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Statistics of a Coherent Lidar Signal in a Turbulent Atmosphere

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Abstract—Absolute and conditional statistical properties of a pulse coherent Doppler lidar signal in a turbulent atmosphere are studied. Upon coherent receiving of optical fields scattered by a large number of particles, the lidar signal is shown to be a nonstationary non-Gaussian random process with Gaussian conditional statistical characteristics. The appearance of non-Gaussian properties of the signal is caused by correlation of turbulent fluctuations of the wind velocity field within the scattering volume. For the considered signal model, which corresponds to the single scattering approximation and is a sum of a large number of random variables, the central limit theorem is found to be untrue due to the statistical dependence of particles' positions in a turbulent atmosphere. The results of numerical calculations show that, for a homogeneous and isotropic turbulence, the behavior of the signal statistics significantly depends on the size of the scattering volume and on the state of atmospheric turbulence. A Gaussian statistics is observed at small heights; with an increase in height, the non-Gaussian component becomes considerable in fluctuations of the lidar signal.

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INTRODUCTION

Doppler lidar remote sensing of the wind velocity field is one of the dynamically developing branches of modern optics both in theory and in experiment [1]. The significance of these investigations is explained by the possibility of monitoring the wind velocity field by optical methods in local, regional, and global scales in the interests of meteorology, ecology, and studying climatic changes. At present, one topical problem in the branch of Doppler lidar remote sensing is the problem of studying the signal statistic upon coherent detection in a turbulent atmosphere [1].

The necessity of detailed studying statistical properties of the signal in a turbulent atmosphere is caused by the following facts. First, the signal statistics completely determines the behavior of the Doppler shift estimate as a random variable. Therefore, the analysis of statistical properties of the Doppler lidar signal in a turbulent atmosphere is the first and appropriate step in the investigations; it is necessary for correct formulation of the problem and for choosing the investigation methods.

Second, the aerosol component of the pulse Doppler lidar signal is a sum of a large number of random values, which depend on the position of particles in the scattering volume. It is well known that turbulence leads to chaos; therefore, it seems logical to assume that particles are distributed in the scattering volume uniformly and statistically independent from each other. Such logic of reasoning gives rise to a desire to apply the central limit theorem and to consider the signal statistics as Gaussian. At the same time, the scattered light statistics is non-Gaussian in turbulent flows [2, 3]. In connection with this, there arises a problem regarding the influence of non-Gaussian properties on the performance of a pulse Doppler lidar in a turbulent atmosphere.

Third, spectroscopy of intensity fluctuations [2, 3] and Doppler lidar remote sensing are closely connected with each other as two branches of optics but radically differ in problem statements. The problem of spectroscopy of intensity fluctuations restricts itself to the study of absolute statistical characteristics of the scattered field [2, 3]. The problem of Doppler lidar remote sensing is interesting in terms of frequency shift, not intensity; from the theoretical viewpoint, it requires performing a more detailed statistical analysis of the signal. It was shown in [4] that the choice of absolute statistical characteristics as a zero approximation in the analysis of the Doppler shift estimate in a turbulent atmosphere leads to the appearance of secular terms, i.e., to a nonuniform approximation of the perturbation theory series. Consistent application of renormalization methods shows that uniform approximation is achieved in the case wherein the zero approximation depends on the random wind velocity field. This means that problems of Doppler lidar remote sensing need a more detailed study, namely, studying not only absolute, but also conditional, statistical characteristics.

In the first part of this work, a model of the aerosol component of the pulse Doppler lidar signal is described in the single scattering approximation. The second, third, and fourth parts of the work present the results of studying absolute and conditional statistical characteristics of the aerosol component of the pulse

Doppler lidar signal. Problems connected with the central limit theorem are presented in the fifth part of the work. In the end, the main conclusions are presented.

PULSE DOPPLER LIDAR SIGNAL

It is well known [5–7] that the Doppler lidar signal $j(t)$ can be represented as a sum of two statistically independent parts in the form $j(t) = j_s(t) + j_n(t)$, where $j_s(t)$ and $j_n(t)$ are the aerosol and noise components of the signal. Noise of the receiver of the Doppler lidar $j_n(t)$ is assumed to be Gaussian white noise; therefore, the statistics of $j(t)$ is completely determined by aerosol component $j_s(t)$ of the signal.

