Response to reviewer 1:

Thank you for your letter and valuable comments and questions concerning our manuscript entitled "Error analyses of a multistatic meteor radar system to obtain a 3-dimensional spatial resolution distribution" [MS no. amt-2020-353]. These comments are all valuable and very helpful for revising and improving our manuscript, as well as the important guiding significance to our researches. We have found a mistake in equation 10 and corrected it. We have studied the valuable comments from you carefully, and tried our best to revise the manuscript. These changes in the revise manuscript have been marked in the track changes version manuscript, as well as the point to point responses have listed as following:

1) The reviewer had difficulties to follow some part of the error propagation due to the various introduced coordinate systems denoted as prime without prime and so forth. This made the manuscript very hard to read and one gets easily lost.

Response: We apologize for our unreasonable article structure and denominations to make you confusion. Inspired by your comments, we had removed all denoted primes in coordinate system in revised manuscript. This makes our expression in manuscript more concise and readable. And the three introduced coordinate system can be well distinguished by only using subscripts.

Following your comments, we had carefully rearranged our article structure and reorganized our languages to try to make our article easy to read. We do those changes in revised version:

1. In original manuscript, we established left-hand coordinate systems with right-hand screw rule which is not idiomatical for most readers. Thus, in revised version, we change the coordinate systems to idiomatically right-hand coordinate systems with right-hand screw rule. We hope this change may increase the readability of our manuscript.
2. Section 2 is the main body of this manuscript and we divide it into four parts to make its structure more clearly. We add a brief conclusion of the analytical

process in the end of section 2 (line 275-284). And we add a flow chart to descript our analytical process (Figure 5(a)). In Figure 5(a), the variables and equations in section 2 are all included. We hope that reading section 2 while seeing Figure 5(a) will help readers understand the tedious analytical process.

3. some units or quantitative expressions examples: use the specific angle and distance values to help readers understand the parameters settings in our program (line 282-293); use the specific location error values and resolution values to explain their relationships (line 298-301); use specific rotation angle values to explain the slant of the receiver antennas plane (line 334).

4. Apart from correct the grammar and spelling mistakes you suggested in minor concern, we reread our manuscript to carefully check the spelling, grammar and wording. For example, "traditional meteor radars" is corrected as "classic meteor radars"; "wind retrieving" as "wind retrievals"; "AoA" and "AoAs" are unified as "AoAs"; "clockwise rotation is" as "clockwise rotation satisfies " et.al.

5. We have found that equation 10 in original version is not correct. We have corrected it and reorganized the relative content in section 2 (line 237-268), figures and code et.al. In corrected version, except there is no "good horizontal resolution area split when baseline is long" , other results are the same.

If you have any confusion, comments or suggestions in revised manuscript, don't hesitate to feedback to us. And we would very pleasure to revise our manuscript and try to make our manuscript better. Thanks for your precious comment.

2） Although there are some schematics outlining the coordinate systems the reviewer was not able to follow what actually is shown in Figure 5-7. The reviewer was not able to understand the plots reading the figure caption or the corresponding passage in the text. So please describe the color bars in the text or in the caption what they actually mean.

Response: We apologize for Figure 5-7's poor plots to make you having difficulties in reading the manuscript. Following your suggestion, we carefully replotted original manuscript's Figure 5-7 and the new figures are Figure 6-8 in revised manuscript (because we add an algorithm flow chart and is shown in Figure 5 in new manuscript , the results figures are start from Figure 6). However, due to our rearrange of original manuscript, the new figures do not correspond to original one to one. In original version, we only label the axes with coordinate axes, which is not intuitionistic. And in revised version we label the axes with noun of locality: altitude, east, north and horizontal distance. We hope this change would make readers understand the figures at a glance. In original version, there lack figure captions or corresponding text which makes the figures hard to understand. Therefore, in new version, we add more descriptions in figure captions . Because the deducing process in the section 2 is tedious, we try to provide information as much as possible in figure captions. Moreover, in Figure 6-8 we add subplots titles and colorbar unit (km) to help understand the pictures. For the reason that E_2 related resolution is very smaller comparing with E_1 related and total resolution, we change the colorbar of E_2 related to make this difference visible at a glance, which is not shown well in original one. Thanks very much for your comments and suggestions about our figures. If you have any other confusion, comments or suggestions about revised figures, don't hesitate to feedback to us. And we would very pleasure to carefully revise our manuscript and try to make our pictures more intuitional.

