



1 **Radiative transfer acceleration based on the**
2 **Principal Component Analysis and Look-Up**
3 **Table of corrections: Optimization and**
4 **application to UV ozone profile retrievals**
5

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16
17 **Abstract**

18 In this work, we apply a principal component analysis (PCA)-based approach combined with look-up
19 tables (LUTs) of corrections to accelerate the VLIDORT radiative transfer (RT) model used in the
20 retrieval of ozone profiles from backscattered ultraviolet (UV) measurements by the Ozone Monitoring
21 Instrument (OMI). The spectral binning scheme, which determines the accuracy and efficiency of the
22 PCA-RT performance, is thoroughly optimized over the spectral range 265 to 360 nm with the
23 assumption of a Rayleigh-scattering atmosphere above a Lambertian surface. The high level of accuracy
24 (~ 0.03 %) is achieved from fast-PCA calculations of full radiances. In this approach, computationally
25 expensive full multiple scattering (MS) calculations are limited to a small set of PCA-derived optical
26 states, while fast single scattering and 2-stream multiple scattering calculations are performed, for every
27 spectral point. The number of calls to the full MS model is only 51 in the application to OMI ozone
28 profile retrievals with the fitting window of 270-330 nm where the RT model should be called at fine
29 intervals (~0.03 nm with ~ 2000 wavelengths) to simulate OMI native measurements at 229
30 wavelengths (spectral resolution: 0.4-0.6 nm). We also developed a Look Up Table (LUT) to correct RT
31 approximations performed using a scalar RT model with 4 streams (discrete ordinates) and 24 layers,
32 thereby achieving the accuracy at the level attainable from simulations with a vector model with 12
33 streams and 72 layers; this speeds up the RT calculations by more than 2 orders of magnitude when
34 ignoring other overhead. Overall, we speed up our OMI retrieval by a factor of 3.3 over the previous
35 version, which has already been significantly sped up over line-by-line calculations due to various RT
36 approximations. Improved treatments for RT approximation errors using LUT corrections improve



37 spectral fitting (2-5 %) and hence retrieval errors, especially for tropospheric ozone by up to ~10%; the
38 remaining errors due to the forward model errors are within 5 % in the troposphere and 3 % in the
39 stratosphere.

40 1. Introduction

41 Optimal estimation-based inversions have become standard for the retrieval of atmospheric ozone
42 profiles from atmospheric chemistry UV-Vis backscatter instruments. This inversion model requires
43 iterative simulations of not only radiances, but also of Jacobians with respect to atmospheric and surface
44 variables, until the simulated radiances are sufficiently matched with the measured radiances. These
45 ozone profile algorithms face a computational challenge for use in global processing of high
46 spatial/temporal resolution satellite measurements, due to on-line radiative transfer (RT) computations
47 at many spectral points from 270 to 330 nm; it is computationally very expensive to perform full
48 multiple-scattering (MS) simulations with the polarized RT model. To reduce the computational cost, a
49 scalar RT model can be applied together with a polarization correction scheme based on a LUT (Kroon
50 et al., 2011; Miles et al., 2015). Another approach is to carry out on-line vector calculations at a few
51 wavelengths (Liu et al., 2010) together with other approximations (e.g., low-stream, coarse vertical
52 layering, Lambertian reflectance for surface and cloud, no aerosol treatment). However, the
53 computational speed is still insufficient to process one day of measurements from the Aura Ozone
54 Monitoring Instrument (OMI) within 24 h (30 cross-track pixels \times 1644 along-track pixels \times 14
55 orbits) with reasonable computational resources. Consequently, only 20 % of the available OMI pixels
56 are processed to generate the operational ozone profile (OMO3PR) product (Kroon et al., 2011), and
57 the spatial resolution is degraded by a factor of 4 to produce the research ozone profile (OMPROFOZ)
58 product (Liu et al., 2010). With the advent of sophisticated inversion techniques and superior
59 spaceborne remote sensing instruments, computational budgets have increased rapidly in recent years.
60 Joint retrievals combining UV and thermal infrared ($\sim 9.6 \mu\text{m}$) have been investigated to better
61 distinguish between upper- and lower-tropospheric ozone abundances from multiple instruments, e.g.,
62 OMI + TES, OMI +AIRS, and GOME-2 + IASI (Fu et al., 2013; 2018; Cuesta et al., 2013). The
63 geostationary satellite instrument Tropospheric Emissions: Monitoring of Pollution (TEMPO),
64 scheduled for Launch in 2022, is specially designed for joint retrievals combining UV and visible (540-
65 740 nm) radiances to enhance the performance of retrievals for ground-level ozone (Zoogman et al.,
66 2017). Moreover, the temporal and spatial resolutions of upcoming geostationary satellite instruments
67 are being improved, leading to a tremendous increase in the data volume to be processed; for example,
68 daily measurements of TEMPO (with ~ 2000 N/S cross-track pixels \times ~ 1200 E/W mirror steps \times ~ 8



69 times a day) are ~30 times greater in volume than those of OMI. Therefore, accelerating RT simulations
70 is one of the highest priority tasks to assure operational capability. For speed-up, LUTs have often been
71 used in trace gas retrieval algorithms to serve as proxies for RT modeling or to perform corrections to
72 on-line RT approximations. In recent years, applying neural network techniques and principal
73 component analysis (PCA) to RT computational performance has received quite a lot of attention (e.g.,
74 Natraj et al., 2005; Spurr et al., 2013;2016; Liu et al., 2016; Yang et al., 2016; Loyola et al., 2018; Nanda
75 et al., 2019; Liu et al., 2020).

76 The goal of this paper is to improve both computational efficiency and accuracy of RT
77 simulations in the OMI ozone profile algorithm (Liu et al., 2010) by combining a fast PCA-based RT
78 model with two kinds of correction techniques. The application of PCA to RT simulations was first
79 proposed by Natraj et al. (2005) by demonstrating a computational improvement of intensity simulation
80 in the O_2A band by a factor of 10 and with ~ 0.3 % accuracy compared to full line-by-line (LBL)
81 calculations. This scheme has been deployed to the UV-backscatter, thermal emission, and cross-over
82 régimes, and has been extended for the derivation of analytic Jacobians, for vector RT applications, and
83 for bidirectional surface reflectances (Kopparla et al., 2016; 2017; Natraj et al., 2010; Somkuti et al.,
84 2017; Spurr et al., 2013). The RT performance enhancement arises from a reduction in the number of
85 expensive full multiple scattering calculations; the PCA scheme uses spectral binning of the
86 wavelengths into several bins based on the similarity of their optical properties and the projection to
87 every spectral point of these full MS calculations which are executed for a small number of PCA-
88 derived optical states. In addition to the adaption of a PCA-based RTM for our ozone profile retrieval,
89 we have adopted the undersampling correction from our previous implementation (Kim et al., 2009;
90 Bak et al., 2019); this enables us to use fewer wavelengths for further speed-up without much loss of
91 accuracy. Furthermore, we have developed a LUT-based correction to accelerate on-line RT simulations,
92 by starting with a lower-accuracy configuration (scalar RT with no polarization, 4 streams, 24 layers)
93 and then correcting the accuracy to the level attainable by means of a computationally more expensive
94 configuration (vector RT, 12 streams, 72 layers). The stream value refers to the number of discrete
95 ordinates in the full polar space; thus, for example, the term “12 streams” indicates the use of 6
96 upwelling and 6 downwelling polar cosine discrete ordinate directions. In previous work, PCA-based
97 RT calculations were assessed mostly against LBL calculations, independently from the inverse model.
98 Therefore, the PCA performance is likely to be overestimated in terms of operational capability, because
99 operational algorithms have their own speed-up strategies with many approximations; this is the case
100 for our ozone profile algorithm. As mentioned above, the PCA-based RT model is employed in this
101 work to make forward-model simulations of OMI measurements for the retrieval of ozone profiles.



102 Therefore, we evaluate the operational capability of our retrieval algorithm in terms of the retrieval
103 efficiency as well as the accuracy, and assess these relative to the current operational implementation.

104 This paper is structured as follows. Section 2 describes the current forward model scheme and
105 evaluates the approximations made in RT calculations, with the determination of the configuration
106 parameters for accurate simulations. The updated forward model scheme is introduced for the PCA-
107 based RT model in Section 3.1, and the two kinds of correction schemes to use less spectral sampling
108 and less accurate RT configuration are detailed in Section 3.2. The evaluation is performed in Section
109 4 and then we summarize and discuss the results in Section 5.