In the single scattering approximation [7, 8], the aerosol component of the signal can be written in the form

$$j_s(t) = \sum_{m=1}^{N_p} A_m P(t, \mathbf{r}_m) e^{2ikR_m(t)}, \tag{1}$$

$$R_m(t) = R_m + u_r(R_m, \phi, \theta)(t - t_0), \tag{2}$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, $R_m(t) = \mathbf{r}_m(t)\mathbf{n}$ is the projection of the vector $\mathbf{r}_m(t)$ on the sensing direction $\mathbf{n} = \{n_1, n_2, n_3\}$, $R_m = R_m(t_0)$, $\mathbf{r}_m(t)$ is the coordinate of the mth particle, $u_r(R_m, \phi, \theta) = \mathbf{u}(R_m, \phi, \theta)\mathbf{n}$ is the radial wind velocity at point $\mathbf{r}_m = \mathbf{r}_m(t_0) = \{R_m, \phi, \theta\}$, $\mathbf{u}(R_m, \phi, \theta) = \{u(R_m, \phi, \theta),$ $v(R_m, \varphi, \theta)$, $w(R_m, \phi, \theta)\}$ is the wind velocity field, ϕ is the azimuth, θ is the angle of sight, A_m is the scattering amplitude of the mth particle, N_p is the number of particles, $P(t, R_m)$ is the Doppler lidar pattern, and t_0 and t are time readings corresponding to the beginning and to the end of measurements of the radial wind velocity by an single pulse.

The Doppler lidar pattern in sensing direction $\mathbf{n}$ is determined by the form of the sensing pulse; with respect to the transverse coordinate $\mathbf{r}_{m\perp}$, this intersection has the form of the δ-function:

$$P(t, \mathbf{r}_m) = P_0(t - 2R_m/c)\,\delta(\mathbf{r}_{m\perp}),$$

where

$$P_0(t) = P_0 \exp(-t^2/2\tau_0^2)$$

is the shape of the probing pulse and $2\tau_0$ is the pulse duration, which is determined at e^{-1} level of the maximal value of the function $|P_0(t)|^2$ [7, 9]. Note that the coefficient P_0 includes quantities that are connected with the signal decay depending on the sensing distance, optical radiation absorption, quantum efficiency of the photodetector, amplitude of fields of the reference heterodyne and sensing radiation, and other parameters. In this work, only the influence of atmospheric turbulence on the signal statistics is studied; therefore, the functional dependence of P_0 on these parameters is not specified.

In studying statistical properties of the signal, we assume that the coordinates $\mathbf{r}_m$ of particles' positions at the initial time instant, as well as their velocities $\mathbf{u}(R_m, \phi, \theta)$ and number N_p, are statistically independent random parameters. At the initial time instant, the particles are uniformly distributed in volume V and their positions are also statistically independent. Fluctuations of number of particles N_p ($N_p \gg 1$) are described using the Poisson law, and the law of velocity distribution obeys the normal probability distribution [2, 3, 10–12]. Statistical analysis of $j_s(t)$ is performed using the averaging technique presented in [2, 3, 7, 10].

It is commonly known that a random process is called Gaussian if all its probability distributions are normal for all time instants. In studying conditional and absolute statistical characteristics, we use another, equivalent definition of the normal random process. It is based on factorization of correlation functions by the law of the Gaussian statistics [2, 3].

CONDITIONAL STATISTICAL PROPERTIES OF THE SIGNAL

We define conditional correlation functions of the second and fourth orders as a result of averaging the quantities $j_s(t_1) j_s^*(t_2)$ and $|j_s(t_1)|^2 |j_s(t_2)|^2$ over all random parameters, excluding fluctuations of the turbulent flow velocity. We assume that the particles are homogeneous in composition and have similar scattering cross sections $|A_m|^2 = |A|^2$. Under these assumptions, the conditional correlation functions of the second and fourth orders of the aerosol component of the signal (the functions were obtained as a result of averaging the parameters $j_s(t_1) j_s^*(t_2)$ and $|j_s(t_1)|^2 |j_s(t_2)|^2$ by distribution laws of random coordinates and number of particles using expression (1)) have the form