3) The reviewer understands that the authors intended to keep things as general as possible, but some units or quantitative expressions are helpful.

Response: Thanks very much for this very precious suggestion. We apologize for our negligence of taking some specific examples to explain some deducing processes or results that are hard to descript or understand. Using some units or quantitative expressions are a very helpful way to increase readability. Following your suggestion, we add some units or quantitative expressions examples: use the specific angle and distance values to help readers understand the parameters settings in our program (line 282-293); use the specific location error values and resolution values to explain their relationships (line 298-301); use specific rotation angle values to explain the slant of the receiver antennas plane (line 334). If you had any other suggestions about adding some specific quantitative expressions, we would very pleasure to revise our manuscript again.

4) In particular, section 2 after line 140 is very hard to read and to follow. This is also partly the case as the Figures are only found at the end of manuscript and one has always to scroll forth and back.

Response: We apologize for our poor structure and presentation in section 2. Following your suggestion, we try our best to rearrange and revise section 2. Section 2 is the main body of this manuscript and we divide it into four parts to make it structure more clearly. We add a brief conclusion of the analytical process in the end of section 2 (line 275-284). We add a flow chart to descript our analytical process (Figure 5(a)). In Figure 5(a), the variables and equations in section 2 are all included. We hope that reading section 2 while seeing Figure 5(a) will help readers understand the tedious analytical process.

5) Another important point that should be discussed is that the algebraic errors are just one source that plays a role. The authors should mention in the discussion that there are other error sources as well, originating from the scattering itself or from the experimental set up due to a potential mutual antenna coupling or other obstacles in the surrounding. The later one introduces further biases in the measurements as the angle of arrivals can be significantly altered. Usually, HFSS simulation are required to investigate actually the limits of trustworthiness for the interferometry.

Response: Thanks very much for your suggestion. Inspired by your comments, we mention and discuss the issues of other error sources (line 348-353). The antenna design and site selection are important for meteor radars and HFSS is a powerful tool to study those issues. We only discuss the mathematic error propagation starting from phase difference measuring errors and put emphasis on multistatic configurations. We try to induce things in general, thus the discussion of some specific case of the interferometry maybe beyond the scope of our text. However, if substitute the phase difference measuring errors in our text (set as constant) to values in specific case, our method will still work(line 338-347). There are many detailed works in discuss the interferometry and their AoAs measuring errors in a more specific case, such as (Kang, 2008; Vaudrin et al.,Younger and Reid, 2017). These results of AoAs error distribution can be taken into our method to study a more specific case.

6) Furthermore, the authors should mention in the discussion that the scattering occurs not really at a singular point. The radio wave is bounced back from at least a few Fresnel zones of several kilometer length along the trajectory, which is actually most relevant for the altitude resolution as the radar signal is scattered from an extended volume (1D) and, thus, probes a volume.

Response: Thanks very much for your suggestions. Following your suggestions, we had carefully thought this issue. The fact that radio wave scattered from a few Fresnel zones around specular point will cause an antenna pair's phase difference deviation from the theoretical expectant value. The theoretical expectant value will resolve a AoAs pointing to specular point. This phase difference deviation is one error source of phase difference measuring errors and is included in phase difference measuring errors ($\delta(\Delta\Psi_1)$ and $\delta(\Delta\Psi_2)$). However, this issue is not clearly point out in our manuscript. Thus, we mention this issue briefly in new version (185-190, 348-350 and Figure 1-2's caption). The details of this issue can be seen in the **RC1 supplement.**

7) It is also worth to mention and discuss the issues of the sampling volume in the context of the trustworthiness of the interferometry. The schematic in Figure 2 provides a nice example of a multistatic geometry resulting in a less good measurement response compared to a monostatic radar of the same measurement volume, although the set up appears to have a multistatic geometry. The measurement response provides a measure of how well a bragg vector can be inverted to still derive reliable wind speeds (u,v,w). Ideally, all three variables can be estimated with similar measurement response, otherwise biases in one of the wind components are not avoidable. The receiver array in Figure 2 defines the sampling volume. Meteors below a certain elevation angle have to be excluded from the analysis due to the mutual antenna coupling or other ground obstacles causing issues in the interferometry.