110 2. Current forward model scheme based on Vector LIDORT (VLIDORT) only

111 We first describe the current v1 SAO OMI ozone profile algorithm that was implemented in OMI
112 Science Investigator-led Processing Systems (SIPS) to generate the research OMPROFOZ ozone
113 profile product, publicly available at the Aura Validation Data Center (AVDC,
114 <https://avdc.gsfc.nasa.gov/index.php?site=1620829979&id=74>). It employs the OMI UV channel that
115 is divided into UV1 (270-310 nm) and UV2 (310-380 nm). The spatial resolution of UV1 is degraded
116 by a factor of 2 in order to increase the signal to noise ratio (SNR) in this spectral region. The full width
117 at half maximum (FWHM) of the instrument spectral response function (ISRF) is ~0.63 nm for UV1
118 and ~0.42 nm for UV2, with corresponding spectral intervals of 0.33 nm and 0.14 nm, respectively. The
119 total number of OMI wavelengths used in our spectral fitting for ozone profiles is 229, from 270-308
120 nm (UV1) and 312-330 nm (UV2). The RT model needs to simulate sun-normalized radiances as well
121 as their derivatives with respect to the ozone profile elements and surface albedo. This simulation is
122 iteratively performed to ingest the atmospheric and surface variables adjusted through the physical
123 fitting between measured and simulated spectra and simultaneously the statistical fitting between the
124 state vector and the *a priori* vector. The retrieval is optimized within typically 2-3 iterations (up to 10
125 is permitted). The vertical grids of the retrieved ozone profiles in 24 layers are initially spaced in log
126 (pressure) at $P_i = 2^{-\frac{i}{2}} atm$ for 0 (surface), 23 (~55 km) and with the top of atmosphere set for P_{24}
127 (~65 km). Each layer is thus approximately 2.5-km thick, except for the top layer (~ 10 km). A number
128 of RT approximations have already been applied in the current forward model to speed up the processing.
129 In the remainder of this section, the current forward model scheme is described, with its flow chart
130 depicted in the left panel of Fig. 1. An error analysis is performed for optimizing the RT model
131 configuration to maximize the simulation accuracy.



132 In the first step, we select 93 effective wavelengths with variable sampling intervals, 1.0 nm below
 133 295 nm, 0.4 nm from 295-310 nm, and 0.6 nm above 310 nm. The number of the wavelengths is smaller
 134 than the OMI native pixels (229 from 270-330 nm) by more than a factor of 2. The on-line radiative
 135 transfer model is run to generate the full radiance spectrum (single + multiple scattering) at these
 136 wavelengths in the scalar mode, with 8 streams and a Rayleigh atmosphere divided into 25 layers – a
 137 grid that is similar to that for the retrieval, except for the top layer (~ 55 km to 65 km) which is further
 138 divided into two layers. Note that the Vector Linearized Discrete Ordinate Radiative Transfer VLIDORT
 139 model Version 2.8 (Spurr and Christi, 2019) is implemented in this study. In step 2, a polarization
 140 correction is applied to the scalar calculations done in step 1 using the on-line vector calculation at
 141 fourteen wavelengths (visually shown with the vertical lines in Fig. 3.a.2). In step 3 the simulation at
 142 the effective wavelength grid is interpolated into 0.05 nm intervals with the undersampling correction,
 143 and the result is finally interpolated/convolved into OMI native grids in step 4.

144 Figure 2.b shows approximation errors related to undersampling from 0.02 nm to 0.1 nm compared
 145 to the simulated radiance at the sampling rate of the ozone cross sections (0.01 nm) (Fig. 2.a). This
 146 illustrates that current forward model calculation has trivial errors (less than 0.01 %) except for 0.02 %
 147 around 310 nm if there is no error after undersampling correction to 0.05 nm. The correction applied in
 148 step 3 allows relaxation of the sampling rate without loss of the accuracy. This correction is based on
 149 the adjustment of the radiance due to the difference of the optical depth profiles between fine (λ_h) and
 150 coarse (λ_c) spectral grids assisted by application of the weighting functions ($\frac{dI}{dx}$) as follows:

$$\begin{aligned}
 152 \quad I(\lambda_h) & \\
 153 \quad &= I(\lambda_c) \\
 154 \quad &+ \sum_{l=1}^N \frac{\partial I(\lambda_c)}{\partial \Delta_l^{gas}} \left(\Delta_l^{gas}(\lambda_h) - \Delta_l^{gas}(\lambda_c) \right) \\
 155 \quad &+ \frac{\partial I(\lambda_c)}{\partial \Delta_l^{ray}} \left(\Delta_l^{ray}(\lambda_h) - \Delta_l^{ray}(\lambda_c) \right), \quad (1)
 \end{aligned}$$

156 where Δ_l^{gas} and Δ_l^{ray} are the optical depth profiles for trace gas absorption and Rayleigh scattering,
 157 $l = 1, \dots, N_L$ (the number of atmospheric layers). Figure 2.c demonstrates that the undersampling
 158 correction works well for simulations at 0.2 nm intervals or less over the entire spectral range, but it
 159 can cause large errors when the simulations are performed at intervals of 1.0 nm, 0.4, and 0.6 nm for
 160 the spectral ranges, 270-295 nm, 295-310 nm, and 310-330 nm, respectively. Figure 3 shows the



161 approximations applied to on-line VLIDORT calculations, including (a.1) neglect of the polarization
162 effect; (b) use of 8 streams; and (c) use of a coarse 24-layer height grid. As we mentioned above, in the
163 v1 forward model the scalar model is used for all wavelengths, with the vector model at 14 wavelengths
164 for correcting the scalar simulations. However, Figure 3.a.2 illustrates that second order of polarization
165 correction errors ($\sim 0.2\%$) could remain due to neglecting the dependence of polarization effects on the
166 fine structures of ozone absorption. Using 8 streams causes errors of $\sim 0.05\%$ above 320 nm, whereas
167 using the 24 layers causes 1 % errors at shorter UV wavelengths. Based on the results shown in Fig 3,
168 we conclude that there is room for improving the simulation accuracy by increasing the number of
169 streams to 12, dividing the atmosphere into 72 layers and using more wavelengths in the polarization
170 correction.

171 **3. The improved forward model scheme based on PCA-VLIDORT**

172 The right panel of Fig. 1.2 illustrates the flow chart of the new forward model scheme (v2) which
173 employs the PCA-based RT model to perform on-line scalar simulations using 4 streams and a 24-layer
174 atmosphere for RT performance enhancement (step 1) and two kinds of correction schemes for
175 accounting for approximation errors (steps 2 and 3). Section 3.1.1 gives an overview on how the PCA
176 tool is combined with the VLIDORT Version 2.8 model; full theoretical details may be found in Spurr
177 et al. (2016) and Kopparla et al. (2017). Here, our paper gives details on how the PCA-based RT
178 configuration is optimized for the application to UV ozone profile retrievals for maximizing the speed-
179 up in the section 3.1.2. Section 3.2 specifies the step 2 wherein the LUT-based correction is applied to
180 simulation errors due to the use of a scalar model, a smaller number of streams and coarser-resolution
181 vertical grid. In the step 3 the undersampling correction is adopted from the v1 implementation, but the
182 Rayleigh scattering term of the equation 1 is neglected for the speed up with trivial loss of accuracy.

183 **3.1.1 General PCA procedure**

184 The PCA-based RT process begins with a grouping of spectral points into several bins; atmospheric
185 profile optical properties within each bin are similar. PCA is a mathematical transformation that
186 converts a correlated mean-subtracted dataset into a series of principal components (PCs). To enhance
187 RT performance, PCA is used to compress a binned set of correlated optical profile data into a small set
188 of atmospheric profiles which capture the vast majority of the data variance within the bin. The layer
189 extinction optical thickness Δ_{ni} and the single scattering albedos ω_{ni} are generally subjected to PCA,
190 where n and i are indices for atmospheric layers ($n = 1, \dots, N_L$) and spectral points ($i = 1, \dots, N_S$),



191 respectively. For each bin, the optical profiles $\{\ln \Delta_{ni}, \ln \omega_{ni}\}$ is composed of $2N_L \times N_S$ matrix G in
 192 log-space ($G_{n,i} = \ln \Delta_{ni}$, $G_{n+N_L,i} = \ln \omega_{ni}$). The mean-removed $2N_L \times 2N_L$ covariance matrix Y is
 193 then:

$$194 \quad Y = [G - \langle G \rangle]^T [G - \langle G \rangle], \quad (2)$$

195 where $\langle \rangle$ denotes a mean-value over all grid points in a bin. This covariance matrix Y is decomposed
 196 into eigenvalues ρ_k and unit eigenvectors X_k through solution of the eigenvalue problem $YX_k =$
 197 $\rho_k X_k$. The scaled eigenvectors of the covariance matrix are defined as the empirical orthogonal
 198 function (EOFs), $W_k = \sqrt{\rho_k} X_k$, where the index k is ranked from 1 to $2N_L$ in descending order
 199 starting with the largest eigenvalues. The principal components (PCs) are the projections of the original
 200 data onto the eigenvectors, $P_k = \frac{1}{\sqrt{\rho_k}} G W_k$. The original data set can then be expanded in terms of the
 201 mean value and a sum over all EOFs. As inputs to the RT simulation, the PCA-defined optical states are
 202 defined as $F_o = \exp[\langle G \rangle]$ and $F_k^\pm = F_o \exp[\pm W_k]$, corresponding respectively to the mean value and
 203 to positive and negative perturbations from the mean value by an amount equal to the magnitude of k^{th}
 204 EOF. Therefore, $\Delta_{n,i}$ and $\omega_{n,i}$ ($i=1 \dots N_S$) are expressed as followings:

$$205 \quad F_o = \begin{Bmatrix} \Delta_{n,o} \\ \omega_{n,o} \end{Bmatrix} \equiv \begin{Bmatrix} \exp\left[\frac{1}{N_S} \sum_{i=1}^{N_S} \ln \Delta_{ni}\right] \\ \exp\left[\frac{1}{N_S} \sum_{i=1}^{N_S} \ln \omega_{ni}\right] \end{Bmatrix}; \quad F_k^\pm = \begin{Bmatrix} \Delta_{n,\pm k} \\ \omega_{n,\pm k} \end{Bmatrix} \equiv \begin{Bmatrix} \Delta_{n,o} \exp[\pm W_{n,k}] \\ \omega_{n,o} \exp[\pm W_{n+N_L,k}] \end{Bmatrix}. \quad (3)$$

206 For those optical quantities not included in the PCA reduction but still required in the RT simulations,
 207 the spectral mean values for the bin are assumed, as long as they have smooth monotonic spectral
 208 dependency or else are constant over the bin range. In our application, the phase functions and phase
 209 matrices for Rayleigh scattering are derived from bin-average values of the depolarization factor.
 210 Surface Lambertian albedos are constant in the RT simulation, but the calculated radiance is later
 211 adjusted to account for 1th order wavelength dependency using surface albedo weighting functions. For
 212 larger bins, it is possible to include the depolarization ratio or the Lambertian albedo as additional
 213 elements in the optical data set subject to PCA; this has been investigated in another context by Somkuti
 214 et al. (2017).