$$\begin{aligned} &\overline{j_s(t + \tau/2) j_s^*(t - \tau/2)} = S\exp(-\tau^2/4\tau_0^2) \\ &\times \int dR_m |p_0(t - 2R_m/c)|^2 e^{2ik[\langle u_r(R_m,\phi,\theta)\rangle + u_r'(R_m,\phi,\theta)]\tau}, \end{aligned} \tag{3}$$

$$\begin{aligned} \overline{|j_s(t_2)|^2 |j_s(t_1)|^2} &= \overline{j_s(t_2) j_s^*(t_2)}\;\overline{j_s(t_1) j_s^*(t_1)} \\ &+ \overline{j_s(t_2) j_s^*(t_1)}\;\overline{j_s(t_1) j_s^*(t_2)}, \end{aligned} \tag{4}$$

where $S = |A|^2 \bar{N}_p |P_0|^2$ is the signal power, $\bar{N}_p$ is the concentration of particles, $|P_0|^2 = \int |P_0(R_m)|^2 dR_m$, $|p_0(R_m)|^2 = |P_0(R_m)|^2 / |P_0|^2$ is the normalized Doppler lidar pattern, $\langle u_r(R_m, \phi, \theta)\rangle$ is the average radial wind velocity, $u'_r(R_m, \phi, \theta)$ are fluctuations of the radial wind velocity, $t = (t_1 + t_2)/2$, $\tau = t_1 - t_2$, and the bar is the operator of averaging over random coordinates $\mathbf{r}_m$ and fluctuations of number of particles N_p.

It follows from expressions (3) and (4) that the conditional correlation function of the fourth order is factorized by the law of a Gaussian statistics [2, 3]. Therefore, the pulse Doppler lidar signal is a random process with Gaussian conditional statistical characteristics.

ABSOLUTE STATISTICAL PROPERTIES OF THE SIGNAL

The absolute correlation functions of the second and fourth orders are the result of averaging the quantities $j_s(t_1) j_s^*(t_2)$ and $|j_s(t_1)|^2 |j_s(t_2)|^2$ over all random parameters—position at the initial time instant $\mathbf{r}_m$, velocity $\mathbf{u}(R_m, \phi, \theta)$, and number of particles N_p. Application of the averaging technique presented in [2, 3, 7, 10] to expressions $j_s(t_1) j_s^*(t_2)$ and $|j_s(t_1)|^2 |j_s(t_2)|^2$ using formula (1) leads to the following form:

$$\begin{aligned} &\left\langle j_s(t+\tau/2) j_s^*(t-\tau/2)\right\rangle = S\exp\left(-\tau^2/4\tau_0^2\right) \\ &\times \int dR_m \left|p(t-2R_m/c)\right|^2 e^{2ik\langle u_r(R_m,\phi,\theta)\rangle\tau - 2k^2\left\langle u_r'^2(R_m,\phi,\theta)\right\rangle\tau^2}, \end{aligned} \tag{5}$$

$$\begin{aligned} &\left\langle \left|j_s(t+\tau/2)\right|^2 \left|j_s(t-\tau/2)\right|^2\right\rangle \\ &= \left\langle \left|j_s(t+\tau/2)\right|^2\right\rangle \left\langle \left|j_s(t-\tau/2)\right|^2\right\rangle + S^2\exp\left(-\tau^2/2\tau_0^2\right) \\ &\times \int dR_m dR_n \left|p(t-2R_m/c)\right|^2 \left|p(t-2R_n/c)\right|^2 \\ &\times e^{-2k^2\left\langle [u_r'(R_m,\phi,\theta) - u_r'(R_n,\phi,\theta)]^2\right\rangle\tau^2}, \end{aligned} \tag{6}$$

where $u'_r(R_m, \phi, \theta)$ is the fluctuation part of the radial wind velocity. It follows from expressions (5) and (6) that the absolute correlation function of the fourth order is not factorized by the law of a Gaussian statistics; therefore, the signal statistics is non-Gaussian. It is seen from these expressions, as well as from formulas (3) and (4), that the absolute and conditional correlation functions depend both on t and on τ. This means that the pulse Doppler lidar signal is nonstationary.

Thus, the pulse Doppler lidar signal is a nonstationary non-Gaussian random process with Gaussian conditional statistical characteristics.

