
37 in the troposphere and 3 % in the stratosphere.

38 **1. Introduction**

39 Optimal estimation-based inversions have become standard for the retrieval of atmospheric ozone
40 profiles from atmospheric chemistry UV-Vis backscatter instruments. This inversion model requires
41 iterative simulations of not only radiances, but also of Jacobians with respect to atmospheric and surface
42 variables, until the simulated radiances are sufficiently matched with the measured radiances. These
43 ozone profile algorithms face a computational challenge for use in global processing of high
44 spatial/temporal resolution satellite measurements, due to on-line radiative transfer (RT) computations
45 at many spectral points from 270 to 330 nm; it is computationally very expensive to perform full
46 multiple-scattering (MS) simulations with the polarized RT model. To reduce the computational cost, a
47 scalar RT model can be applied together with a polarization correction scheme based on a Look Up
48 Table (LUT) (Kroon et al., 2011; Miles et al., 2015). Another approach is to carry out on-line vector
49 calculations at a few wavelengths (Liu et al., 2010) together with other approximations (e.g., low-stream,
50 coarse vertical layering, Lambertian reflectance for surface and cloud, no aerosol treatment). However,
51 the computational speed is still insufficient to process one day of measurements from the Aura Ozone
52 Monitoring Instrument (OMI) within 24 h (30 cross-track pixels \times 1644 along-track pixels \times 14
53 orbits) with reasonable computational resources. Consequently, only 20 % of the available OMI pixels
54 are processed to generate the operational ozone profile (OMO3PR) product (Kroon et al., 2011), and
55 the spatial resolution is degraded by a factor of 4 to produce the research ozone profile (OMPROFOZ)
56 product (Liu et al., 2010). With the advent of sophisticated inversion techniques and superior
57 spaceborne remote sensing instruments, computational budgets have increased rapidly in recent years.
58 Joint retrievals combining UV and thermal infrared ($\sim 9.6 \mu\text{m}$) have been investigated to better
59 distinguish between upper- and lower-tropospheric ozone abundances from multiple instruments, e.g.,
60 OMI and TES (Fu et al., 2013), OMI and AIRS (Fu et al., 2018), or GOME-2 and IASI (Cuesta et al.,
61 2013). The geostationary Tropospheric Emissions: Monitoring of Pollution (TEMPO) instrument,
62 scheduled for Launch in 2022, is specially designed for joint retrievals combining UV and visible (540-
63 740 nm) radiances to enhance the performance of retrievals for ground-level ozone (Zoogman et al.,
64 2017). Moreover, the temporal and spatial resolutions of upcoming geostationary satellite instruments
65 are being improved, leading to a tremendous increase in the data volume to be processed; for example,
66 daily measurements of TEMPO (with ~ 2000 N/S cross-track pixels \times ~ 1200 E/W mirror steps \times ~ 8
67 times a day) are ~ 30 times greater in volume than those of OMI. Therefore, accelerating RT simulations

68 is one of the highest priority tasks to assure operational capability. For speed-up, LUTs have often been
69 used in trace gas retrieval algorithms to serve as proxies for RT modeling or to perform corrections to
70 on-line RT approximations. In recent years, applying neural network techniques and principal
71 component analysis (PCA) to RT computational performance has received quite a lot of attention (e.g.,
72 Natraj et al., 2005; Spurr et al., 2013;2016; Liu et al., 2016; Yang et al., 2016; Loyola et al., 2018; Nanda
73 et al., 2019; Liu et al., 2020).

74 The goal of this paper is to improve both computational efficiency and accuracy of RT
75 simulations in the OMI ozone profile algorithm (Liu et al., 2010) by combining a fast PCA-based RT
76 model with two kinds of correction techniques. The application of PCA to RT simulations was first
77 proposed by Natraj et al. (2005) by demonstrating a computational improvement of intensity simulation
78 in the O₂A band by a factor of 10 and with ~ 0.3 % accuracy compared to full line-by-line (LBL)
79 calculations. This scheme has been deployed to the UV-backscatter, thermal emission, and cross-over
80 régimes, and has been extended for the derivation of analytic Jacobians, for vector RT applications, and
81 for bidirectional surface reflectances (Kopparla et al., 2016; 2017; Natraj et al., 2010; Somkuti et al.,
82 2017; Spurr et al., 2013). The RT performance enhancement arises from a reduction in the number of
83 expensive full multiple scattering (MS) calculations; the PCA scheme uses spectral binning of the
84 wavelengths into several bins based on the similarity of their optical properties and the projection to
85 every spectral point of these full MS calculations which are executed for a small number of PCA-
86 derived optical states. In addition to the adaption of a PCA-based RTM for our ozone profile retrieval,
87 we have adopted the undersampling correction from our previous implementation (Kim et al., 2009;
88 Bak et al., 2019); this enables us to use fewer wavelengths for further speed-up without much loss of
89 accuracy. Furthermore, we have developed a LUT-based correction to accelerate on-line RT simulations,
90 by starting with a lower-accuracy configuration (scalar RT with no polarization, 4 streams, 24 layers)
91 and then correcting the accuracy to the level attainable by means of a computationally more expensive
92 configuration (vector RT, 12 streams, 72 layers). The stream value refers to the number of discrete
93 ordinates in the full polar space; thus, for example, the term “12 streams” indicates the use of 6
94 upwelling and 6 downwelling polar cosine discrete ordinate directions. In previous work, PCA-based
95 RT calculations were assessed mostly against LBL calculations, independently from the inverse model.
96 Therefore, the PCA performance is likely to be overestimated in terms of operational capability, because
97 operational algorithms have their own speed-up strategies with many approximations; this is the case
98 for our ozone profile algorithm. As mentioned above, the PCA-based RT model is employed in this
99 work to make forward-model simulations of OMI measurements for the retrieval of ozone profiles.
100 Therefore, we evaluate the operational capability of our retrieval algorithm in terms of the retrieval

101 efficiency as well as the accuracy, and assess these relative to the current operational implementation.

102 This paper is structured as follows. Section 2 describes the current forward model scheme and
103 evaluates the approximations made in RT calculations, with the determination of the configuration
104 parameters for accurate simulations. The updated forward model scheme is introduced for the PCA-
105 based RT model in Section 3.1, and the two kinds of correction schemes to use fewer spectral samples
106 and less accurate RT configurations are detailed in Section 3.2. The evaluation is performed in Section
107 4 and then we summarize and discuss the results in Section 5.

108 2. Current forward model scheme

109 We first describe the current v1 SAO OMI ozone profile algorithm that was implemented in OMI
110 Science Investigator-led Processing Systems (SIPS) to generate the research OMPROFOZ ozone
111 profile product, publicly available at the Aura Validation Data Center (AVDC,
112 <https://avdc.gsfc.nasa.gov/index.php?site=1620829979&id=74>). It employs the OMI UV channel that
113 is divided into UV1 (270-310 nm) and UV2 (310-380 nm). The spatial resolution of UV1 is degraded
114 by a factor of 2 in order to increase the signal to noise ratio (SNR) in this spectral region. The full width
115 at half maximum (FWHM) of the instrument spectral response function (ISRF) is ~ 0.63 nm for UV1
116 and ~ 0.42 nm for UV2, with spectral intervals of 0.33 nm and 0.14 nm, respectively. The total number
117 of OMI wavelengths used in our spectral fitting for ozone profiles is 229, from 270-308 nm (UV1) and
118 312-330 nm (UV2). The RT model needs to simulate sun-normalized radiances as well as their
119 derivatives with respect to the ozone profile elements and surface albedo. The Vector Linearized Discrete
120 Ordinate Radiative Transfer (VLIDORT) model Version 2.4 (Spurr, 2006) was employed as the forward
121 model in the v1 OMI ozone profile algorithm (Liu et al., 2010) implemented at SIPS. We have updated
122 VLIDORT to use the most recent Version 2.8 for this study (Spurr and Christi, 2019), as well as in the PCA-
123 VLIDORT described in Sect. 3. Note that there is little difference between these two VLIDORT versions in
124 terms of simulation accuracy. The RT simulation is performed iteratively to ingest the atmospheric and
125 surface variables adjusted through the physical fitting between measured and simulated spectra and
126 simultaneously the statistical fitting between the state vector and the *a priori* vector. The retrieval is
127 optimized typically after 2-3 iterations (up to 10 is permitted). The vertical grids of the retrieved ozone
128 profiles in 24 layers are initially spaced in log (pressure) at $P_i = 2^{-\frac{i}{2}} atm$ (in atm, 1 atm = 1013.25
129 hPa) for 0 (surface), 23 (~ 55 km) and with the top of atmosphere set for P_{24} (~ 65 km). Each layer is
130 thus approximately 2.5-km thick, except for the top layer (~ 10 km). A number of RT approximations
131 have already been applied in the current forward model to speed up the processing. In the remainder of

132 this section, the current forward model scheme is described, with its flow chart depicted in the left panel
 133 of Fig. 1. An error analysis is performed for optimizing the RT model configuration to maximize the
 134 simulation accuracy.

135 In the first step, we select 93 wavelengths with variable sampling intervals: 1.0 nm below 295 nm,
 136 0.4 nm from 295-310 nm, and 0.6 nm above 310 nm. The number of these wavelengths is smaller than
 137 the number of OMI native pixels (229 from 270-330 nm) by more than a factor of 2. The on-line
 138 radiative transfer model is run to generate the full radiance spectrum (single + multiple scattering) at
 139 these wavelengths in the scalar mode, with 8 streams and a Rayleigh atmosphere divided into 25 layers
 140 – a grid that is similar to that for the retrieval, except for the top layer (~ 55 km to 65 km) which is
 141 further divided into two layers. In step 2, the scalar calculations done in step 1 are corrected using the
 142 on-line vector calculation at fourteen wavelengths (shown visually with the vertical lines in Fig. 3.b).
 143 In step 3 individual calculations are interpolated to 0.05 nm intervals with the undersampling correction,
 144 and the result is finally interpolated/convolved on to OMI native grids in step 4.