215 In the PCA-based RT package, three independent RT models are combined in order to generate the
 216 full scattering intensity field (I_{Full}) at each spectral point λ_i in a single bin



217
$$I_{\text{full}}(\lambda_i) \cong [I_{2s}(\lambda_i) + I_{\text{FO}}(\lambda_i)]C(\lambda_i). \quad (4)$$

218 Two fast RT models, the “First-Order” (FO) and 2STREAM (2S), are used to generate an accurate
 219 single scatter (SS) field (I_{FO}) and an approximate multiple scatter (MS) field (I_{2s}), respectively, for
 220 every spectral point. The scalar 2S model computes the radiation field with 2 discrete ordinates only.
 221 To derive the correction factors $C(\lambda_i)$, we first compute (logarithmic) ratios of the full-scatter and 2S-
 222 based intensity fields calculated with PCA-derived optical states F_o and F_k^\pm :

223
$$J_o = \ln \left[\frac{I_{\text{VLD}}(F_o) + I_{\text{FO}}(F_o)}{I_{2s}(F_o) + I_{\text{FO}}(F_o)} \right]; J_k^\pm = \left[\frac{I_{\text{VLD}}(F_k^\pm) + I_{\text{FO}}(F_k^\pm)}{I_{2s}(F_k^\pm) + I_{\text{FO}}(F_k^\pm)} \right]. \quad (5)$$

224 Intensity ratios at the original spectral points $J(\lambda_i)$ are then obtained using a second-order central
 225 difference expansion based on the PCA principal components P_{ki} :

226
$$J(\lambda_i) = J_o + \sum_{k=1}^{N_{\text{EOF}}} \frac{(J_k^+ - J_k^-)}{2} P_{ki} + \frac{1}{2} \sum_{k=1}^{N_{\text{EOF}}} (J_k^+ - 2J_o + J_k^-)^2 P_{ki}^2. \quad (6)$$

227 The correction factors $C(\lambda_i) = \exp[J(\lambda_i)]$ are then applied to the approximate simulation $[I_{2s}(\lambda_i) +$
 228 $I_{\text{FO}}(\lambda_i)]$ according to Equation 4 above. More details can be found in the literature (Natraj et al., 2005,
 229 2010; Spurr et al., ;2013; 2016; Kopparla et al., 2017).

230 So far, we have discussed generation of total *intensity* field, using values $I_{\text{FO}}(\lambda_i)$ and $I_{2s}(\lambda_i)$
 231 from full-spectrum FO and 2S model calculation, and PCA-derived values $I_{\text{VLD}}(F)$, $I_{2s}(F)$ and
 232 $I_{\text{FO}}(F)$ based on PCA-derived optical states $F = \{F_o, F_k^\pm\}$. The above procedure works with
 233 VLIDORT operating in scalar or vector mode; however, the 2S model is purely scalar, and cannot be
 234 used if we want to establish PCA-RT approximations to the Q and U components of the Stokes vector
 235 with polarization present. Instead, we rely on just the VLIDORT and FO models, and develop a PCA-
 236 RT scheme based on the differences between the VLIDORT and FO Q/U values for monochromatic and
 237 PCA-derived calculations, with an additive correction factor instead of the logarithmic ratios in
 238 Equation (6) above. This was first introduced in Natraj et al. (2010), and is discussed in detail in Spurr
 239 et al. (2016).

240 Of greater importance for us is the need to derive PCA-RT approximations to profile Jacobians
 241 (weighting functions of the total intensity with respect to ozone profile optical depths). A PCA-RT



242 Jacobians scheme was developed by Spurr et al. (2013) for total column Jacobians in connection with
 243 the retrieval of total ozone; this scheme involved formal differentiation of the entire PCA-RT system as
 244 outlined above for the intensity field. This is satisfactory for bulk property Jacobians, but for profile
 245 Jacobians it is easier to write (Efremenko et al., 2014; Spurr et al., 2016):

$$246 \quad K_{Full}^{(\xi)}(\lambda_i) \cong \left[K_{2S}^{(\xi)}(\lambda_i) + K_{FO}^{(\xi)}(\lambda_i) \right] D^{(\xi)}(\lambda_i), \quad (7)$$

247 Here, $K_{2S}^{(\xi)}(\lambda_i) \equiv \frac{\partial I_{2S}(\lambda_i)}{\partial \xi}$, with similar definitions for the FO and VLIDORT partial derivatives. The
 248 Jacobian correction factor $D^{(\xi)}(\lambda_i) = \exp[L^{(\xi)}(\lambda_i)]$ is determined using the same central-difference
 249 expansion as that in Equation (6), but with quantities

$$250 \quad L_0^{(\xi)} = \ln \left[\frac{K_{VLD}^{(\xi)}(F_o) + K_{FO}^{(\xi)}(F_o)}{K_{2S}^{(\xi)}(F_o) + K_{FO}^{(\xi)}(F_o)} \right]; \quad L_{\pm k}^{(\xi)} = \left[\frac{K_{VLD}^{(\xi)}(F_k^{\pm}) + K_{FO}^{(\xi)}(F_k^{\pm})}{K_{2S}^{(\xi)}(F_k^{\pm}) + K_{FO}^{(\xi)}(F_k^{\pm})} \right] \quad (8)$$

251 in place of J_o and J_k^{\pm} in Equation (5).

252 3.1.2 The binning scheme

253 The major performance saving is achieved by limiting full-MS VLIDORT calculations to those
 254 based on the reduced set of PCA-derived optical states F_o and F_k^{\pm} . A general binning scheme has been
 255 developed over the shortwave region from 0.29 to 3.0 μm (Kopparla et al. 2016), whereby the entire
 256 region is divided into 33 specially-chosen sub-windows encompassing the major trace-gas absorption
 257 signatures; in each such sub-window there are 11 bins for grouping optical properties, and up to four
 258 EOFs for each PCA bin treatment; with this scheme, radiance accuracies of 0.1% can be achieved
 259 throughout the region. However, the binning scheme should be tuned to the specific application to get
 260 additional computational saving, and here, we investigate the optimal set for spectral binning and the
 261 number of EOFs in the Hartley and Huggins ozone bands (265-360 nm).

262 Optical properties within each bin must be strongly correlated to reduce the number of EOFs
 263 required to attain a given accuracy. According to Kopparla et al. (2016), the UV region is divided at 340
 264 nm, beyond which O_2 - O_2 absorption must be considered. In our application, the spectral region 340-
 265 360 nm is further divided at 350 nm: in the first sub-window, ozone absorption is much stronger than
 266 O_2 - O_2 , while for the second (350-360 nm), O_2 - O_2 absorption becomes dominant. The binning criteria



267 are generally determined by similarities in total optical depth of gas absorption profiles τ_{ij} as defined
268 below:

$$269 \quad \Gamma_g = -\ln \sum_{n=1}^{N_L} \sum_{j=1}^{N_g} \tau_{nj}, \quad (9)$$

270 where N_L and N_g denote the number of atmospheric layers and atmospheric trace gases.

271 To evaluate the performance of the PCA approximation, the “exact-RT” model is executed in order
272 to calculate a fully accurate multiple-scattering spectrum using the FO model for an accurate single
273 scattering field and VLIDORT model for an accurate multiple scattering field:

$$275 \quad I_{\text{exact}}(\lambda_i) = I_{\text{VLD}}(\lambda_i) + I_{\text{FO}}(\lambda_i). \quad (10)$$

274

276 We first evaluate the impact of applying different binning steps and numbers of EOFs in Fig. 4.
277 where the residuals ($I_{\text{PCA}} - I_{\text{EXACT}}$) are plotted as a function of Γ_g for the spectral window 265-340
278 nm at small and large SZAs, respectively. In this evaluation, the bins are equally spaced in Γ_g for the
279 five steps from 0.20 to 1.0. For $\Gamma_g < 1$, where the extinction is strong enough that radiances are very
280 small, the residuals are effectively reduced by having more bins rather than increasing the number of
281 EOFs. In this optical range, using the first EOF is enough to capture the vast majority of the spectral
282 variance, with the optimization of the binning step. However, the bins should be narrowly spaced with
283 Γ_g intervals of at least 0.3-0.4 for those spectral grids for which Γ_g is less than -2. These spectral grids
284 are correlated with the Hartley band above ~300 nm, where radiance values rapidly increase due to
285 decreasing ozone absorption, but the spectral variations are almost unstructured. The rest of our
286 spectral region corresponds to the Huggins band above 310 nm, where spectral variations are distinctly
287 influenced by local maxima and minima of ozone absorption. In this spectral region, PCA
288 approximation errors can be greatly reduced by increasing the number of EOFs. However, it is
289 interesting to note that the PCA approximation is not further improved by using 4 EOFs instead of 3
290 (not shown here). Figure 4 also illustrates the dependence of the PCA performance on SZA in the
291 spectral range below 340 nm: For example, when 2 EOFs are applied with the binning step 0.4, errors
292 are within $\pm 0.02\%$ at smaller SZA, but increase up to $\pm 0.03\%$ at larger SZA. Therefore, as listed
293 in Table 1, two sets of binning criteria are determined to keep the accuracy within 0.05 % for any



294 viewing geometry. Based on the experiments shown in Fig. 5, the binning criteria are determined for
 295 the other sub-windows listed in Table 1, namely 340-350 nm and 350-360 nm: The former is set with
 296 bins at intervals of 1 and using the first two EOFs, while the latter is divided into a single bin with the
 297 first four EOFs. Figure 6 illustrates the binning criteria thus determined, demonstrating that the PCA
 298 performance keeps accuracies within 0.03 % when various sets of SZAs, ozone profiles, and vertical
 299 layers are implemented.