Response: Very kind of you for your comments. After carefully thinking about your comments, your comments inspired us to add an important discussion about our results (354-363) to mention the issues of sampling volume and measurement response briefly. The measurement response is one of the things that affect the accuracy of Doppler shift. The location error, Doppler shift errors and other issues will determine the accuracy of the wind retrievals. We intend to discuss this in a future work. Following your suggestion, we add two black lines to represent the 30° elevation angle limit in revised figures.

8) Further, it is obvious that the angular diversity of the three links inside the remaining sampling volume is less diverse (all are located in a certain sector relative to the receiver) than a monostatic radar and could systematic bias the wind retrievals. This is the nature of the forward scatter ellipse. As all three forward scatter ellipses have the receiver site in the one of their foci points and the bragg vectors always points towards a point along the distance vector between Rx and Tx. It is further obvious that the longer the total path Rt+Rx becomes the less spatial diversity these vectors have, or with other words – all three links start to see the same geometry as it would

be the case for a monostatic radar. However, building three receiver sites and using one transmitter would increase the sampling volume and if well-distributed compensates some of this sampling effect on the wind analysis (at least partially), but still has a less good measurement response compared to a monostatic system. I suggest that they add in Figure 5-8 a line or shading area indicating the angular limit of the receiver/transmitter array by using a truncation elevation angle of about maybe 30◦. The actual limit depends on the array set up.

Response: Thanks very much for your comments. Following your comments, we briefly mention the issue of angular diversity (364-369). However, the impact of angular diversity of Bragg vector on wind retrievals also exceed the topic of our manuscript. We intend to discuss this in a future work. Following your suggestion, we add two black lines to represent the 30° elevation angle limit in revised figures and also mention this issue (line 319-320)

Reference

Ceplecha, Z., Borovička, J., Elford, W. G., ReVelle, D. O., Hawkes, R. L., Porubčan, V., and Šimek, M.: Meteor Phenomena and Bodies, Space Science Reviews, 84, 327-471, 10.1023/A:1005069928850, 1998.

Holdsworth, D. A., Reid, I. M., and Cervera, M. A.: Buckland Park all-sky interferometric meteor radar, Radio Science, 39, https://doi.org/10.1029/2003RS003014, 2004.

Hocking, W. K.: Spatial distribution of errors associated with multistatic meteor radar, Earth, Planets and Space, 70, 93, 10.1186/s40623-018-0860-2, 2018.

Kang, C.: Meteor radar signal processing and error analysis, 2008.

Vaudrin, C. V., Palo, S. E., and Chau, J. L.: Complex Plane Specular Meteor Radar Interferometry, Radio Science, 53, 112-128, 10.1002/2017rs006317, 2018.

Younger, J. P., and Reid, I. M.: Interferometer angle-of-arrival determination using precalculated phases, Radio Science, 52, 1058-1066, 10.1002/2017rs006284, 2017.

RC1 Supplement

1. The issue of the radio wave scattered from Fresnel zones

Specular meteor radars (SMR) usually utilize undersense meteor trails. (Ceplecha et al., 1998) discussed radio wave backscatter process with meteors passing though the SMR . In short, for idealized case that ignoring diffusion of meteor trail and assuming that secondary radiative and absorptive effects can be neglected, the return signal received by one antenna can be expressed as:

$$E_{R1}(x_t) = E_0 e^{i(\omega t - 2kR_0)} \int_{-\infty}^{x_t} e^{i(-\pi x^2/2)} dx \quad (1)$$