145 Fig. 2.a shows the reference spectrum for which Gaussian smoothing to 0.4 nm has been applied to
 146 LBL calculations at the sampling rate (0.01 nm) of the ozone cross sections (Brion et al., 1993); this
 147 reference spectrum is used to evaluate approximation errors related to undersampling. Fig. 2.b
 148 demonstrates that LBL calculations need to be performed at intervals of 0.03 nm or finer. The
 149 undersampling correction applied in step 3 allows relaxation of the sampling rate without loss of
 150 accuracy. This correction is based on the adjustment of the radiance due to the difference of the optical
 151 depth profiles between fine (λ_h) and coarse (λ_c) spectral grids as follows:

$$\begin{aligned}
 153 \quad & I(\lambda_h) \\
 154 \quad & = I(\lambda_c) \\
 155 \quad & + \sum_{l=1}^N \frac{\partial I(\lambda_c)}{\partial \Delta_l^{gas}} \left(\Delta_l^{gas}(\lambda_h) - \Delta_l^{gas}(\lambda_c) \right) \\
 156 \quad & + \frac{\partial I(\lambda_c)}{\partial \Delta_l^{ray}} \left(\Delta_l^{ray}(\lambda_h) - \Delta_l^{ray}(\lambda_c) \right), \quad (1)
 \end{aligned}$$

157 Here, $\frac{\partial I}{\partial \Delta_l}$ are weighting functions with respect to optical depth profiles Δ_l^{gas} and Δ_l^{ray} for gas
 158 absorption and Rayleigh scattering, for $l = 1, \dots, N_L$ (the number of atmospheric layers). However, as
 159 shown in Fig. 2.c, the sampling rates (1.0, 0.4, 0.6 nm) used in the v1 forward model are too coarse to
 160 be corrected and hence we have decided to use resolutions 0.3 nm and 0.1 nm in the v1 forward model.

161 Figure 3 shows the errors due to RT approximations. As we mentioned above, the v1 forward model
162 performs scalar simulations for all wavelengths, with errors up to $\sim 10\%$ compared with vector
163 simulations (Fig. 3.a). For adjusting vector/scalar differences, vector simulations are performed
164 additionally at 14 wavelengths. However, as shown in Fig. 3.b, second order errors ($\sim 0.2\%$) remain
165 due to neglect of the dependence of polarization effects on the fine structures of ozone absorption. Using
166 8 streams leads to errors of $\sim 0.05\%$ above 320 nm (Fig. 3.c), whereas using 24 layers causes errors at
167 the 1% level for shorter UV wavelengths (Fig. 3c). Moreover, to improve the v2 simulations we decided
168 to use 12 streams and 72 layers in the RT model, as well as running the model at more wavelengths for
169 the polarization correction.

170 **3. The improved forward model scheme**

171 The right panel of Fig. 1 presents the flow chart for the updated forward model scheme (v2), which
172 employs the PCA-based RT model to perform on-line scalar simulations using 4 streams and a 24-layer
173 atmosphere for RT performance enhancement (step 1), and two kinds of correction schemes for
174 accounting for approximation errors (steps 2 and 3). Section 3.1.1 gives an overview on how the PCA
175 procedure is combined with the VLIDORT version 2.8 model; full theoretical details may be found in
176 Spurr et al. (2016) and Kopparla et al. (2017). Here, our paper gives details on how the PCA-based RT
177 configuration is optimized for the application to UV ozone profile retrievals for maximizing the speed-
178 up in the section 3.1.2. Section 3.2 describes the step 2, wherein the LUT-based correction is applied to
179 remedy approximation errors due to use of a scalar model, a smaller number of streams and coarser-
180 resolution vertical grid. In step 3 the undersampling correction is adopted from the v1 implementation,
181 but the Rayleigh scattering term of the equation 1 is neglected for the speed up, with trivial loss of
182 accuracy.

183 **3.1.1 General PCA procedure**

184 The PCA-based RT process begins with a grouping of spectral points into several bins; atmospheric
185 profile optical properties within each bin are similar. PCA is a mathematical transformation that
186 converts a correlated mean-subtracted dataset into a series of principal components (PCs). To enhance
187 RT performance, PCA is used to compress a binned set of correlated optical profile data into a small set
188 of atmospheric profiles which capture the vast majority of the data variance within the bin. The layer
189 extinction optical thickness $\Delta_{n,i}$ and the single scattering albedos $\omega_{n,i}$ are generally subjected to PCA,
190 where n and i are indices for atmospheric layers ($n = 1, \dots, N_L$) and spectral points ($i = 1, \dots, N_S$),

191 respectively. For each bin, the optical profiles $\{\ln \Delta_{n,i}, \ln \omega_{n,i}\}$ are assembled in the $2N_L \times N_S$ matrix
 192 G in log-space ($G_{n,i} = \ln \Delta_{n,i}$, $G_{n+N_L,i} = \ln \omega_{n,i}$). The mean-removed $2N_L \times 2N_L$ covariance matrix
 193 Y is then:

$$194 \quad Y = [G - \langle G \rangle]^T [G - \langle G \rangle], \quad (2)$$

195 where $\langle \rangle$ denotes a mean-value over all grid points in a bin. This covariance matrix Y is decomposed
 196 into eigenvalues ρ_k and unit eigenvectors X_k through solution of the eigenvalue problem $YX_k =$
 197 $\rho_k X_k$. The scaled eigenvectors of the covariance matrix are defined as the empirical orthogonal
 198 functions (EOFs), $W_k = \sqrt{\rho_k} X_k$, where the index k is ranked from 1 to $2N_L$ in descending order of
 199 the eigenvalues. The principal components (PCs) are the projections of the original data onto the
 200 eigenvectors, $P_k = \frac{1}{\sqrt{\rho_k}} G W_k$. The original data set can then be expanded in terms of the mean value
 201 and a sum over all EOFs. As inputs to the RT simulation, the PCA-defined optical states are defined as
 202 $F_o = \exp[\langle G \rangle]$ and $F_k^\pm = F_o \exp[\pm W_k]$, corresponding respectively to the mean value and to positive
 203 and negative perturbations from the mean value by an amount equal to the magnitude of the k^{th} EOF.
 204 In other words, $\Delta_{n,i}$ and $\omega_{n,i}$ ($i=1 \dots N_S$) are expressed as follows:

$$205 \quad F_o = \begin{Bmatrix} \Delta_{n,o} \\ \omega_{n,o} \end{Bmatrix} \equiv \begin{Bmatrix} \exp\left[\frac{1}{N_S} \sum_{i=1}^{N_S} \ln \Delta_{n,i}\right] \\ \exp\left[\frac{1}{N_S} \sum_{i=1}^{N_S} \ln \omega_{n,i}\right] \end{Bmatrix}; \quad F_k^\pm = \begin{Bmatrix} \Delta_{n,\pm k} \\ \omega_{n,\pm k} \end{Bmatrix} \equiv \begin{Bmatrix} \Delta_{n,o} \exp[\pm W_{n,k}] \\ \omega_{n,o} \exp[\pm W_{n+N_L,k}] \end{Bmatrix}. \quad (3)$$

206 For those optical quantities not included in the PCA reduction but still required for the RT simulations,
 207 the spectral mean values for the bin are assumed, as long as they have smooth monotonic spectral
 208 dependency or else are constant over the bin range. In our application, the phase functions and phase
 209 matrices for Rayleigh scattering are derived from bin-averaged values of the depolarization factor.
 210 Although surface Lambertian albedos are constant in the RT simulations, the calculated radiances are
 211 later adjusted to account for a linear wavelength dependency, by means of the surface albedo weighting
 212 functions. For larger bins, it is possible to include the depolarization ratio or the Lambertian albedo as
 213 additional elements in the optical data set subject to PCA; this has been investigated in another context
 214 by Somkuti et al. (2017).

215 In the PCA-based RT package, three independent RT models are combined in order to generate the
 216 full scattering intensity field (I_{full}) at each spectral point λ_i in a single bin, as in the following:

217 $I_{\text{full}}(\lambda_i) \cong [I_{2S}(\lambda_i) + I_{FO}(\lambda_i)]C(\lambda_i). (4)$

218 Two fast RT models, the ‘‘First-Order’’ (FO) and 2STREAM (2S), are used to generate an accurate
 219 single scatter (SS) field (I_{FO}) and an approximate multiple scatter (MS) field (I_{2S}), respectively, for
 220 every spectral point. The scalar 2S model computes the radiation field with 2 discrete ordinates only.
 221 To derive the correction factors $C(\lambda_i)$, we first compute (logarithmic) ratios of the full-scatter and 2S-
 222 based intensity fields calculated with PCA-derived optical states F_o and F_k^\pm :

223
$$J_o = \ln \left[\frac{I_{VLD}(F_o) + I_{FO}(F_o)}{I_{2S}(F_o) + I_{FO}(F_o)} \right] ; J_k^\pm = \left[\frac{I_{VLD}(F_k^\pm) + I_{FO}(F_k^\pm)}{I_{2S}(F_k^\pm) + I_{FO}(F_k^\pm)} \right]. (5)$$

224 Intensity ratios at the original spectral points $J(\lambda_i)$ are then obtained using a second-order central
 225 difference expansion based on the PCA principal components P_{ki} :

226
$$J(\lambda_i) = J_o + \sum_{k=1}^{N_{EOF}} \frac{(J_k^+ - J_k^-)}{2} P_{ki} + \frac{1}{2} \sum_{k=1}^{N_{EOF}} (J_k^+ - 2J_o + J_k^-)^2 P_{ki}^2 . (6)$$

227 The correction factors $C(\lambda_i) = \exp[J(\lambda_i)]$ are then applied to the approximate simulation ($I_{2S}(\lambda_i) +$
 228 $I_{FO}(\lambda_i)$) according to eq. 4 above. More details can be found in the literature (Natraj et al., 2005, 2010;
 229 Spurr et al., ;2013; 2016; Kopparla et al., 2017).