300 3.2 LUT-based correction

301 Two sets of LUTs are created, for high accuracy (LUT_H: vector/12 streams/72 layers), and low
 302 accuracy (LUT_L: scalar/4 streams/24 layers) configurations. The on-line PCA-VLIDORT model is
 303 configured to run in the “LUT_L” mode. The correction spectrum is straightforwardly calculated as
 304 the ratio of the LUT-based spectrum (LUT_H/LUT_L), but the radiance correction term is additionally
 305 adjusted to account for the different gas optical depth profiles used in on-line and LUT simulations. The
 306 RT results are corrected as follows.

$$308 \quad I_{on} = I_{on,L} \times \exp \left(\ln(I_{LUT_H}/I_{LUT_L}) + \sum_{n=1}^{N_L} \left[\left(\frac{\partial \ln I}{\partial \tau_{LUT_H}} - \frac{\partial \ln I}{\partial \tau_{LUT_L}} \right) \times (\tau_{on} - \tau_{LUT}) \right] (n) \right); \quad (11a)$$

307

$$309 \quad \frac{\partial I}{\partial A_{son}} = \frac{\partial I}{\partial A_{son,L}} \times \frac{\frac{\partial I}{\partial A_{sLUT_H}}}{\frac{\partial I}{\partial A_{sLUT_L}}}; \quad (11b)$$

$$310 \quad \frac{\partial I}{\partial \tau_{on}} = \frac{\partial I}{\partial \tau_{on,L}} \times \frac{\frac{\partial I}{\partial \tau_{LUT_H}}}{\frac{\partial I}{\partial \tau_{LUT_L}}}, \quad (11c)$$

311 where A_s and τ_n represent the surface albedos and gas absorption optical depths (n is the layer index).
 312 To construct LUTs, RT calculations are performed using the VLIDORT version 2.8 model for sets of
 313 geometrical configurations (θ, θ_o ; solar zenith angle, viewing zenith angle), surface pressures for
 314 22 climatological ozone profiles and 92 wavelengths (265-345 nm) as listed in Table 2. The azimuth
 315 dependence is treated exactly using the 0-2 Fourier intensity components in a Rayleigh scattering
 316 atmosphere in conjunction with the associated cosine-azimuth expansion of the full intensity; see the



317 discussion below. The 22 ozone profiles are constructed from the GOME ozone profile product (Liu
 318 et al. 2005), where the ozone profile shapes vary according to three latitude regimes and with the total
 319 column ozone amounts at 50 DU intervals. The 92 wavelengths are regularly sampled at 5 nm intervals
 320 below 295 nm and at 1.0 nm intervals up to 310 nm in the Hartley band, while irregularly sampled
 321 based on the local minima and maxima of the ozone absorption structures in the Huggins band. The
 322 results of these RT calculations are separated into two components: the path radiance I_{atm} and the
 323 surface reflectance term I_{sfc} according to Chandrasekhar (1960), so that the following relationship
 324 is employed to recover the full radiance:

$$325 \quad I(\theta, \theta_o, \varphi - \varphi_o, A_s) = I_{atm}(\theta, \theta_o, \varphi - \varphi_o) + I_{sfc}(\theta, \theta_o, A_s). \quad (12)$$

326

327 I_{atm} represents the purely atmospheric contribution to the radiance in the presence of a dark surface
 328 (zero albedo), and in a Rayleigh scattering atmosphere, this is given as a Fourier expansion in the cosine
 329 of the relative azimuth angle.

$$331 \quad I_{atm}(\theta, \theta_o, \varphi - \varphi_o) = I_o(\theta, \theta_o) + \cos(\varphi - \varphi_o) I_1(\theta, \theta_o) + \cos 2(\varphi - \varphi_o) I_2(\theta, \theta_o). \quad (13)$$

330

332 However, it is more convenient to write this in the form:

$$334 \quad I_{atm} = I_o(\theta, \theta_o) \left(1 + aq_1 \cos(\varphi - \varphi_o) Z_1(\theta, \theta_o) + aq_2 \cos 2(\varphi - \varphi_o) Z_2(\theta, \theta_o) \right); \quad (14a)$$

333

$$335 \quad Z_1(\theta, \theta_o) = \frac{1}{aq_1} \frac{I_1(\theta, \theta_o)}{I_o(\theta, \theta_o)}; \quad Z_2(\theta, \theta_o) = \frac{1}{aq_2} \frac{I_2(\theta, \theta_o)}{I_o(\theta, \theta_o)}; \quad (14b)$$

$$337 \quad aq_1 = \frac{3}{8} \cos \theta \sin \theta \sin \theta_o; \quad aq_2 = \frac{3}{32} \frac{(\sin \theta \sin \theta_o)^2}{\cos \theta_o}. \quad (14c)$$

336

338 In the LUTs, the three coefficients (I_o , Z_1 , and Z_2) are stored instead of I_{atm} . Note that the use
 339 of terms aq_1 and aq_2 is taken from Dave (1964); most of the angular variability in
 340 components I_1 and I_2 are captured analytically with these functions. In other words, Z_1 and
 341 Z_2 are angularly smooth and well-behaved (non-singular) functions, which helps improve



342 angular interpolation accuracy with fewer points in the angular grids. The surface term is

343
$$I_{sfc}(\theta, \theta_o, A_s) = \frac{A_s T(\theta, \theta_o)}{1 - A_s s^*}. \quad (15)$$

344 In the LUTs, we store the transmission term $T(\theta, \theta_o)$, which is the product of the atmosphere
 345 downwelling flux transmittance for a solar source with the upwelling transmittance from a
 346 surface illuminated isotropically from below, and the geometry-independent term s^* which is
 347 the spherical albedo from such a surface. This is the so-called “planetary problem” calculation
 348 (Chandrasekhar, 1960), and the code to obtain T and s^* is now implemented in VLIDORT
 349 Version 2.8 (Spurr, 2019). One of the key features of the VLIDORT code is its ability to
 350 generate simultaneously (along with the Stokes vector radiation field) any set of Jacobians with
 351 respect to atmospheric and surface optical properties. VLIDORT also contains an analytical
 352 linearization of the planetary problem. Indeed, in our Rayleigh-based application, we require
 353 Jacobians with respect to the albedo A_s and the ozone profile elements τ . First for the albedo
 354 weighting function we have straightforward differentiation from Equation (15) as following

355
$$\frac{\partial I}{\partial A_s} = T(\theta, \theta_o) \left(\frac{qr}{A_s} \right)^2; \quad qr = A_s / (1 - A_s s^*). \quad (16)$$

356

357 For the optical depth derivative, $\partial I / \partial \tau$ is calculated from

359
$$\frac{\partial I}{\partial \tau} = \frac{\partial I_o}{\partial \tau} + aq_1 \cos(\varphi - \varphi_o) \frac{\partial Z_1}{\partial \tau} + aq_2 \cos 2(\varphi - \varphi_o) \frac{\partial Z_2}{\partial \tau} + qr \cdot \frac{\partial T}{\partial \tau} + T \cdot (qr)^2 \cdot \frac{\partial s^*}{\partial \tau}. \quad (17)$$

358

360 All partial derivatives in this expression are returned automatically by VLIDORT. For a given
 361 ozone profile, wavelength, and surface pressure, the number of the LUT values specified in
 362 Table 3 is 770 ($n_{Var} \times n_{\theta} \times n_{\theta_o} + S_b + \frac{dS_b}{d\tau}$, $n_{Var} = 8$: $I_o, Z_1, Z_2, T, \frac{dI_o}{d\tau}, \frac{dZ_1}{d\tau}, \frac{dZ_2}{d\tau}, \frac{dT}{d\tau}$), which is
 363 much smaller than that of a LUT with dependence on 8 relative azimuth angles and 5 surface
 364 albedo values ($11,520 = n_{Var} \times n_{\theta} \times n_{\theta_o} \times n(\varphi - \varphi_o) \times nA_s$, $n_{Var} = 3$: $I, \partial I / \partial \tau, \partial I / \partial A_s$). LUT-based
 365 simulated radiances are evaluated against on-line simulations: The LUT interpolation errors



366 are mostly less than 0.2-0.3 % (not shown here), except for extreme path length scenarios (e.g.,
367 $\sim 1\%$ at $\theta_o = 87.0^\circ$) as shown in Fig. 7 a,b, However the interpolation errors are quite similar
368 to each other for LUT_H and LUT_L . Therefore, those errors are canceled out when performing
369 corrections using the two LUTs and thereby the overall error after LUT correction is much
370 smaller than $\sim 0.05\%$ (Fig. c). Note that the accuracy is completely maintained with respect to
371 both $\varphi - \varphi_o$ and A_s , while the size of a LUT is reduced by a factor of 15. However, LUT
372 corrections still contain ozone profile shape errors due to the use of 22 representative total
373 ozone-dependent ozone profiles in the LUT. Figure 8 shows an example of the correction
374 spectrum as a function of SZA, showing that polarization errors are mostly dominant, except
375 at the high SZAs above 310 nm, where errors due to use of a low number of streams become
376 significant, and for wavelengths below 300 nm where the use of the coarse vertical layering
377 scheme becomes the main source of uncertainty.