See figure 1, R_0 is the distance from this antenna 1 to the specular point, or the orthogonal point (t_0-point hereafter) in other words. $x = \sqrt{\frac{4}{\lambda R_0}} S$ and $k = \frac{2\pi}{\lambda}$. If origin time is when meteor arrives at t_0 point, it will get that $x_t = 2(\lambda R_0)^{-\frac{1}{2}} V t$ (V is meteor velocity). $\int_{-\infty}^{x_t} e^{i(-\pi x^2/2)} dx$ is a complex Fresnel integral and can be expressed as $C - iS$,where:

$$C(x_t) = \int_{-\infty}^{x_t} \cos(\pi x^2/2) dx$$

$$S(x_t) = \int_{-\infty}^{x_t} \sin(\pi x^2/2) dx \quad (2)$$

Thus, apart from ideal specular reflection signal term "$e^{i(\omega t - 2kR_0)}$", there is a complex Fresnel modulation term $C - iS$. This modulation will cause amplitude occasion ($\sqrt{C^2 + S^2}$) and phase variation ($\phi_{add} = \arctan\frac{S}{C}$) in the period a meteor passing through. See figure 2, curve A represent the process based on eq. (1) and curve B, C, D show the effect of including an increasing degree of diffusion of the trail.

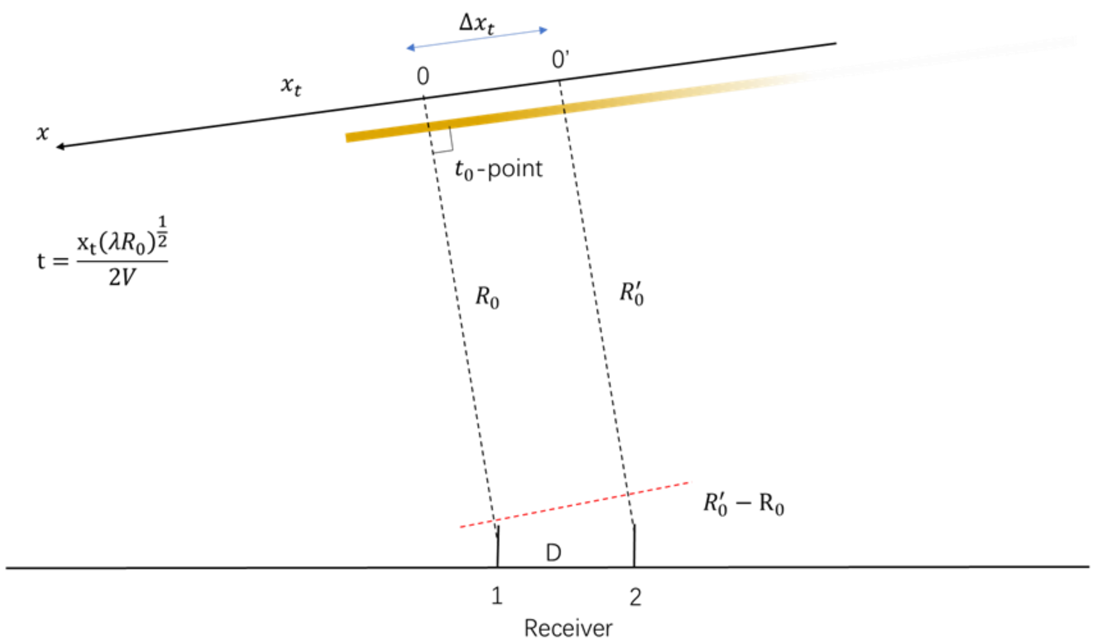

Figure 1

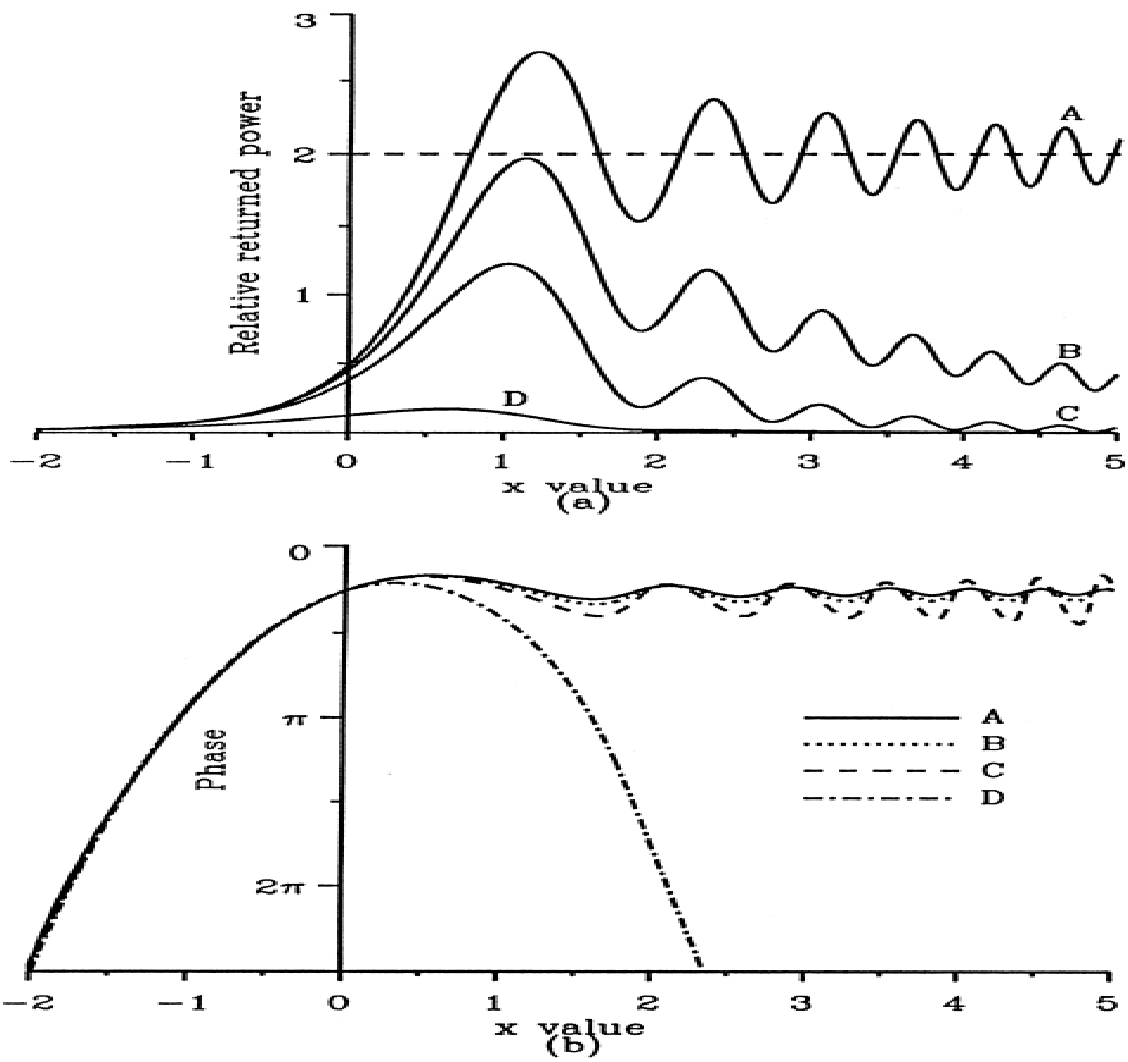

Figure 2(pick from (Ceplecha et al., 1998))

Similarly, the return signal received by antenna 2 is

$$\mathrm{E}_{\mathrm{R2}}(x_t) = E_0'\mathrm{e}^{i(\omega t - kR_0 - kR_0')}\int_{-\infty}^{x_t+\Delta x_t} e^{i(-\pi x^2/2)}dx \tag{3}$$

See eq. (1) and (3), the phase difference between two antennas is from second term and third term in right side of the equations. The phase difference caused by second term is $k(R_0' - R_0)$ which is the theoretical basis of interferometer to obtain AoAs. And this phase difference will solve an AoAs pointing to specular point. However, the third term, which is related to the radio wave scattered from a few Fresnel zone, will cause additional phase difference between two antennas. This additional phase difference is caused by a delay integer length Δx_t between two antennas. For:

$$\Delta x_t = \sqrt{\frac{4}{\lambda R_0}}\ Dsin\alpha \tag{4}$$

Take a 30MHz meteor radar for example, since $Dsin\alpha \leq 4.5\lambda$ and R_0 is about 100km, Δx_t will not exceed 0.1. The major concern is how big this additional phase difference is. The change rate of the Fresnel modulation phase Φ, i.e. the derivative function of $\arctan(\frac{\mathrm{S}}{\mathrm{C}})$, will determine the magnitude of this additional phase difference. The Phase changes dramatically in pre-t_0 period and in small concussion after t_0. The additional phase difference is $\Delta x_t \frac{\mathrm{d}\Phi}{\mathrm{dx_t}}$ and it's no more than 25 degree around $x_t = -1$ (figure 3). Furthermore, a meteor radar system generally set an amplitude threshold to judge a meteor event and thus IQ analyze is nearly in post-t_0 period which additional phase is very small.

Multistatic meteor radars utilizing the forward scatter is a more general case. The effect of Fresnel zone scatter on measuring errors is nearly the same as monostatic case. See figure 4, t_0-point is the point where the radio wave path is shortest. Thus t_0-point is also the specular point where the angle of incidence equals the angle of reflection. T_x' is the symmetry point of T_x about meteor trail (axis-x). For a scatter point x_i alongside the trail, the radio wave propagation path length is the sum of the length from T_x' to x_i and from x_i to an antenna. Therefore t_0 point is the intersection of the trail path and the line from T_x' to an antenna, which represents shortest path length. t_0 point is also specified as the origin of axis-x (or time) . For a scatter point x_i which is

S away from t_0, the radio wave propagation path length can be expressed as:

$$R = \sqrt{R_i^2 + S^2 - 2R_i S cos(90° + \theta)} + \sqrt{R_s^2 + S^2 - 2R_s S cos(90° - \theta)} \quad (5)$$

R_i and R_s are specular reflection path length for incident and reflection wave. θ is the incident angle (or reflection angle). Eq. (5) can be expanded to second order because S is very small compared to R_i and R_s. Thus, R can be expressed as:

$$R = R_i + R_s + \left(\cos^2\theta\,(\frac{1}{R_i} + \frac{1}{R_s})\right) S^2 \quad (6)$$

$R_i + R_s$ correspond to $2R_0$ in monostatic case which represents the shortest path for the radio wave. If substitute $x = \sqrt{\frac{4\cos^2\theta(R_i+R_s)}{\lambda R_i R_s}} S$, other process is the same as monostatic case.