230 So far, we have discussed generation of the total *intensity* field, using values $I_{FO}(\lambda_i)$ and $I_{2S}(\lambda_i)$
 231 from full-spectrum FO and 2S model calculations, and PCA-derived values $I_{VLD}(F)$, $I_{2S}(F)$ and
 232 $I_{FO}(F)$ based on simulations with PCA-derived optical states $F = \{F_o, F_k^\pm\}$. The above procedure
 233 works with VLIDORT operating in scalar or vector mode; however, the 2S model is purely scalar, and
 234 cannot be used if we want to establish PCA-RT approximations to the Q and U components of the
 235 Stokes vector with polarization present. Instead, we rely only on the VLIDORT and FO models, and
 236 develop a PCA-RT scheme based on the differences between the Q/U values calculated by VLIDORT
 237 and FO for monochromatic and PCA-derived calculations, with an additive correction factor instead of
 238 the logarithmic ratios in Equation (6). This Additive procedure was first introduced in Natraj et al.
 239 (2010), and is discussed in detail in Spurr et al. (2016).

240 Of great importance for us is the need to derive PCA-RT approximations to profile Jacobians
 241 (weighting functions of the total intensity with respect to ozone profile optical depths). A PCA-RT

242 Jacobian scheme was developed by Spurr et al. (2013) for total column weighting functions in
 243 connection with the retrieval of total ozone; this scheme involved formal differentiation of the entire
 244 PCA-RT system as outlined above for the intensity field. This is satisfactory for bulk property (total
 245 atmosphere) Jacobians, but for profile Jacobians it is easier to write (Efremenko et al., 2014; Spurr et
 246 al., 2016):

$$247 \quad K_{Full}^{(\xi)}(\lambda_i) \cong \left[K_{2S}^{(\xi)}(\lambda_i) + K_{FO}^{(\xi)}(\lambda_i) \right] D^{(\xi)}(\lambda_i), \quad (7)$$

248 Here, $K_{2S}^{(\xi)}(\lambda_i) \equiv \frac{\partial I_{2S}(\lambda_i)}{\partial \xi}$, with similar definitions applicable to the FO and VLIDORT partial
 249 derivatives. The Jacobian correction factor $D^{(\xi)}(\lambda_i) = \exp[L^{(\xi)}(\lambda_i)]$ is determined using the same
 250 central-difference expansion as that in Equation (6), but with quantities

$$251 \quad L_0^{(\xi)} = \ln \left[\frac{K_{VLD}^{(\xi)}(F_o) + K_{FO}^{(\xi)}(F_o)}{K_{2S}^{(\xi)}(F_o) + K_{FO}^{(\xi)}(F_o)} \right]; L_{\pm k}^{(\xi)} = \left[\frac{K_{VLD}^{(\xi)}(F_k^{\pm}) + K_{FO}^{(\xi)}(F_k^{\pm})}{K_{2S}^{(\xi)}(F_k^{\pm}) + K_{FO}^{(\xi)}(F_k^{\pm})} \right] \quad (8)$$

252 in place of J_o and J_k^{\pm} in Equation (5).

253 3.1.2 The binning scheme

254 The major performance-saving enhancement is achieved by limiting the number of full-MS
 255 VLIDORT calculations to those based on the reduced set of PCA-derived optical states F_o and F_k^{\pm} . A
 256 general binning scheme has been developed over the shortwave region from 0.29 to 3.0 μm (Kopparla
 257 et al. 2016), whereby this entire region is divided into 33 specially-chosen sub-windows encompassing
 258 the major trace-gas absorption signatures; in each such sub-window there are 11 bins for grouping
 259 optical properties, and up to four EOFs for each PCA bin treatment; with this scheme, radiance
 260 accuracies of 0.1% can be achieved throughout the entire region. However, this general binning scheme
 261 should be fine-tuned to specific applications in order to achieve additional computational saving, and
 262 here, we investigate the optimal settings for spectral binning and the number of EOFs in the Hartley
 263 and Huggins ozone bands (265-360 nm).

264 Optical properties within each bin must be strongly correlated to reduce the number of EOFs
 265 required to attain a given accuracy. According to the general binning scheme noted above in Kopparla
 266 et al. (2016), the UV region is divided at 340 nm, beyond which O_2 - O_2 absorption must be considered.

267 In our application, the spectral region 340-360 nm is further divided at 350 nm: in the first sub-window,
 268 ozone absorption is much stronger than O₂-O₂, while for the second (350-360 nm), O₂-O₂ absorption
 269 becomes dominant. The binning criteria are generally determined by similarities in total optical depths
 270 of gas absorption profiles $\{\tau_{ij}\}$ as defined next:

$$271 \quad \Gamma_g = -\ln \sum_{i=1}^{N_L} \sum_{j=1}^{N_g} \tau_{ij}, \quad (9)$$

272 where N_L denotes the number of atmospheric layers, and N_g the number of atmospheric trace gases.

273 To evaluate the PCA approximation, the “exact-RT” model is performed, whereby accurate full-MS
 274 VLIDORT calculations are performed at all wavelengths in addition to accurate SS calculations; thus:

$$276 \quad I_{\text{exact}}(\lambda_i) = I_{\text{VLD}}(\lambda_i) + I_{\text{FO}}(\lambda_i). \quad (10)$$

277 We first evaluate the impact of applying different binning steps and numbers of EOFs in Fig. 4 where
 278 the residuals $(I_{\text{PCA}} - I_{\text{EXACT}})$ are plotted as a function of Γ_g for the spectral window 265-340 nm at
 279 small and large SZAs, respectively. In this evaluation, the bins are equally spaced in Γ_g for the five
 280 intervals from 0.20 to 1.0. For $\Gamma_g < 1$, where the extinction is strong enough that radiances are very
 281 small, the residuals are effectively reduced by using more bins rather than increasing the number of
 282 EOFs. In this optical range, using the first EOF is enough to capture the vast majority of the spectral
 283 variance, with the optimization of the binning step. However, the bins should be narrowly spaced with
 284 Γ_g intervals of at least 0.3-0.4 for those spectral grids for which Γ_g is less than -2. These spectral grids
 285 are correlated with the Hartley band above ~300 nm, where radiance values rapidly increase due to
 286 decreasing ozone absorption, but the spectral variations are almost unstructured. The rest of our spectral
 287 region corresponds to the Huggins band above 310 nm, where spectral variations are distinctly
 288 influenced by local maxima and minima of ozone absorption. In this spectral region, PCA
 289 approximation errors can be greatly reduced by increasing the number of EOFs. However, it is
 290 interesting to note that the PCA approximation is not further improved by using 4 EOFs instead of 3
 291 (not shown here). Figure 4 also illustrates the dependence of the PCA performance on SZA in the
 292 spectral range below 340 nm: For example, when 2 EOFs are applied with the binning step 0.4, errors
 293 are within $\pm 0.02\%$ at smaller SZA, but increase up to $\pm 0.03\%$ at larger SZA. Therefore, as listed

294 in Table 1, two sets of binning criteria are determined to keep the accuracy within 0.05 % for any
 295 viewing geometry. Based on the experiments shown in Fig. 5, the binning criteria are determined for
 296 the other sub-windows listed in Table 1, namely 340-350 nm and 350-360 nm: the former is set with
 297 bins at intervals of 1 and using the first two EOFs, while the latter is divided into a single bin with the
 298 first four EOFs. Figure 6 illustrates the binning criteria thus determined, demonstrating that the PCA
 299 performance maintains accuracies to within 0.03 % overall, when various sets of SZAs, ozone profiles,
 300 and vertical layers are implemented.