378 4. Evaluation

379 The PCA-RT model developed as described in this paper is implemented as the forward model
380 component of an iterative OE based inversion for retrieving ozone profile from OMI measurements. In
381 previous studies, the PCA-RT performance was evaluated against a suite of exact monochromatic
382 baseline of fully accurate VLIDORT simulations. However, such exact RT calculations cannot be
383 applied in the operational data processing system, especially when thousands of spectral points are
384 involved; in other words, the operational capability of the PCA-RT approach has been overestimated in
385 previous studies. Therefore, we evaluate the RT model developed against the existing forward model
386 where many RT approximations are applied to meet the computational budget in the operational system.

387 Table 4 contains sets of configurations for 7 forward models. OMI spectra are simulated at the under-
388 sampled (“US”) intervals specified in the first column of this table and then interpolated at high-
389 resolution (“HR”) intervals (second column) with the undersampling correction before convolution with
390 OMI slit functions. In the v1 forward model, the US spectral intervals were set at 1.0 nm/0.4 nm
391 intervals below/above 295 nm and 0.6 nm above 310 nm, while the HR spectral interval was set at 0.05
392 nm. In the updated RT model, the spectral points are selected at 0.3 nm (0.1 nm) intervals below (above)
393 305 nm and the HR interval is set as 0.03 nm, which enables us to achieve very high-accuracy, better
394 than 0.01 %, as shown in Fig. 2.c. In the reference configuration (abbreviated to “Ref”), VLIDORT is
395 run in vector mode with 12 streams and 72 atmospheric layers, so that the RT approximation errors are



396 significantly eliminated. The VLIDORT-based forward model is run with five sets of configurations
397 (abbreviated to “VLD” in Table 4) to quantify the impact of RT approximations on ozone retrievals.
398 Figure 10 compares the mean biases of the retrieved ozone profiles between VLD/PCA and Ref for
399 three SZA regimes. VLD⁰ represents the v1 forward model configuration, demonstrating that the ozone
400 retrieval errors due to the entire forward model errors range from ~ 2 % for the large SZA regime to ~
401 5 % for the small SZA regime at the lower atmospheric layers, but ~ 1 % at the upper layers. The
402 configuration VLD¹ assesses the impact of undersampling errors on the retrievals, causing negative
403 biases of up to 2.0 % below ~ 20 km. Compared to the use of 12 streams, using 8 streams causes
404 negligible impacts on ozone retrievals (VLD²) as the corresponding RTM approximation errors are
405 negligible, except for extreme viewing geometries where the ozone retrieval errors are overwhelmed
406 by instrumental measurement errors (a few %), rather than the forward model errors of ~ 0.05 % as
407 shown in Fig. 3. The VLD³ based RT calculation is applied to ozone retrievals for evaluating on-line
408 polarization correction, showing that the corresponding errors in tropospheric ozone retrievals are
409 estimated as ± 2 % at small SZAs. The evaluation for VLD⁴ demonstrates that the use of coarse
410 atmospheric layering causes the largest errors (~4.5 % in the troposphere, ~ 1.5 % in the stratosphere).
411 PCA⁰ represents the v2 forward model configuration while PCA¹ is done in the highest accurate
412 configuration except for PCA approximation. Retrieval errors due to PCA approximation are negligible
413 except for the bottom few layers at smaller solar zenith angles (up to ~ 1.5 %). Differences between
414 PCA⁰ and PCA¹ represents the ozone retrieval errors due to LUT errors, mostly related to the profile
415 shape errors between LUT and on-line calculations. In Fig. 10, the comparison between VLD0 (v1
416 PROFOZ) and PCA0 (v2 PROFOZ) is performed for individual ozone profile retrievals. The large
417 systematic errors of ~ 5- 15 % due to v1 forward model errors are greatly eliminated below 30 km. In
418 addition, the random-noise errors are significantly eliminated over the entire layers at high solar zenith
419 angles. However, there are still some remaining retrieval errors up to - 5% in troposphere and 3 % in
420 stratosphere due to v2 forward model simulation errors. Figure 11 further evaluates the v2
421 implementation. First of all, the comparison of the runtime (Fig 11.a) demonstrates a 3.3-fold-increase
422 in speed on average, thanks to switching the forward model from v1 to v2. Some spectral fit residuals
423 are eliminated in the UV 1 band over the middle area of the swath (low latitudes), where the SZAs are
424 relatively small, by up to ~ 2 %; the corresponding improvements are found in the stratospheric column
425 ozone. The amount of the stratospheric column ozone deviated from the reference is reduced by ~ 0.2 %
426 with v2 implementation. On the other hand, the tropospheric column ozone retrievals show
427 improvements for most cases, whereas the fit residuals of the UV2 band are slightly worse in the low
428 latitudes, but significantly better (2-5 %) in the Northern high latitudes (OMI along-track number > ~



429 1300).

430 5. Summary and Conclusions

431 We have extended the PCA-based fast RT method to overcome computational challenges for OE-
432 based SAO OMI ozone profile retrievals from ultraviolet measurements requiring iterative calculations
433 of the radiance and its Jacobian derivatives, to match the simulated spectrum to the measured spectrum
434 The PCA-RT model is designed to perform MS calculations for a few EOF-derived optical states which
435 are developed from spectrally binned sets of inherent optical properties that possess some redundancy.
436 To maximize the performance enhancement, we carefully tuned the binning scheme for the UV ozone
437 fitting window from 265 nm to 360 nm in such a way as to choose the number of EOFs to be as small
438 as possible for each bin, rather than always using the first four EOFs for all bins selected in previous
439 studies. The spectral windows are divided into three sub-windows: 1) 265-340 nm, 2) 340-350, and 3)
440 350-360 nm. Then, optical profiles are grouped into bins according to criteria based on the total gas
441 optical depth, as specified in Table 1. Spectral bins correlated to the Hartley ozone band use only the
442 first EOF, but 2 or 3 EOFs are required for those bins related to the Huggins ozone band. The MS model
443 is executed 85 times for the entire wavelength range (265-360 nm), and only 51 times for the OMI
444 ozone fitting window (270-330 nm). We demonstrated that the PCA approximation errors are within
445 0.03 % for any viewing geometry, optical depth profile, and vertical layering. The existing (v1) forward
446 model calculations are evaluated to determine the optimal configuration for the v2 forward model. RT
447 approximation errors exist due to the use of 24 quite coarse vertical layers (2.5 km thick), which can
448 cause radiance simulation errors of up to ~ 1 % below 320 nm and this leads to ozone retrieval errors of 2-
449 4 % in the troposphere and 1 % in the stratosphere. Eight-stream calculations can result in radiance residuals
450 of ~ 0.05 % or less except at extreme viewing geometries, which causes trivial errors on ozone retrievals
451 compared to other error factors. In spite of accounting for polarization errors using vector and scalar
452 differences at 14 wavelengths, the retrieval accuracies are systematically worse by ~ 2 % due to neglecting
453 second-order polarization errors which are strongly correlated with ozone absorption features. We found that
454 72 atmospheric layers (~ 0.7 km thick) and 12 streams should be used at least for fully accurate RT
455 calculations comparable to those with 99 atmospheric layers and 32 streams. The OMI spectral fit uses 229
456 wavelengths at OMI native grids, but the existing RT model simulates 93 wavelengths and is then
457 interpolated onto 0.05 nm grids with undersampling correction. However, we found room to improve our
458 retrievals (~ 1.5 % on average) by simulating 244 wavelength grids selected at intervals of 0.3 nm/0.1 nm
459 below/above 305 nm and then performing the undersampling correction to 0.03 nm. Applying the PCA-RT
460 approach allows us to reduce the number of MS calculations from the high-resolution optical dataset to 51



461 sets of EOF-derived optical states, but the performance savings are not enough to improve over previous RT
462 approximations. To improve both efficiency and accuracy, we have developed a LUT-based correction for
463 eliminating the RT approximation errors arising from the vector vs scalar, 12 vs. 4 streams, and 72 vs. 24
464 layers. In conclusion, the updated PCA-based RT model combined with LUT corrections makes ozone
465 profile retrievals faster than the v1 forward model by a factor of 3.3 on average. Improvements in fitting
466 accuracies are also achieved in the UV1 band by 2 % and in the UV2 band by 2-5 %. Correspondingly,
467 the ozone profile retrievals are significantly improved, especially in the troposphere by ~ up to 10 %.
468 However, there are still some remaining retrieval errors of up to - 5% in troposphere and 3 % in
469 stratosphere due to the LUT correction errors and PCA approximation errors in the v2 implementation.
470 The updated forward model is in preparation for reprocessing all OMI measurements (2004 -current)
471 for the next version of the PROFOZ product.