STATISTICS OF THE SIGNAL FOR A HOMOGENEOUS AND ISOTROPIC TURBULENCE

The appearance of non-Gaussian properties of the pulse Doppler lidar signal is most clearly pronounced in the case of a homogeneous and isotropic turbulence. It is well known [11, 12] that the structural tensor

$$K_{ij}(|\mathbf{r}_m - \mathbf{r}_n|) = \left\langle u'_i(\mathbf{r}_m) u'_j(\mathbf{r}_n)\right\rangle$$

of fluctuations of the wind velocity field for a homogeneous and isotropic turbulence has the form

$$K_{ij}(r) = K(r)\delta_{ij} + (r/2)dK(r)/dr\left(\delta_{ij} - r_i r_j/r^2\right),$$

where $K(r)$ is the longitudinal component of the tensor and r_i are components of the vector $\mathbf{r}$.

For the exponential model $K(r) = (2e/3)\exp(-r/l)$, where e and l are the kinetic energy and turbulence scale [11, 12], expressions (5) and (6) take the form

$$\begin{aligned} &\left\langle j_s(t+\tau/2) j_s^*(t-\tau/2)\right\rangle \\ &= S\exp\left(-\tau^2/4\tau_0^2 - 4k^2 e\tau^2/3 + 2iku_r\tau\right), \end{aligned} \tag{7}$$

$$\begin{aligned} &\left\langle \left|j_s(t+\tau/2)\right|^2 \left|j_s(t-\tau/2)\right|^2\right\rangle \\ &= \left\langle \left|j_s(t+\tau/2)\right|^2\right\rangle \left\langle \left|j_s(t-\tau/2)\right|^2\right\rangle \\ &+ S^2\exp\left(-\tau^2/2\tau_0^2 - 8k^2 e\tau^2/3\right) NG, \end{aligned} \tag{8}$$

where

$$NG = \frac{1}{\sqrt{2\pi}d_{1/2}} \int dr\exp\left(-r^2/2d_{1/2}^2 + 4k^2 K(r)\tau^2\right)$$

is the factor that describes the deviation of the statistics of the Doppler lidar signal from the Gaussian statistics and $2d_{1/2} = c\tau_0$ is the minimal length of the scattering volume. It follows from expressions (7) and (8) that the correlation function of the fourth order is factorized by the law of a Gaussian statistics only for $NG = 1$.

Figure 1 presents the calculation results for the factor NG as a function of the parameter $\tau_* = 2k\tau\sqrt{2e/3}$ for different values of the ratio $l/d_{1/2}$. It follows from expressions (7) and (8), as well as from Fig. 1, that the Gaussian statistics of the Doppler lidar signal is observed in the limit case where the dimensions of the scattering volume $2d_{1/2}$ unboundedly increase and begin to exceed the turbulence scale l, i.e., for $l/2d_{1/2} \ll 1$. For finite values of the ratio $l/2d_{1/2}$, the pulse Doppler lidar signal is a non-Gaussian random process. The non-Gaussian properties of the pulse Doppler lidar signal manifest themselves in a turbulent atmosphere as strongly as the ratio $l/2d_{1/2}$ is large.

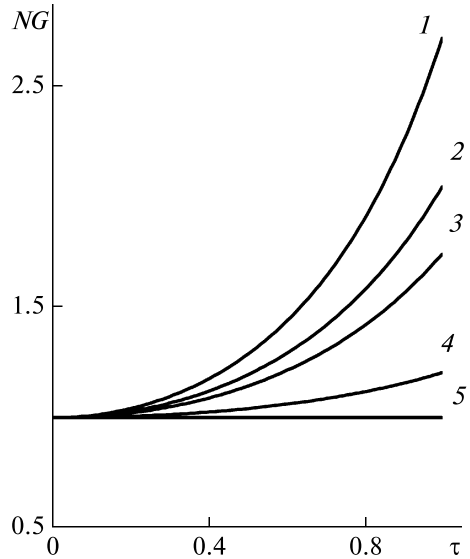

Fig. 1. Calculation results for the factor *NG* depending on the parameter $\tau_* = 2k\tau\sqrt{2e/3}$: (*1*) $l/d_{1/2} \gg 1$, (*2*) $l/d_{1/2} = 2$, (*3*) $l/d_{1/2} = 1$, (*4*) $l/d_{1/2} = 0.2$, and (*5*) $l/d_{1/2} \ll 1$.