301 3.2 LUT-based correction

302 Two sets of LUTs are created, one for high accuracy (LUT_H : vector, 12 streams, 72 layers), and the
 303 other for lower accuracy (LUT_L : scalar, 4 streams, 24 layers) configurations. The on-line PCA-
 304 VLIDORT model is configured to run in the “ LUT_L ” mode. The correction spectrum is straightforwardly
 305 calculated as the ratio of the LUT-based spectra (LUT_H/LUT_L), but the radiance correction term is
 306 additionally adjusted to account for the different gas optical depth profiles used in on-line and LUT
 307 simulations. The RT results are corrected as follows.

$$309 \quad I_{on} = I_{on,L} \times \exp \left(\ln(I_{LUT_H}/I_{LUT_L}) + \sum_{n=1}^{N_L} \left[\left(\frac{\partial \ln I}{\partial \tau_{LUT_H}} - \frac{\partial \ln I}{\partial \tau_{LUT_L}} \right) \times (\tau_{on} - \tau_{LUT}) \right] (n) \right); \quad (11a)$$

308

$$310 \quad \frac{\partial I}{\partial A_{son}} = \frac{\partial I}{\partial A_{son,L}} \times \frac{\frac{\partial I}{\partial A_{sLUT_H}}}{\frac{\partial I}{\partial A_{sLUT_L}}}; \quad (11b)$$

$$311 \quad \frac{\partial I}{\partial \tau_{on}} = \frac{\partial I}{\partial \tau_{on,L}} \times \frac{\frac{\partial I}{\partial \tau_{LUT_H}}}{\frac{\partial I}{\partial \tau_{LUT_L}}}, \quad (11c)$$

312 where the subscripts “on” and “LUT” stand for on-line and LUT-based calculations, respectively; A_s
 313 and τ_n represent the surface albedos and gas absorption optical depths (n is the layer index). To
 314 construct LUTs, RT calculations are performed using the VLIDORT version 2.8 model for sets of
 315 geometrical configurations (θ_0, θ ; solar zenith angle, viewing zenith angle), surface pressures for 22
 316 climatological ozone profiles and 92 wavelengths (265-345 nm) as listed in Table 2. The azimuth
 317 dependence is treated exactly using the Fourier-0, -1 and -2 intensity components in a Rayleigh

318 scattering atmosphere in conjunction with the associated cosine-azimuth expansion of the full intensity;
 319 see the discussion below. The 22 ozone profiles are constructed from the GOME ozone profile product
 320 (Liu et al. 2005), where the ozone profile shapes are classified according to three latitude regimes and
 321 for total column ozone amounts at 50 DU intervals. The 92 wavelengths are regularly sampled at 5.0
 322 nm intervals below 295 nm and at 1.0 nm intervals up to 310 nm in the Hartley band, and irregularly
 323 sampled at the local minima and maxima of the ozone absorption structures in the Huggins band. The
 324 results of these RT calculations are separated into two components: the path radiance I_{atm} and the
 325 surface reflectance term I_{sfc} according to Chandrasekhar (1960), so that the following relationship is
 326 employed to recover the full radiance:

$$327 \quad I(\theta_0, \theta, \varphi - \varphi_0, A_s) = I_{atm}(\theta_0, \theta, \varphi - \varphi_0) + I_{sfc}(\theta_0, \theta, A_s). \quad (12)$$

328

329 I_{atm} represents the purely atmospheric contribution to the radiance in the presence of a dark surface
 330 (zero albedo), and in a Rayleigh scattering atmosphere, this is given as a Fourier expansion in the cosine
 331 of the relative azimuth angle.

$$333 \quad I_{atm}(\theta_0, \theta, \varphi - \varphi_0) = I_0(\theta_0, \theta) + \cos(\varphi - \varphi_0) I_1(\theta_0, \theta) + \cos 2(\varphi - \varphi_0) I_2(\theta_0, \theta). \quad (13)$$

332

334 However, it is more convenient to write this in the form:

$$336 \quad I_{atm} = I_0(\theta_0, \theta) \left(1 + k_1 \cos(\varphi - \varphi_0) Z_1(\theta_0, \theta) + k_2 \cos 2(\varphi - \varphi_0) Z_2(\theta_0, \theta) \right); \quad (14a)$$

335

$$337 \quad Z_1(\theta_0, \theta) = \frac{1}{k_1} \frac{I_1(\theta_0, \theta)}{I_0(\theta_0, \theta)}; \quad Z_2(\theta_0, \theta) = \frac{1}{k_2} \frac{I_2(\theta_0, \theta)}{I_0(\theta_0, \theta)}; \quad (14b)$$

$$339 \quad k_1 = -\frac{3}{8} \cos \theta_0 \sin \theta_0 \sin \theta; \quad k_2 = \frac{3}{32} \frac{(\sin \theta_0 \sin \theta)^2}{\cos \theta}. \quad (14c)$$

338

340 In the LUTs, the three coefficients (I_0 , Z_1 , and Z_2) are stored instead of I_{atm} . Note that the use
 341 of terms k_1 and k_2 is taken from Dave (1964); most of the angular variability in components
 342 I_1 and I_2 are captured analytically with these functions. In other words, Z_1 and Z_2 are

343 angularly smooth and well-behaved (non-singular) functions, which helps improve angular
 344 interpolation accuracy with fewer points in the angular grids. The surface term is given by:

$$345 \quad I_{sfc}(\theta_0, \theta, A_s) = \frac{A_s T(\theta_0, \theta)}{1 - A_s s^*}. \quad (15)$$

346 In the LUTs, we store the transmission term $T(\theta_0, \theta)$, which is the product of the atmosphere
 347 downwelling flux transmittance for a solar source with the upwelling transmittance from a
 348 surface illuminated isotropically from below, and the geometry-independent term s^* which is
 349 the spherical albedo from such a surface. This is the so-called ‘‘planetary problem’’ calculation
 350 (Chandrasekhar, 1960), and the code to obtain T and s^* is now implemented in VLIDORT
 351 Version 2.8 (Spurr, 2019). One of the key features of the VLIDORT code is its ability to
 352 generate simultaneously (along with the Stokes vector radiation field) any set of Jacobians with
 353 respect to atmospheric and surface optical properties. VLIDORT also contains an analytical
 354 linearization of the planetary problem. Indeed, in our Rayleigh-based application, we require
 355 Jacobians with respect to the albedo A_s and the ozone profile elements. The albedo Jacobian
 356 is obtained through straightforward differentiation of eq. 15 as follows:

$$357 \quad \frac{\partial I}{\partial A_s} = T(\theta_0, \theta) \left(\frac{q}{A_s} \right)^2; \quad q = A_s / (1 - A_s s^*). \quad (16)$$

358

359 For the optical depth derivative, $\partial I / \partial \tau$ is calculated from

$$361 \quad \frac{\partial I}{\partial \tau} = \frac{\partial I_0}{\partial \tau} + k_1 \cos(\varphi - \varphi_0) \frac{\partial Z_1}{\partial \tau} + k_2 \cos 2(\varphi - \varphi_0) \frac{\partial Z_2}{\partial \tau} + q \frac{\partial T}{\partial \tau} + T(q)^2 \frac{\partial s^*}{\partial \tau}. \quad (17)$$

360

362 All partial derivatives in this expression are returned automatically by VLIDORT. For a given
 363 ozone profile, wavelength, and surface pressure, the number of LUT values specified in Table
 364 3 is 770 ($n_{\text{Var}} \times n_{\theta_0} \times n_{\theta} + S_b + \frac{dS_b}{d\tau}$, $n_{\text{Var}} = 8$: $I_0, Z_1, Z_2, T, \frac{dI_0}{d\tau}, \frac{dZ_1}{d\tau}, \frac{dZ_2}{d\tau}, \frac{dT}{d\tau}$), which is much
 365 smaller than that of a LUT with dependence on 8 relative azimuth angles and 5 surface albedo
 366 values ($11,520 = n_{\text{Var}} \times n_{\theta_0} \times n_{\theta} \times n(\varphi - \varphi_0) \times nA_s$, $n_{\text{Var}} = 3$: $I, \partial I / \partial \tau, \partial I / \partial A_s$). LUT-based

367 simulated radiances are evaluated against on-line simulations: The LUT interpolation errors
368 are mostly less than 0.2-0.3 % (not shown here), except for extreme path-length scenarios (e.g.,
369 $\sim 1\%$ at $\theta_0 = 87.0^\circ$) as shown in Fig. 7 a,b; however the interpolation errors are quite similar
370 for the two tables LUT_H and LUT_L and these errors tend to get canceled when performing
371 corrections using the two LUTs; the overall errors after LUT correction are much smaller
372 than $\sim 0.05\%$ (Fig. 7c). Note that the accuracy is completely maintained with respect to both
373 $\varphi - \varphi_0$ and A_s , while the size of a LUT is reduced by a factor of 15. However, LUT
374 corrections still contain ozone profile shape errors due to use of the 22-member total ozone-
375 dependent ozone profile climatology in the LUTs. Figure 8 presents an example of the
376 correction spectrum as a function of SZA. This figure shows that polarization errors are mostly
377 dominant, except at high SZAs above 310 nm, where errors due to use of a low number of
378 streams become significant, and for wavelengths below 300 nm where the use of the coarse
379 vertical layering scheme becomes the main source of uncertainty.

380 4. Evaluation

381 The PCA-RT model developed as described in this paper is implemented as the forward model
382 component of an iterative OE based inversion for retrieving ozone profile from OMI measurements. In
383 previous studies, the PCA-RT performance was evaluated against a suite of “exact” fully accurate
384 monochromatic VLIDORT calculations. However, this suite of exact RT calculations cannot be
385 applied in the operational data processing system, especially when thousands of spectral points are
386 involved; in other words, the operational capability of the PCA-RT approach has been overestimated in
387 previous studies. Therefore, we evaluate the developed RT model against the previous forward model
388 where many RT approximations have already been to speed up the operational system.