472

473 **Author contributions.** JB and XL designed the research; RS provided oversight and guidance for using
474 both VLIDORT and PCA-based VLIDORT; KY developed the LUT creation and interpolation scheme;
475 XL contributed to analyzing ozone profile retrievals with different forward model approaches; JB
476 conducted the research and wrote the paper; CN, CM, GA, and KC contributed to the analysis and
477 writing; CM and GA contributed to managing the computational resources.

478 **Competing interests.** The authors declare that they have no conflicts of interest.

479 **Data availability.** OMI Level1b radiance datasets are available at
480 https://aura.gesdisc.eosdis.nasa.gov/data/Aura_OMI_Level1/ (last access: 31 AUG 2020). The LUT
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585 **Table 1. The PCA-RT configuration optimized over the UV spectral range 265-360 nm. The**
 586 **optical depth of the total gas column (Γ_g defined in eq. 9) is used to set the criteria for the spectral**
 587 **binning; for example, one or more bins are created at intervals ($\Delta\Gamma_g$) in the range Γ_g^{min} to Γ_g^{max} .**
 588 **For each bin, the optical states are expanded in terms of the first few number of EOFs (nEOF).**

265-340 nm							
SZA or VZA < 70°				SZA or VZA ≥ 70°			
List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF	List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF
1	∞ to -1.7	2	1	1	∞ to -1.5	2.0	1
2	-1.7 to -1.2	0.5	1	2	-1.5 to -0.7	1.2	1
3	-1.2 to 0.0	0.4	1	3	-0.7 to 0.4	0.35	1
4	0.0 to 0.5	0.5	1	4	0.4 to 0.7	0.3	1
5	0.5 to 3.5	0.6	2	5	0.7 to 2.5	0.6	3
6	3.5 to 4.5	1.0	2	6	2.5 to 3.5	1.0	3
7	4.5 to ∞	2.0	2	7	3.5 to 4.5	1.0	2
8				8	4.5 to ∞	2.0	2
340-350 nm				350-360 nm			
List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF	List	$\Gamma_g^{lower}, \Gamma_g^{upper}$	$\Delta\Gamma_g$	nEOF
1	∞ to ∞	1.0	2	1	∞ to ∞	∞	4

589 **Table 2. LUT parameter specification. Note that the relative azimuth dependence is taken**
 590 **into account explicitly through the Fourier coefficients of path radiance (Table 3) and the**
 591 **surface albedo dependence is taken into account by the planetary problem.**
 592

Parameter	Symbol	N	Grid Values
Ozone Profile ⁺	O_3P	22	<ul style="list-style-type: none"> Low-latitude (30°S-30°N) L200,L250,L300,L350 Mid-latitude (30°-60°N/S) M200,M250,M300,M350, M400,M450, M500,M550 High-latitude (60°-90°N/S) H100,H150,H200,H250,H300,H350, H400,H450, H500,H550
Wavelength	λ	92	265-345 nm
Solar Zenith Angle	θ_o	12	0, 16, 31, 44, 55, 64, 71, 76.5, 80.5, 83.5, 86, 88°
Viewing Zenith Angle	θ	8	0, 15, 30, 43, 53, 61, 67, 72°
Surface albedo	A_s	1	0.0
Surface pressure	P_s	12	100, 150, 200, 300, 400, 500, 600, 700, 800, 900, 1013.25, 1050 hPa

593 ⁺Total ozone-based ozone profiles for three latitude regimes. The grid values represent the



594 amount of total ozone (DU).

595

596 **Table 3. LUT variable specification**

Variable	Dimensions	Variable	Dimensions
I_o^a	$n\lambda, n\theta, n\theta_o, nP_s$	$dI_o/d\tau$	$n\lambda, n\theta, n\theta_o, nz, nP_s$
Z_1^a	$n\lambda, n\theta, n\theta_o, nP_s$	$dZ_1/d\tau$	$n\lambda, n\theta, n\theta_o, nz, nP_s$
Z_2^a	$n\lambda, n\theta, n\theta_o, nP_s$	$dZ_2/d\tau$	$n\lambda, n\theta, n\theta_o, nz, nP_s$
T^b	$n\lambda, n\theta, n\theta_o, nP_s$	$dT/d\tau$	$n\lambda, n\theta, n\theta_o, nz, nP_s$
S_b^c	$n\lambda, nP_s$	$dS_b/d\tau$	$n\lambda, nz, nP_s$
τ^d	$n\lambda, nz^+$		

597 ^a Fourier coefficients of path radiance with respect to relative azimuth angle

598 ^b total transmission of the atmosphere

599 ^c spherical albedo of the atmosphere

600 ^d total gas absorption optical depth profile

601 ⁺ Number of atmospheric layers

602

603

604 **Table 4. List of configurations used in evaluating the different forward model**
 605 **calculations for OMI ozone profile retrievals. The reference, VLIDORT, and PCA-RT**
 606 **models are abbreviated as Ref, VLD, and PCA, respectively.**

RT models	US SI (nm) ^a	HR SI (nm) ^b	Nstream ^c	Nlayer ^d	Polarization ^e	RT corr ^f
Ref	0.3(<305nm) 0.1(≥305 nm)	0.03	12	72	True	False
VLD ⁰	1.0(<295nm) 0.4(between) 0.6(≥310 nm)	0.05	8	24	False	On-line polcorr
VLD ¹	1.0(<295nm) 0.4(between) 0.6(≥310 nm)	0.05	12	72	True	False
VLD ²	0.3(<305nm) 0.1(≥305 nm)	0.03	8	72	True	False
VLD ³	0.3(<305nm) 0.1(≥305 nm)	0.03	12	72	False	On-line polcorr
VLD ⁴	0.3(<305nm) 0.1(≥305 nm)	0.03	12	24	True	False
PCA ⁰	0.3(<305nm) 0.1(≥305 nm)	0.03	4	24	False	LUT
PCA ¹	0.3(<305nm) 0.1(≥305 nm)	0.03	12	72	True	False

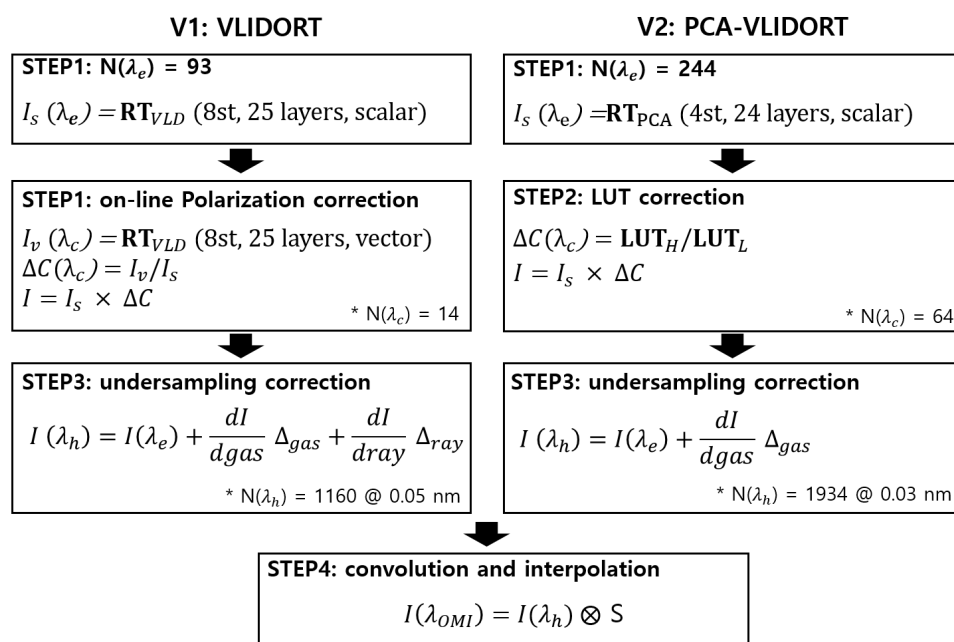
607 ^a Under-sampled (US) spectral intervals (nm) used to define wavelengths at which RT is actually executed.

608 ^b High-resolution (HR) spectral intervals (nm) used to define wavelengths where under-sampled simulations are
 609 interpolated before spectral convolution.



610 ^c the number of discrete ordinates in the full polar space; ^dNumber of atmospheric layers
 611 ^eRT model is run in the vector (scalar) mode if polarization is true (false).
 612 ^fOn-line polarization correction as described in Section 2, which is originally developed from Liu et al. (2010).
 613 LUT-based correction introduced in Section 3.3, which is developed in this work to account for RT approximation
 614 errors due to neglecting polarization as well as using 4 streams and 24 layers.