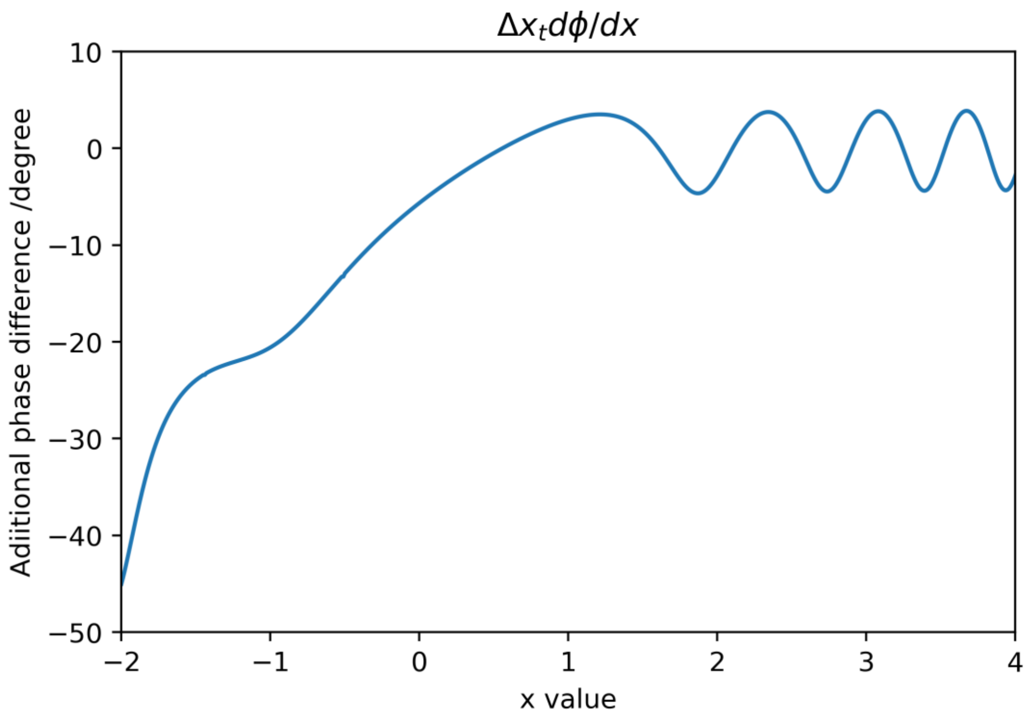

Figure 3

It worth noting that a meteor trail, transmitter and receiver are not always coplanar and a meteor trail and different receiver antenna pairs are not always coplanar too. We only give a semiquantitative analysis.

Additional phase difference and other measuring errors constitute the phase difference measuring errors ($\delta(\Delta\Psi_1)$ and $\delta(\Delta\Psi_2)$). Different radar system set different $\delta(\Delta\Psi_1)$ and $\delta(\Delta\Psi_2)$. For a receiver in Jones configuration which use at least four pairs of antennas to get AoAs, due to the phase difference measuring errors in those antennas pairs, the system should fit those four measured phase differences to get an expectant AoAs. If the RMS phase difference between the fitted and CCF phase exceeds a preselected threshold (default 20 degree) for any receiver pair the candidate is rejected (Holdsworth et al., 2004).In our program, the default value of $\delta(\Delta\Psi_1)$ and $\delta(\Delta\Psi_1)$ is 35 degree and our error propagation starts from this values. That is to say, the error that caused by the radio wave scatter from a few Fresnel zones of several kilometer length along the trajectory is included in the phase difference measuring errors ($\delta(\Delta\Psi_1)$ and $\delta(\Delta\Psi_2)$) in our analytical method .

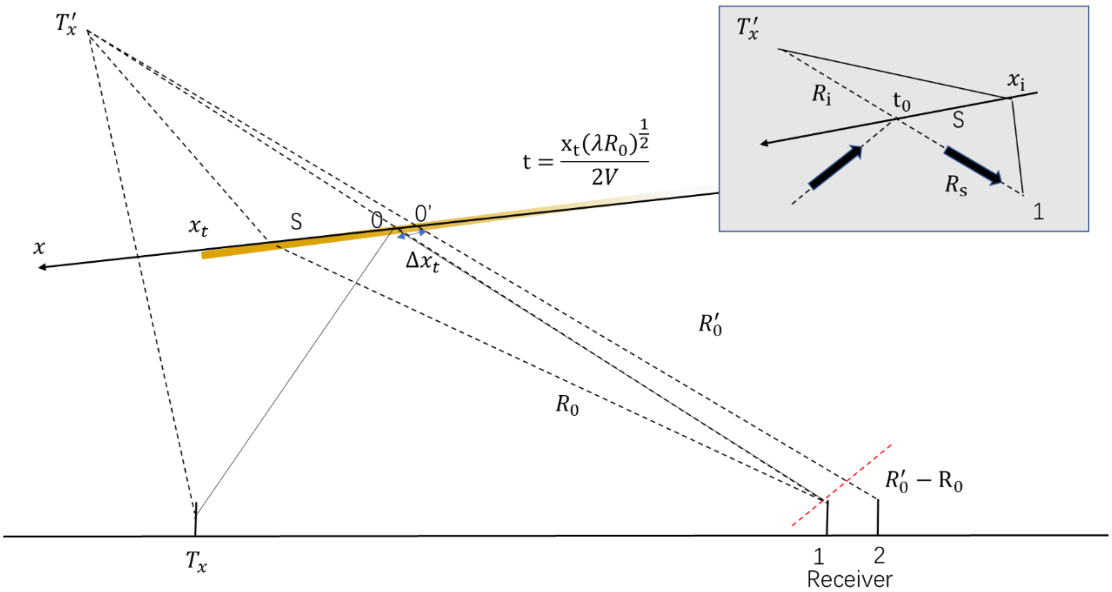

Figure 4