389 Table 4 contains input configuration sets for 7 different forward models. OMI spectra are simulated
390 at the under-sampled (“US”) intervals specified in the first column of this table and then interpolated at
391 high-resolution (“HR”) intervals (second column) with the undersampling correction before
392 convolution with OMI slit functions. In the v1 forward model, the US spectral intervals were set at 1.0
393 nm/0.4 nm intervals below/above 295 nm and 0.6 nm above 310 nm, while the HR spectral interval was
394 set at 0.05 nm. In the updated RT model, the spectral points are selected at 0.3 nm (0.1 nm) intervals
395 below (above) 305 nm and the HR interval is set at 0.03 nm, which enables us to achieve very high-
396 accuracy, better than 0.01 %, as shown in Fig. 2.c. In the reference configuration (abbreviated to “Ref”),

397 VLIDORT is run in vector mode with 12 streams and 72 atmospheric layers, so that the RT
398 approximation errors are significantly reduced. The VLIDORT-based forward model is run with five
399 sets of configurations (abbreviated to “VLD” in Table 4) to quantify the impact of RT approximations
400 on ozone retrievals. Figure 10 compares the mean biases of the retrieved ozone profiles between
401 VLD/PCA and Ref for three SZA regimes. VLD⁰ represents the v1 forward model configuration,
402 demonstrating that the ozone retrieval errors due to the entire forward model errors range from ~ 3.5 %
403 for the large SZA regime to ~ 5.5 % for the small SZA regime at the lower atmospheric layers, but ~
404 2 % at the upper layers. The configuration VLD¹ assesses the impact of undersampling errors on the
405 retrievals, causing negative biases of up to 2.0 % below ~ 20 km. Compared to the use of 12 streams,
406 using 8 streams causes negligible impacts on ozone retrievals (VLD²) as the corresponding RTM
407 approximation errors are negligible, except for extreme viewing geometries where the ozone retrieval
408 errors are overwhelmed by instrumental measurement errors (a few %), rather than the forward model
409 errors of ~ 0.05 % as shown in Fig. 3.c. The VLD³ RT calculation is applied to ozone retrievals for
410 evaluating on-line polarization correction, showing that the corresponding errors in tropospheric ozone
411 retrievals are estimated as ± 2 % at small SZAs. The evaluation for VLD⁴ demonstrates that the use
412 of coarse atmospheric layering causes the largest errors (~4.5 % in the troposphere, ~ 1.5 % in the
413 stratosphere). PCA⁰ represents the v2 forward model configuration while PCA¹ is done with the highest
414 accurate configuration except for PCA approximation. Retrieval errors due to PCA approximation are
415 negligible except for the bottom few layers at smaller solar zenith angles (up to ~ 1.5 %). Differences
416 between PCA⁰ and PCA¹ represent the ozone retrieval errors due to LUT uncertainties, most of these
417 being related to the profile shape errors between LUT and on-line calculations. In Fig. 10, the
418 comparison between VLD0 (v1 PROFOZ) and PCA0 (v2 PROFOZ) is performed for individual ozone
419 profile retrievals. The large systematic errors of ~ 5-15 % due to v1 forward model errors are largely
420 eliminated below 30 km. In addition, the variabilities of individual differences are significantly
421 eliminated over all layers at high solar zenith angles. However, there are still some remaining retrieval
422 errors up to - 5% in the troposphere and 3 % in the stratosphere, these being due to v2 forward model
423 simulation errors. Figure 11 further evaluates the v2 implementation. First, we note that the run-time
424 comparison (Fig 11.a) demonstrates that v2 is faster by a factor of 3.3 on average. Some spectral fit
425 residuals are eliminated in the UV 1 band over the middle area of the swath (low latitudes), where the
426 SZAs are relatively small, by up to ~ 2 %; the corresponding improvements are found in the
427 stratospheric column ozone. The amount by which the stratospheric column ozone deviates from the
428 reference is reduced by ~ 0.2 % with the v2 implementation. On the other hand, the tropospheric column
429 ozone retrievals show improvements for most cases, whereas the fit residuals of the UV2 band are

430 slightly worse in the low latitudes. Note that smaller fitting residuals do not lead directly to better ozone
431 retrievals, thanks to the likely presence of systematic measurement errors.

432 **5. Summary and Conclusions**

433 We have extended the PCA-based fast-RT method to improve computational challenges for OE-
434 based SAO OMI ozone profile retrievals requiring iterative calculations of radiances and Jacobian
435 derivatives. The PCA-RT model is designed to perform MS calculations for a few EOF-derived optical
436 states which are developed from spectrally binned sets of inherent optical properties that possess some
437 redundancy. In this study, the binning scheme is carefully tuned to the UV ozone fitting window from
438 265 nm to 360 nm in such a way as to choose the number of EOFs to be as small as possible for each
439 bin, rather than always using the first four EOFs for all bins selected in previous studies. The spectral
440 windows are divided into three sub-windows: 1) 265-340 nm, 2) 340-350, and 3) 350-360 nm. Optical
441 profiles are then grouped into bins according to criteria based on the total gas optical depth, as specified
442 in Table 1. We demonstrated that the PCA approximation errors for our application are within 0.03 %
443 for any viewing geometry, optical depth profile, and vertical layering.

444 The existing (v1) forward model calculations are evaluated to determine the optimal configuration
445 for the v2 forward model. RT approximation errors are present due to the use of 24 relatively coarse
446 vertical layers (2.5 km thick), which can lead to radiance simulation errors of up to ~ 1 % below 320 nm
447 and this results in ozone retrieval errors of 2-4.5 % in the troposphere and 1.5 % in the stratosphere. Eight-
448 stream calculations can result in radiance residuals of ~ 0.05 % or less except at extreme viewing geometries,
449 which causes trivial errors on ozone retrievals compared to other error factors. Moreover, although we
450 account for polarization errors using vector and scalar differences at 14 wavelengths in v1, retrieval
451 accuracies are systematically worse by up to ~ 2 % due to neglecting second-order polarization errors
452 strongly correlated with ozone absorption features. We found that 72 atmospheric layers (~ 0.7 km thick)
453 and 12 streams should be used to ensure simulation accuracies comparable to those with 99 atmospheric
454 layers and 32 streams. To reduce the impact of undersampling errors, we fine-tuned simulation intervals to
455 0.3 nm below 305 nm and 0.1 nm above this wavelength, thereby reducing the biases of the ozone retrievals
456 by ~ 1.5 % when compared to the results derived from the under-sampled intervals used in the v1 simulation.
457 Applying the PCA-RT approach allows us to reduce the number of MS calculations from the high-resolution
458 optical dataset to 51 sets of EOF-derived optical states, but the performance savings are not significantly
459 better than those obtained from previous RT approximations. To improve both efficiency and accuracy, we
460 have developed a LUT-based correction for eliminating the RT approximation errors arising from the vector
461 vs scalar, 12 vs. 4 streams, and 72 vs. 24 layers. In conclusion, the updated PCA-based RT model combined

462 with LUT corrections makes ozone profile retrievals faster than the v1 forward model by a factor of 3.3
463 on average. Fitting accuracies are significantly improved in the UV1 band by 2 % and at comparable
464 levels in the UV2 band, while the ozone profile retrievals are significantly improved, especially in the
465 troposphere by ~ up to 10 %. However, there are still some remaining retrieval errors of up to - 5% in
466 troposphere and 3 % in stratosphere due to the LUT correction errors and PCA approximation errors in
467 the v2 implementation. The updated forward model is in preparation for reprocessing all OMI
468 measurements (2004 -current) for the next version of the PROFOZ product.

469

470 **Author contributions.** JB and XL designed the research; RS provided oversight and guidance for using
471 both VLIDORT and PCA-based VLIDORT; KY developed the LUT creation and interpolation scheme;
472 XL contributed to analyzing ozone profile retrievals with different forward model approaches; JB
473 conducted the research and wrote the paper; CN, CM, GA, and KC contributed to the analysis and
474 writing; CM and GA contributed to managing the computational resources.

475 **Competing interests.** The authors declare that they have no conflicts of interest.

476 **Data availability.** OMI Level1b radiance datasets are available at
477 https://aura.gesdisc.eosdis.nasa.gov/data/Aura_OMI_Level1/ (last access: 21 DEC 2020). The LUT
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588 **Table 1.** The PCA-RT configuration optimized over the UV spectral range 265-360 nm. The optical
589 depth of the total gas column (Γ_g defined in eq. 9) is used to set the criteria for the spectral binning; for
590 example, one or more bins are created at intervals ($\Delta\Gamma_g$) in the range Γ_g^{lower} to Γ_g^{upper} . For each bin,
591 the optical states are expanded in terms of the first few number of EOFs (nEOF).

265-340 nm							
SZA or VZA < 70°				SZA or VZA ≥ 70°			
	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF	List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF
1	∞ to -1.7	2	1	1	∞ to -1.5	2.0	1
2	-1.7 to -1.2	0.5	1	2	-1.5 to -0.7	1.2	1
3	-1.2 to 0.0	0.4	1	3	-0.7 to 0.4	0.35	1
4	0.0 to 0.5	0.5	1	4	0.4 to 0.7	0.3	1
5	0.5 to 3.5	0.6	2	5	0.7 to 2.5	0.6	3
6	3.5 to 4.5	1.0	2	6	2.5 to 3.5	1.0	3
7	4.5 to ∞	2.0	2	7	3.5 to 4.5	1.0	2
8				8	4.5 to ∞	2.0	2
340-350 nm				350-360 nm			
List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF	List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF
1	∞ to ∞	1.0	2	1	∞ to ∞	∞	4

592
593 **Table 2. LUT parameter specification. Note that the relative azimuth dependence is taken**
594 **into account explicitly through the Fourier coefficients of path radiance (Table 3) and the**
595 **surface albedo dependence is taken into account by the planetary problem.**

Parameter	Symbol	N	Grid Values
Ozone Profile ⁺	O_3P	22	<ul style="list-style-type: none"> • Low-latitude (30°S-30°N) L200,L250,L300,L350 • Mid-latitude (30°-60°N/S) M200,M250,M300,M350, M400,M450, M500,M550 • High-latitude (60°-90°N/S) H100,H150,H200,H250,H300,H350, H400,H450, H500,H550
Wavelength	λ	92	265-345 nm
Solar Zenith Angle (SZA)	θ_0	12	0, 16, 31, 44, 55, 64, 71, 76.5, 80.5, 83.5, 86, 88°
Viewing Zenith Angle (VZA)	θ	8	0, 15, 30, 43, 53, 61, 67, 72°
Surface albedo	A_s	1	0.0
Surface pressure	P_s	12	100, 150, 200, 300, 400, 500, 600, 700, 800, 900, 1013.25, 1050 hPa

596 ⁺Total ozone-based ozone profiles for three latitude regimes. The grid values represent the

597 amount of total ozone (DU).