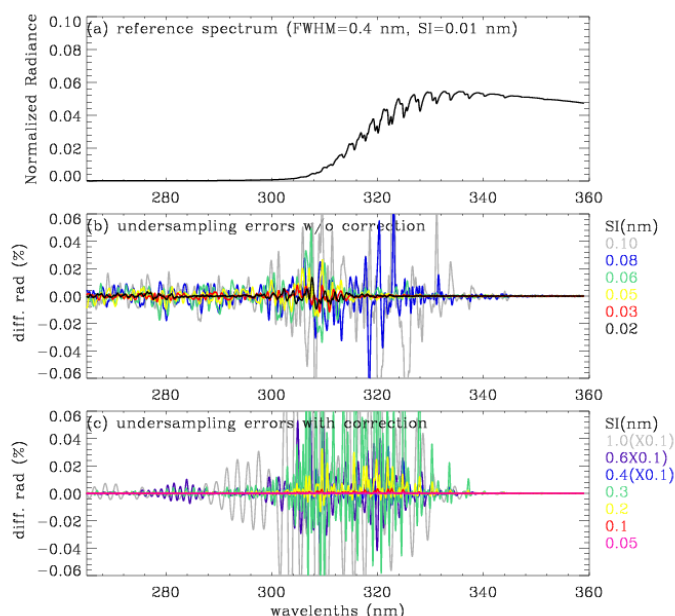
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616

617 **Fig.1.** Schematic flowcharts of VLIDORT (v1) and PCA-VLIDORT (v2) based forward
 618 models, respectively. Note that VLIDORT was used in the generation of the OMPROFOZ v1
 619 dataset, while PCA-VLIDORT is in preparation for OMPROFOZ v2 production. The number
 620 of wavelengths used in each process is denoted as $N(\lambda)$ when the spectral window 270-330
 621 nm is applied. λ_e represents the wavelength grids used for RT calculation, while λ_c and λ_h
 622 are grids used in RT approximation correction and undersampling correction, respectively.
 623 See text for definition of other variables.

624



625

626 **Fig.2.** (a) Reference (truth) normalized radiance spectrum simulated at the spectral intervals
627 (SIs) of 0.01 nm in 265-360 nm (solar zenith angle = 65°, viewing zenith angle = 30°, relative
628 azimuth angle = 120°), which is used for evaluating the simulations in Figs. (b) and (c). (b)
629 Impact of under-sampling on the simulation. (c) is similar to (b), but now the under-sampling
630 correction has been applied. In Fig 2(c), the under-sampling errors are divided by 10 at SIs ≥
631 0.4 nm. Note that individual radiances simulated at different SIs are interpolated to 0.01 nm
632 and then convolved with the Gaussian function (FWHM: 0.4 nm) which represents the OMI
633 instrument spectral response function.

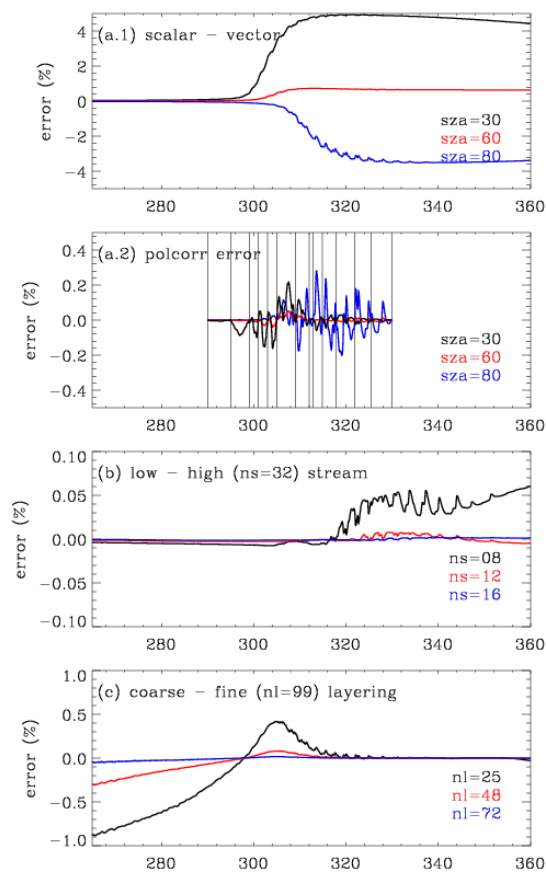
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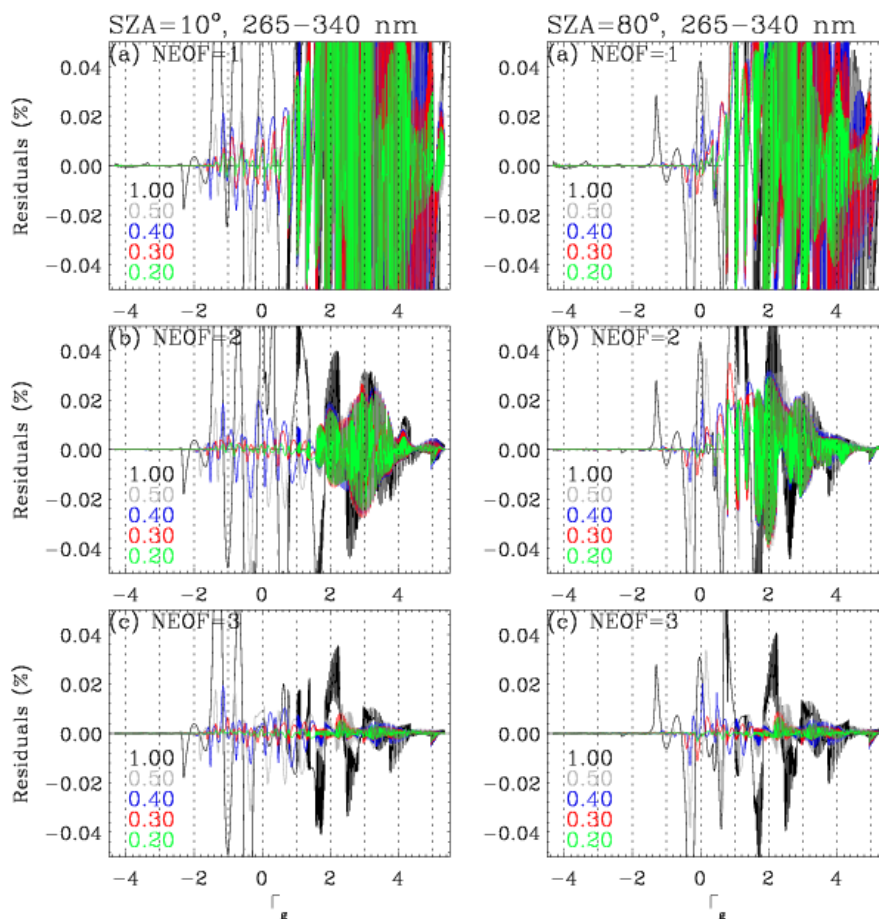


639

640 **Fig. 3.** Errors of the radiance simulation due to the RT approximation used in v1, arising from (a.1) neglecting the
641 polarization effect for different solar zenith angles (sza), (a.2) polarization correction errors, (b) using a low
642 number of streams (ns), and (c) using a coarse number of vertical grids (nl). Note that vertical lines in Fig. 3.a.2
643 indicate wavelengths used in deriving the on-line polarization correction spectrum.

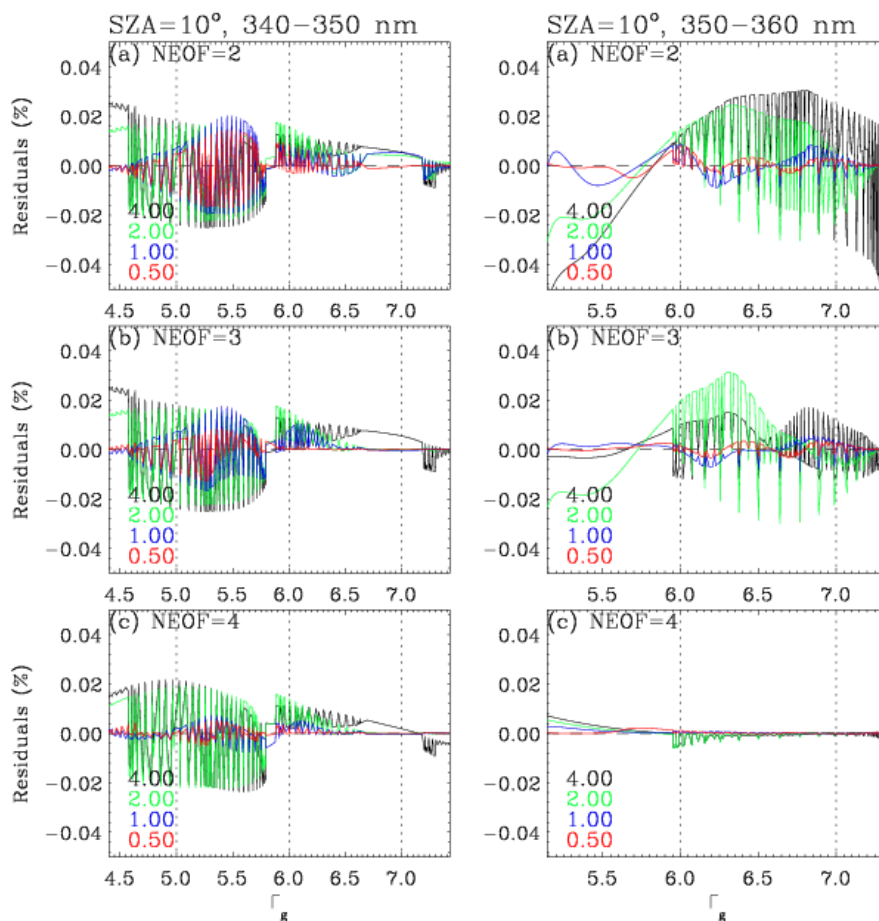
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646

647 **Fig. 4.** Residuals (%) of the PCA-RT radiance in the wavelength range 265-340 nm compared
648 to the exact-RT calculations, for different binning steps (different colors) and number of EOFs
649 (a, b, c). Results are plotted as a function of Γ_g (logarithm of the total gas optical depth), for
650 solar zenith angles (SZAs) of (left) 10° and (right) 80°.