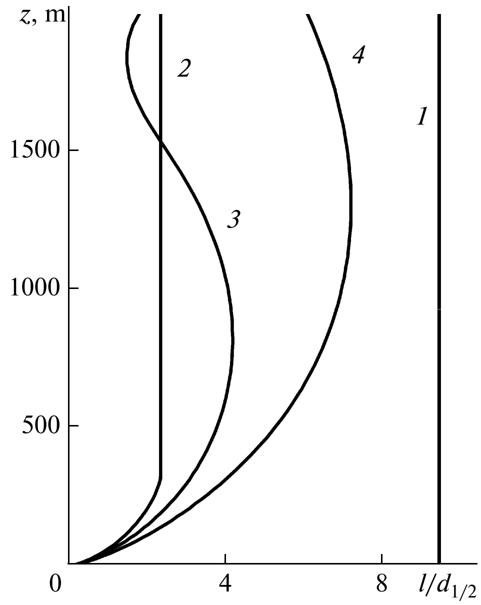

Fig. 2. Daily variations of the profile of the ratio $l/d_{1/2}$: (*1*) 0, (*2*) 10, (*3*) 15, and (*4*) 17 h.

The results of calculating the profile of the ratio $l/d_{1/2}$ presented in Fig. 2 permit one to estimate how the statistics of the pulse Doppler lidar signal changes during a day. Turbulence scale l was calculated for the CASES'99 site for October 6, 1999 [13], using the Mellor–Yamada turbulence model [7, 9]. The value $d_{1/2} = c\tau_0/2 = 18$ m for $\tau_0 = 0.12 \times 10^{-6}$ s. It is seen that the Gaussian statistics is observed only at small heights; in other cases, the non-Gaussian component in signal fluctuations is significant.

Thus, the behavior of the factor *NG* and, correspondingly, the behavior of the Doppler lidar signal statistics significantly depend on the dimensions of scattering volume $2d_{1/2}$ and on the state of atmospheric turbulence which is characterized by turbulence scale l.

SIGNAL STATISTICS AND THE CENTRAL LIMIT THEOREM

As applied to expression (1), which is a sum of a large number of random values, the central limit theorem can be formulated as follows: for a large number of particles that are homogeneous in their composition and are distributed in the scattering volume uniformly and statistically independent from each other, the aerosol component of the Doppler lidar signal has a Gaussian statistics.

Let us consider the question about the statistical dependence of the position of particles in the scattering volume in a turbulent atmosphere. The calculation results for correlations of particles' positions along the sensing axis $\overline{R_m(t)R_n(t)}$ and $\langle R_m(t)R_n(t)\rangle$ that correspond to the absolute and conditional averaging have the form

$$\begin{aligned}\overline{R_m(t)R_n(t)} &= \overline{R_m}\,\overline{R_n} + \overline{R_m}u_r(R_n,\phi,\theta)t \\ &+ \overline{R_n}u_r(R_m,\phi,\theta)t + u_r(R_m,\phi,\theta)u_r(R_n,\phi,\theta)t^2,\end{aligned} \tag{9}$$

$$\begin{aligned}\langle R_m(t)R_n(t)\rangle &= \langle R_m\rangle\langle R_n\rangle \\ &+ \langle R_m\rangle\langle u_r(R_n,\varphi,\theta)\rangle t + \langle R_n\rangle\langle u_r(R_m,\phi,\theta)\rangle t \\ &+ \langle u_r(R_m,\phi,\theta)u_r(R_n,\phi,\theta)\rangle t^2.\end{aligned} \tag{10}$$

In the case of the conditional averaging, as is seen from expression (9), positions of particles along the sensing axis are not correlated, i.e.,

$$\overline{R_m(t)R_n(t)} = \overline{R_m(t)}\,\overline{R_n(t)},$$

and, therefore, they are distributed in the scattering volume statistically independently from each other. Thus, in the case of the conditional averaging, all requirements of the central limit theorem are satisfied and the conditional statistical characteristics of the signal are Gaussian.

For the absolute averaging, as it follows from expression (10), positions of particles are correlated,

$$\langle R_m(t)R_n(t)\rangle \neq \langle R_m(t)\rangle\langle R_n(t)\rangle,$$

and they are statistically dependent from each other in the scattering volume. Based on this, one can conclude that the central limit theorem is not applicable for the statistical analysis of pulse Doppler lidar signal (1) in a turbulent atmosphere. The signal statistics itself is non-Gaussian in the absolute meaning of this word, which agrees with results obtained in previous sections of this work.