598

599 **Table 3. LUT variable specification**

Variable	Dimensions	Variable	Dimensions
I_0^a	$n\lambda, n\theta_0, n\theta, nP_s$	$dI_0/d\tau$	$n\lambda, n\theta_0, n\theta, nz, nP_s$
Z_1^a	$n\lambda, n\theta_0, n\theta, nP_s$	$dZ_1/d\tau$	$n\lambda, n\theta_0, n\theta, nz, nP_s$
Z_2^a	$n\lambda, n\theta_0, n\theta, nP_s$	$dZ_2/d\tau$	$n\lambda, n\theta_0, n\theta, nz, nP_s$
T^b	$n\lambda, n\theta_0, n\theta, nP_s$	$dT/d\tau$	$n\lambda, n\theta_0, n\theta, nz, nP_s$
S_b^c	$n\lambda, nP_s$	$dS_b/d\tau$	$n\lambda, nz, nP_s$
τ^d	$n\lambda, nz^+$		

600 ^a Fourier coefficients of path radiance with respect to relative azimuth angle (AZA)

601 ^b total transmission of the atmosphere

602 ^c spherical albedo of the atmosphere

603 ^d total gas absorption optical depth profile

604 ⁺ Number of atmospheric layers

605 **Table 4. List of configurations used in evaluating the different forward model**
 606 **calculations for OMI ozone profile retrievals. The reference, VLIDORT, and PCA-RT**
 607 **models are abbreviated as Ref, VLD, and PCA, respectively.**

RT models	US SI (nm) ^a	HR SI (nm) ^b	Nstream ^c	Nlayer ^d	Polarization ^e	RT corr ^f
Ref	0.3 0.1	0.03	12	72	True	False
VLD ⁰	1.0 0.4 0.6	0.05	8	24	False	On-line
VLD ¹	1.0 0.4 0.6	0.05	12	72	True	False
VLD ²	0.3 0.1	0.03	8	72	True	False
VLD ³	0.3 0.1	0.03	12	72	False	On-line
VLD ⁴	0.3 0.1	0.03	12	24	True	False
PCA ⁰	0.3 0.1	0.03	4	24	False	LUT
PCA ¹	0.3 0.1	0.03	12	72	True	False

608 ^a Under-sampled (US) spectral intervals (nm) used to define wavelengths at which RT is actually executed. “0.3|0.1”
 609 represents the intervals divided at 305 nm, while for “1.0|0.4|0.6” those are divided at 295 and 310 nm, respectively.

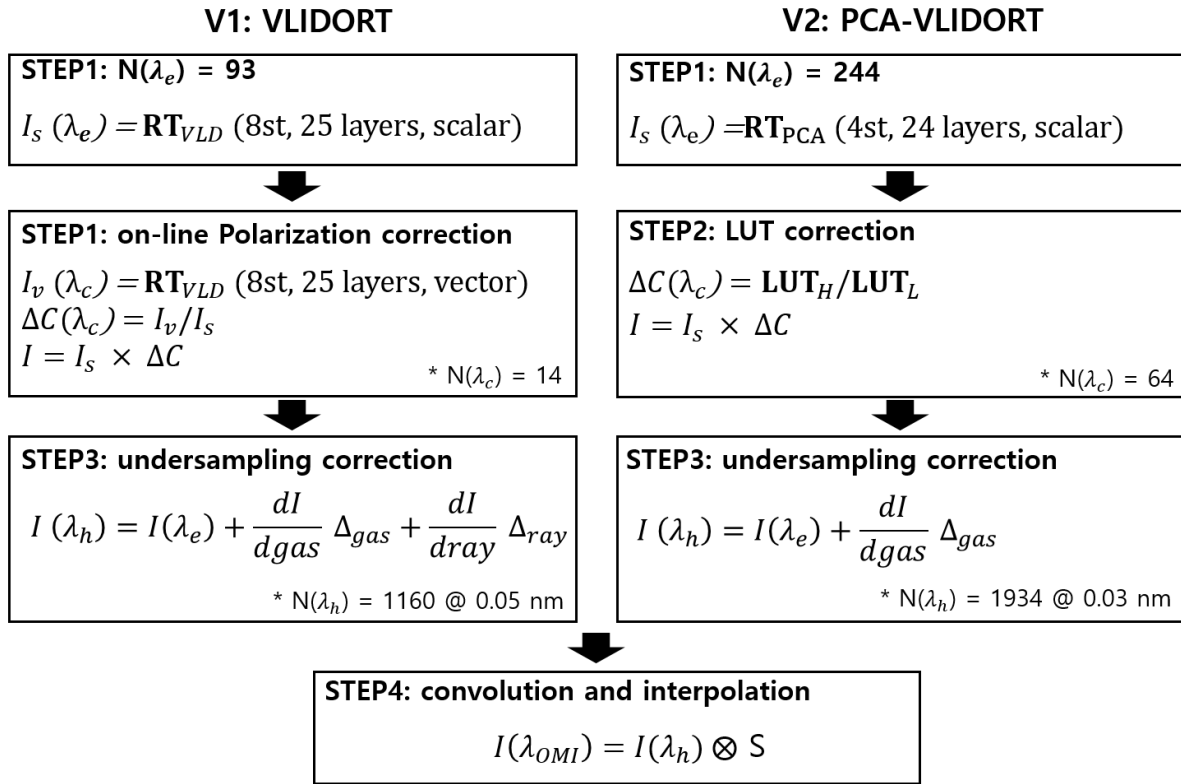
610 ^b High-resolution (HR) spectral intervals (nm) used to define wavelengths where under-sampled simulations are
 611 interpolated before spectral convolution.

612 ^c the number of discrete ordinates in the full polar space; ^d the number of atmospheric layers

613 ^e RT model is run in the vector (scalar) mode if polarization is true (false).

614 ^f On-line correction is performed for polarization errors, based on Liu et al. (2010). LUT-based correction is
 615 performed for RT approximation errors due to neglecting polarization as well as using 4 streams and 24 layers,
 616 developed in this study.

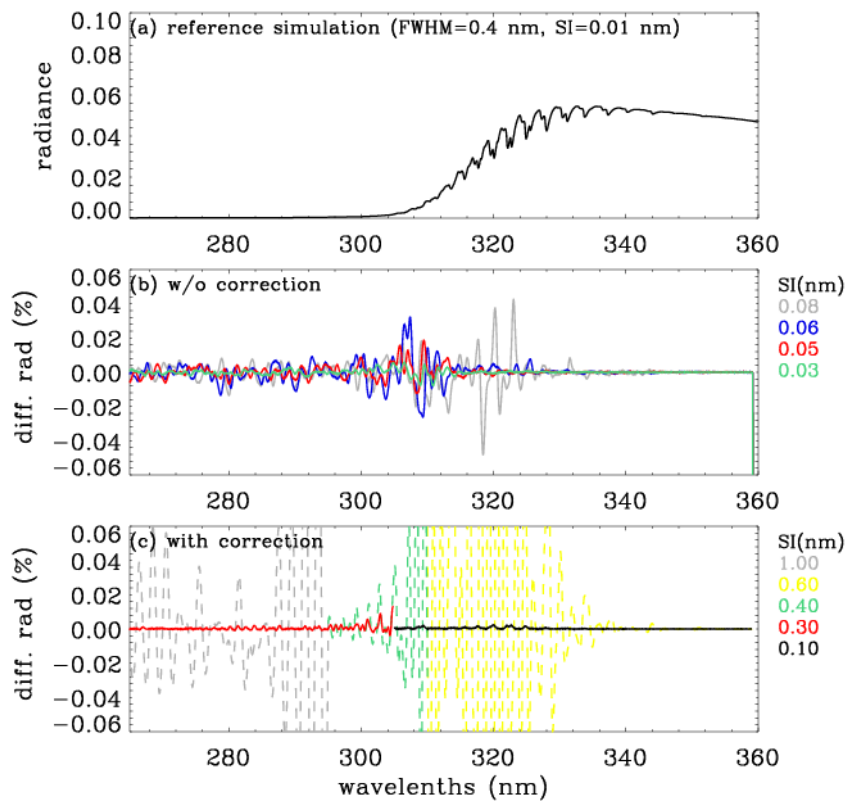
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618

619 **Fig.1.** Schematic flowcharts of the VLIDORT (v1) and PCA-VLIDORT (v2) forward models,
 620 respectively. Note that VLIDORT was used in the generation of the OMPROFOZ v1 dataset,
 621 while PCA-VLIDORT is in preparation for OMPROFOZ v2 production. The number of
 622 wavelengths used in each process is denoted as $N(\lambda)$ when the spectral window 270-330 nm
 623 is applied. λ_e represents the wavelength grids used for RT calculation, while λ_c and λ_h are
 624 grids used in RT approximation correction and undersampling correction, respectively. See
 625 text for definition of other variables.