651

652 **Fig. 5.** Same as Fig. 4, but for different windows, (left) 340–350 nm and (right) 350–360 nm, respectively.

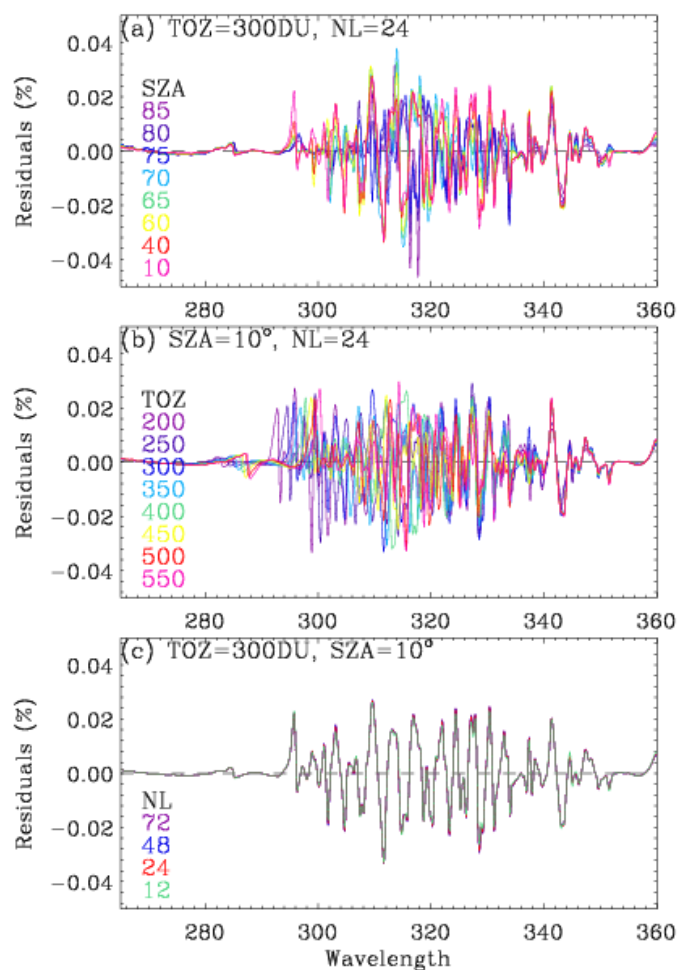
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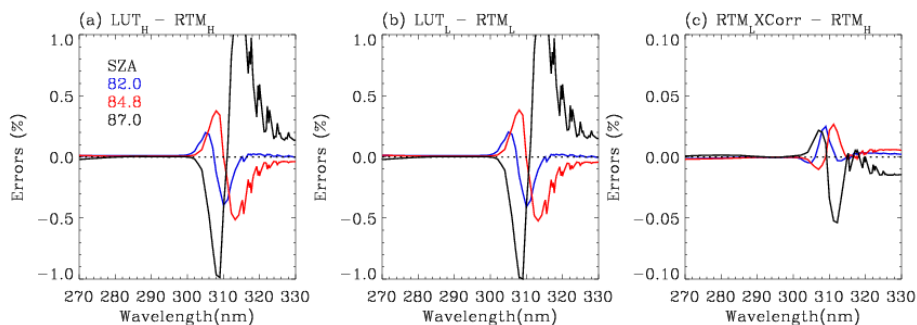
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658

659 **Fig. 6.** Residuals (%) of the PCA-RT radiances with the binning scheme given in Table 1, for
660 various sets of (a) solar zenith angles, (b) ozone profiles with different total ozone columns
661 (TOZs), and (c) number of atmospheric layers.

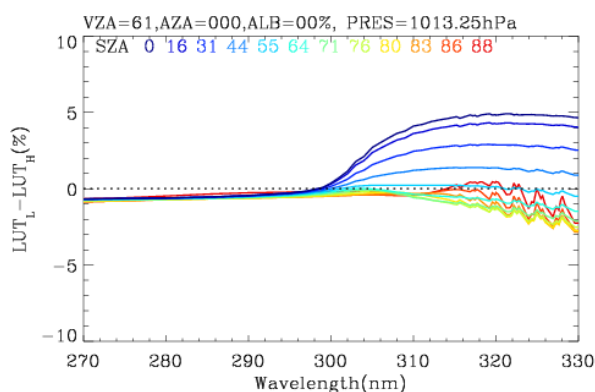
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663

664 **Fig. 7.** Evaluation of simulations with respect to extreme SZAs at VZA = 61°, AZA = 0°,
665 ALB=0 %, and surface pressure = 1013.25 hPa. LUT and RTM represent LUT and on-line
666 radiative transfer model (RTM) based calculations, respectively, with the subscripts H and L
667 indicating high and low accuracy configurations. RTM_L X Corr radiances are simulated using
668 on-line RTM with low accuracy configuration, but corrected using LUT_H/LUT_L.

669

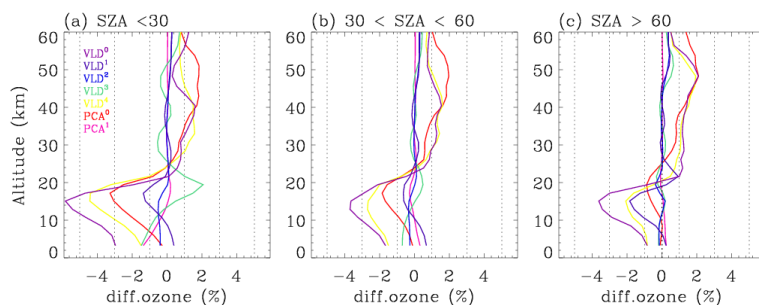


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Fig. 8. Example of LUT-based correction spectrum.

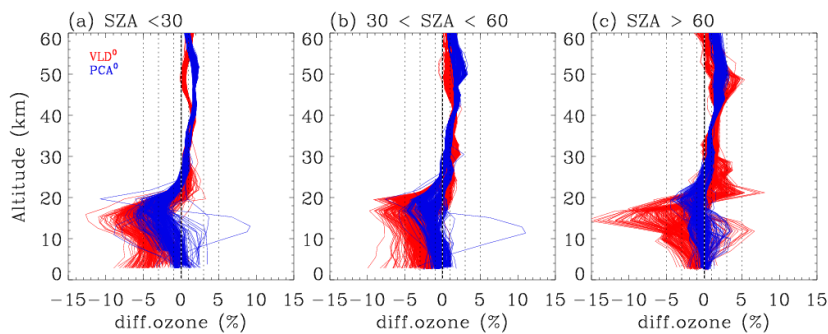


673

674 **Fig. 9.** Mean biases of ozone profile retrievals with different configurations compared to those
675 with the reference configuration. Each configuration is given in Table 4.

676

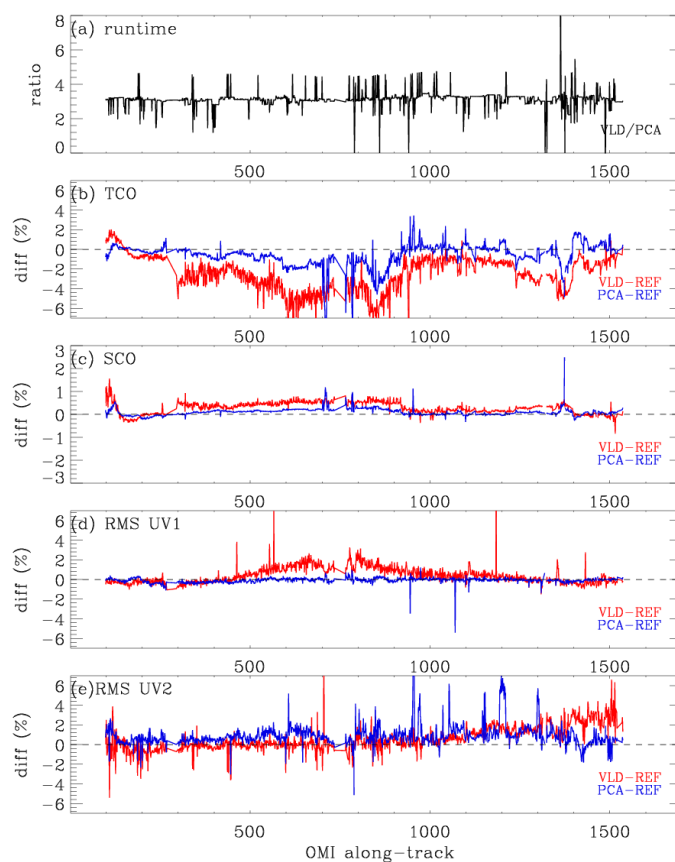
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678

679 **Fig. 10.** Same as Fig.9, but for individual differences. VLD⁰ and PCA⁰ represent v1 and v2 forward model
680 configurations, respectively.

681



682

683 **Fig. 11.** Same as Fig. 10, but for (a) runtime, (b) tropospheric column ozone (TCO), (c)
684 stratospheric column ozone (SCO), and (d) UV1 (270-310 nm)/(e) UV2 (310-330 nm) fitting
685 residuals, along with the OMI along-track position (1-1644) at nadir cross-track. Note that the
686 fitting residuals are estimated as root mean square (RMS) errors for differences between
687 measured and simulated spectra relative to the measurement error. VLD and PCA represent
688 v1 and v2 forward model configurations, respectively.

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