The statistical dependence of particles' positions in the scattering volume and, consequently, the non-Gaussian statistics of the Doppler lidar signal are caused by the correlation of turbulent fluctuations of the wind velocity field within the scattering volume. For a homogeneous and isotropic turbulence and for the exponential model, the correlation of turbulent fluctuations of the radial velocity field has the form

$$\left\langle u_r'(R_m,\phi,\theta)\,u_r'(R_n,\phi,\theta)\right\rangle = (2e/3)\exp(-|R_m - R_n|/l).$$

One can see that

$$\left\langle u_r'(R_m,\phi,\theta)\,u_r'(R_n,\phi,\theta)\right\rangle = 0$$

if the condition $l \ll |R_m - R_n|$ is satisfied; therefore, the positions of particles along the sensing axis are statistically independent. If we take into account that $|R_m - R_n| \sim 2d_{1/2}$, the condition $l \ll |R_m - R_n|$ corresponds to the limit case of large dimensions of the scattering volume ($l/2d_{1/2} \ll 1$) and to a Gaussian statistics of the signal.

With an increase in the ratio $l/2d_{1/2}$, which corresponds to small dimensions of the scattering volume, the correlation of turbulent fluctuations of the field of radial wind velocity significantly increases. This leads to a statistical dependence of the position of particles and to non-Gaussian properties of the pulse Doppler lidar signal. From the physical point of view, the condition $l/2d_{1/2} \gg 1$ means that the same vortex of atmospheric turbulence carries all particles that are within the scattering volume. The positions of particles become correlated, i.e., coordinated, as a result of motion within the limits of the same vortex. As for the terms of sum (1), they become statistically dependent also due to the uniform motion of particles within the same vortex. Thus, conditions of the central limit theorem are not satisfied for the model of aerosol signal (1) and (2), which is a sum of a large number of random values in the single scattering approximation [7, 8].

Analysis of expressions (5)–(8) permits one to draw a conclusion about the statistics of the pulse Doppler lidar signal in a laminar flow. If the kinetic energy of the turbulence tends to zero ($e \to 0$), then

$$\left\langle u_r'(R_m,\phi,\theta)\,u_r'(R_n,\phi,\theta)\right\rangle = 0$$

and the turbulent flow degenerates into a laminar one. It is seen from expressions (5)–(8) that the absolute correlation functions are factorized by the law of a Gaussian statistics as $e \to 0$; therefore, the signal statistics is Gaussian. The same conclusion can be drawn from analysis of expressions (9) and (10). It is seen that the positions of particles along the sensing axis become statistically independent; conditions of the central limit theorem are satisfied, and the statistics of the pulse Doppler lidar signal in a laminar flow becomes Gaussian.

CONCLUSIONS

Based on the performed study of statistical properties of the aerosol component of the pulse Doppler lidar signal, one can come to the following conclusions. During coherent detection of optical fields scattered by a large number of particles ($N_p \gg 1$), the pulse Doppler lidar signal is a nonstationary non-Gaussian random process with Gaussian conditional statistical characteristics. The appearance of non-Gaussian properties of the signal is caused by correlation of turbulent fluctuations of the wind velocity field within the scattering volume.

For the considered model of signal (1) and (2), which corresponds to the single scattering approximation and is a sum of a large number of random values, the central limit theorem is not true due to the statistical dependence of particles' positions in a turbulent atmosphere. For a homogeneous and isotropic turbulence, the behavior of the signal statistics significantly depends on the dimensions of the scattering volume $2d_{1/2}$ and on the state of atmospheric turbulence, which is characterized by turbulence scale l. For finite dimensions of the scattering volume, the signal statistics is non-Gaussian but it tends to a Gaussian statistics as $2d_{1/2}$ increases.

Results of calculations that are based on numerical simulation of the atmospheric turbulence also demonstrate that the statistics of the Doppler lidar signal significantly depends on the turbulence state and on dimensions of the scattering volume. For example, for experiment [13], a Gaussian statistics is observed at small heights; with an increase in height, the non-Gaussian component becomes significant in fluctuations of the Doppler lidar signal, which indicates the necessity of a detailed allowance for statistical properties in the analysis of the estimate of the Doppler shift estimate in a turbulent atmosphere.

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