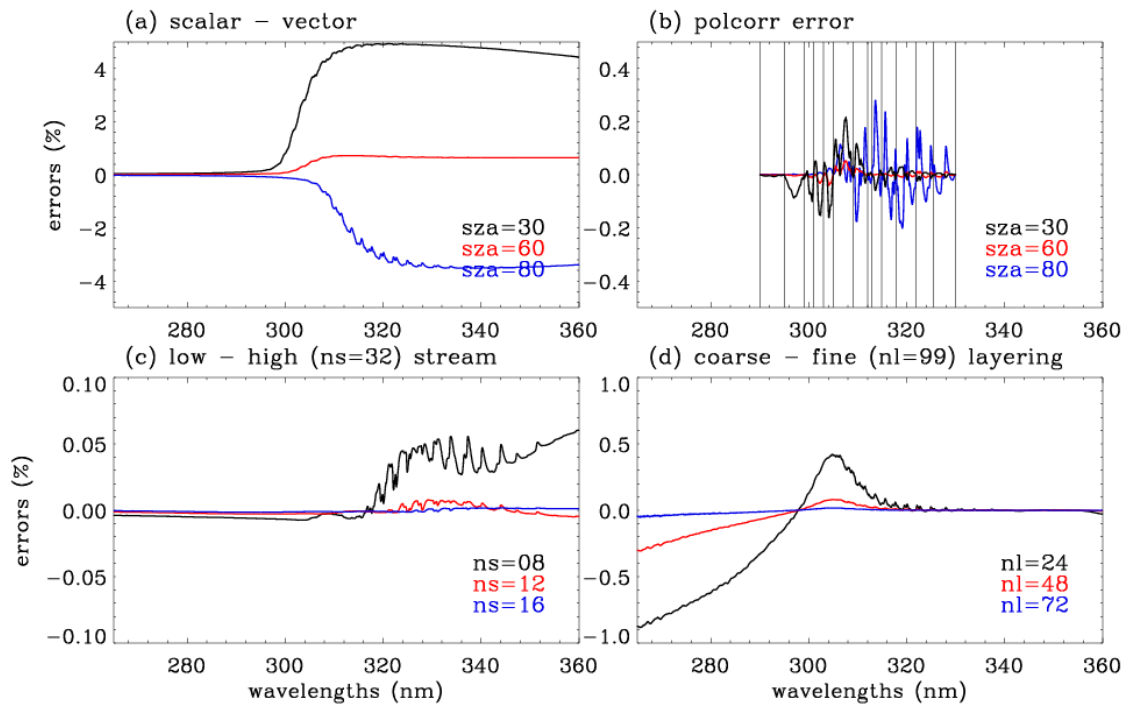
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627

628 **Fig.2.** (a) Reference (truth) normalized radiance spectrum simulated at the spectral intervals
 629 (SIs) of 0.01 nm in 265-360 nm (SZA = 65 °, VZA = 30 °, AZA = 120°), which is used for
 630 evaluating the simulations in Figs. (b) and (c). (b) Impact of undersampling on the simulation.
 631 (c) is similar to (b), but now the undersampling correction has been applied; the dashed and
 632 solid lines represent the sampling rates for v_1 and v_2 , respectively. Note that individual
 633 radiances simulated at different SIs are interpolated to 0.01 nm and then convolved with the
 634 Gaussian function (FWHM: 0.4 nm) which represents the OMI instrument spectral response
 635 function.

636

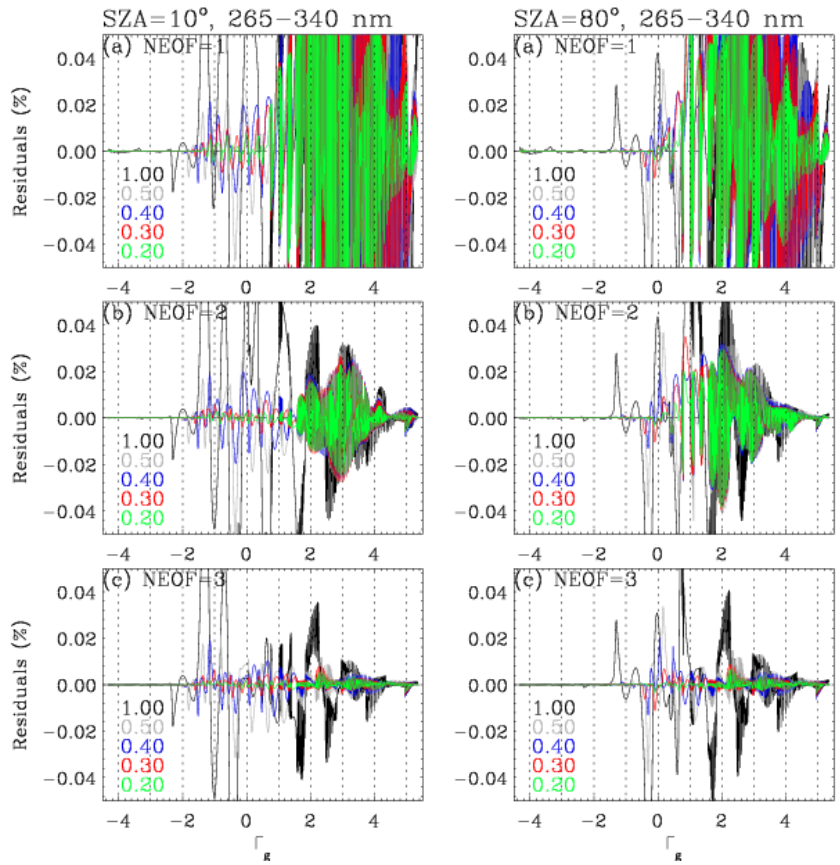


637

638 **Fig. 3.** Errors in radiance simulations due to the RT approximation used in v1, arising from (a) neglecting the
 639 polarization effect, (b) polarization correction errors (vertical lines indicate the 14 wavelengths at which the vector
 640 model is run for deriving the correction spectrum), (c) using a low number of streams (ns), and (d) using a coarse
 641 vertical layering (nl=number of layers). Note that this experiment was done at SZA = 65°, VZA = 30°, and AZA
 642 = 120°.

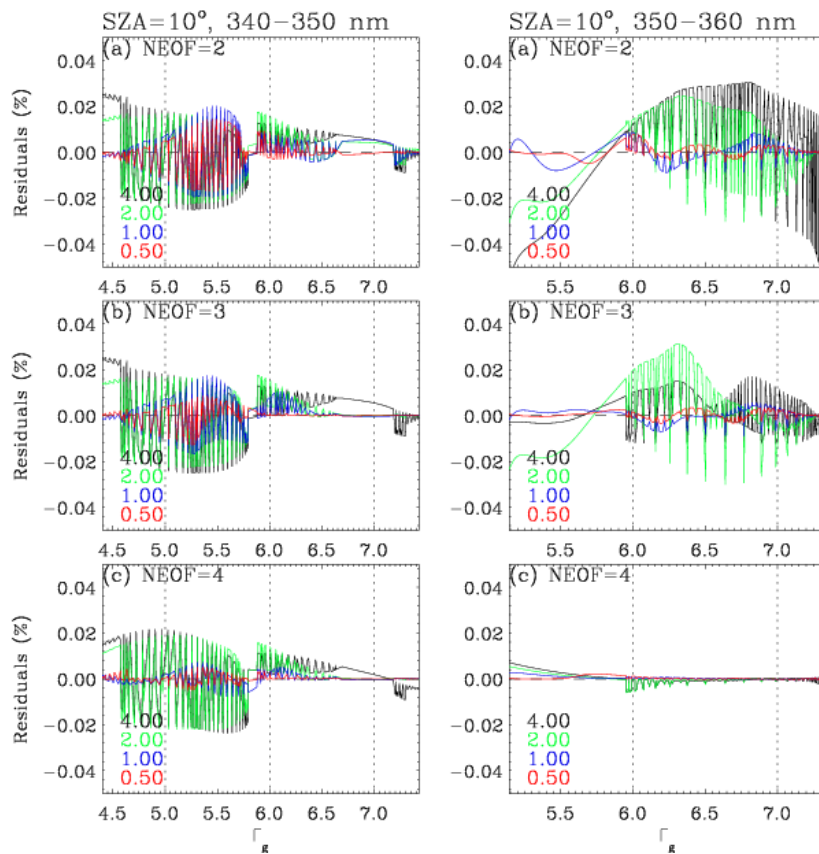
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645

646 **Fig. 4.** Residuals (%) of the PCA-RT radiance in the wavelength range 265-340 nm compared
 647 to the exact-RT calculations, for different binning steps (different colors) and number of EOFs
 648 (a, b, c). Results are plotted as a function of Γ_g (logarithm of the total gas optical depth), for
 649 VZA=30° and AZA=120°, and for SZA 10° (left panels) and 80° (right panels).



650

651 **Fig. 5.** Same as Fig. 4, but for different windows: (left) 340-350 nm and (right) 350-360 nm, respectively.

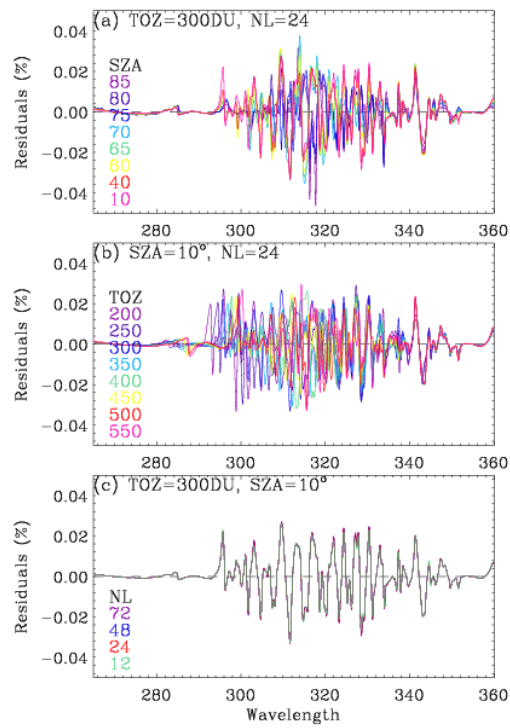
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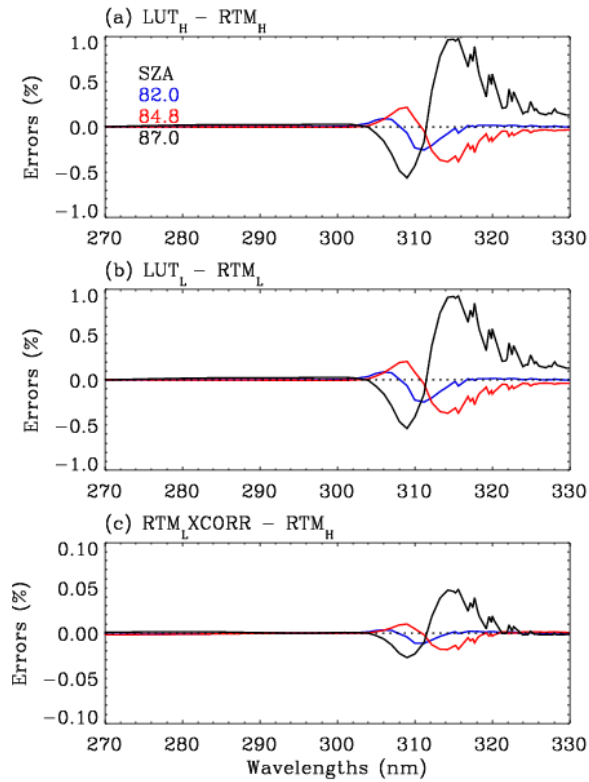
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657

658 **Fig. 6.** Residuals (%) of the PCA-RT radiances with the binning scheme outlined in Table 1,
 659 for various sets of (a) SZAs at VZA=30° and AZA=120°, (b) ozone profiles with different total
 660 ozone columns (TOZs), and (c) number of atmospheric layers.

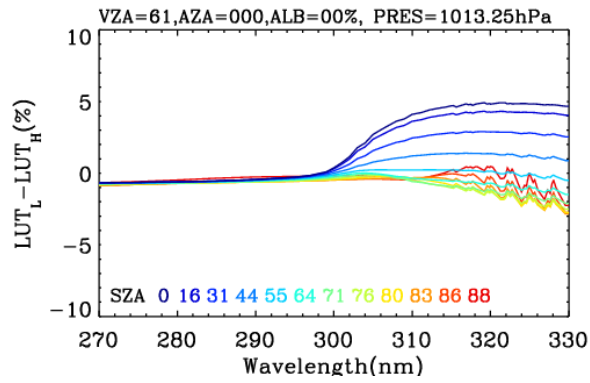
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662

663 **Fig. 7.** Comparisons of radiance simulations at $VZA = 61^\circ$, $AZA = 0^\circ$, for very high SZAs as
 664 indicated. LUT and RTM represent LUT and on-line RT based calculations, respectively, with
 665 the subscripts H and L indicating high and low accuracy configurations, and CORR represents
 666 the correction spectrum taken from the ratio LUT_H/LUT_L .

667

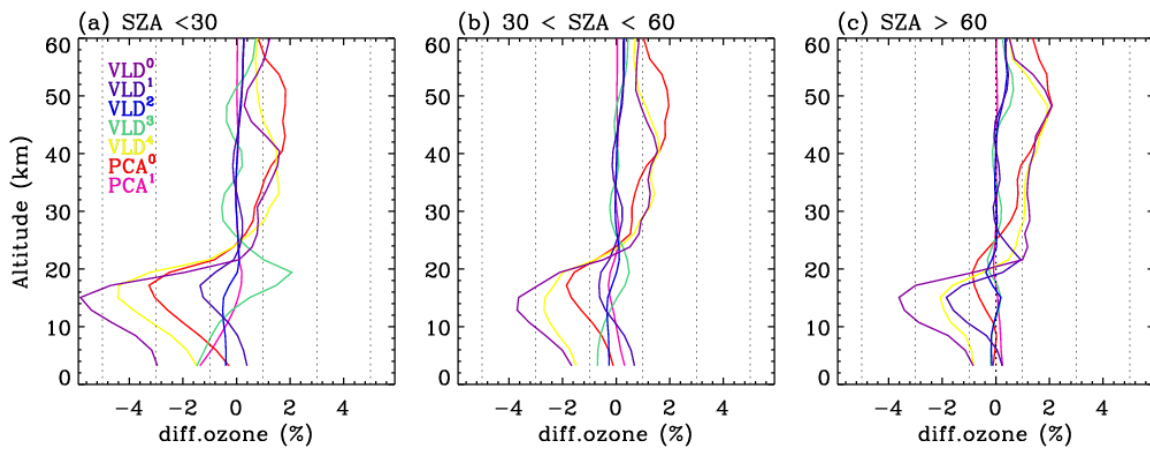


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Fig. 8. Example of LUT-based correction spectrum.

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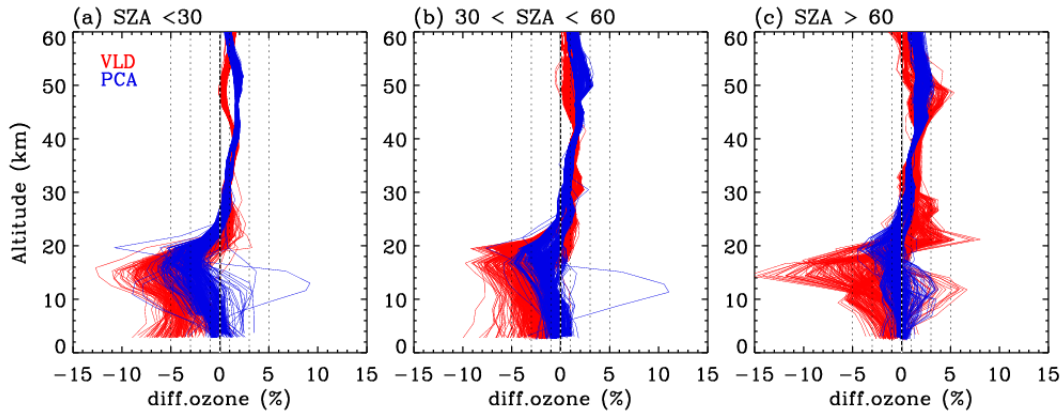
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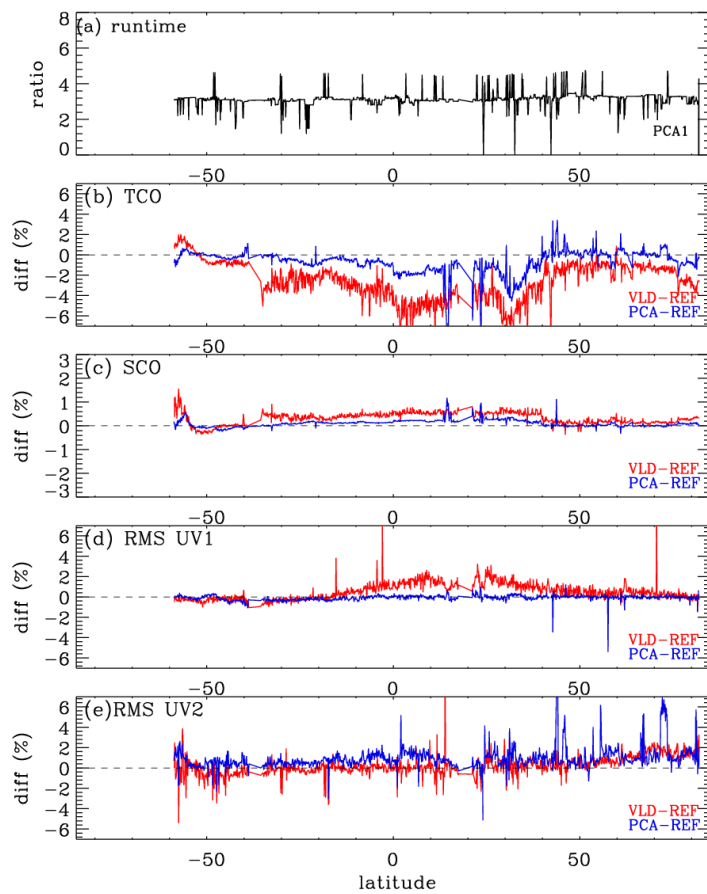
Fig. 9. Mean biases of ozone profile retrievals with different configurations, compared to those with the reference configuration. Each configuration is given in Table 4.

674

675



676
 677 **Fig. 10.** Same as Fig.9, but for individual differences. VLD and PCA represent v1 and v2
 678 forward model configurations, respectively.



679
 680 **Fig. 11.** Same as Fig. 10, but for (a) runtime, (b) tropospheric column ozone (TCO), (c)
 681 stratospheric column ozone (SCO), and (d) UV1 (270-310 nm) and (e) UV2 (310-330 nm)
 682 fitting residuals, as a function of latitude at nadir cross-track. Note that the fitting residuals
 683 are estimated as root mean square (RMS) errors for differences between measured and
 684 simulated spectra relative to the measurement error.