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PHYSICAL OPTICS

Estimate for the Doppler Shift of a Non-Gaussian Signal upon Coherent Detection of Scattered Optical Radiation in a Turbulent Atmosphere

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Abstract—A non-Gaussian model for estimating the radial velocity of turbulent flows in the atmosphere for coherent detection of scattered optical radiation is proposed. The model was obtained based on a theoretical approach that includes results of the statistical analysis of a pulse Doppler lidar signal in a turbulent medium, as well as on the perturbation-theory methods that have been developed in the theory of probability and mathematical statistics. It is shown that the estimate of the Doppler shift in the first-order perturbation theory is a sum of a regular component and two conditional fluctuation components—Gaussian and non-Gaussian ones. In the case of a homogeneous and isotropic turbulence, the estimate of the radial wind velocity is approximately equal to its true average value. The statistical uncertainty in measurements of the average radial wind velocity is determined by the behavior of conditional Gaussian and non-Gaussian components and significantly depends on the state of atmospheric turbulence. It is shown that basic equations of the non-Gaussian model in the limit case coincide with formulas of the local and nonlocal models, as well as with those of the Gaussian model.

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1. INTRODUCTION

In the literature, there are three approaches to studying the behavior of the estimate obtained for the radial velocity of turbulent flows based on coherent optical measurements. These approaches are based on the local and nonlocal models, as well as on the Gaussian model of the radial-velocity estimate.

The local model is based on the well-known formula that describes the Doppler effect and is used in solving problems of Doppler anemometry and active laser interferometry [1, 2]. The formulation of these problems assumes that the dimensions of the scattering volume are small and particles in it move with similar velocities; i.e., turbulent flow velocities are measured in a local region. Physically, the locality of measurements is achieved in the case in which the dimensions of the scattering volume are small compared to the correlation scale of turbulent-flow velocity fluctuations and, therefore, the velocity is not averaged over the scattering volume.

When sensing turbulent flows in the atmosphere, the dimensions of the scattering volume can considerably exceed the characteristic scales of the wind-velocity variation. As a result, the particles in the scattering volume move with different velocities, which leads to averaging of the velocity over the scattering volume. When studying such a situation, a nonlocal model of the estimate for the radial wind velocity is used that was proposed in [3] for the case of a Doppler radar. In [4], a similar problem is studied from the point of view of the influence of scattering volume dimensions in the case of a Doppler lidar. The main drawback of the nonlocal model is the absence of the dependence of the Doppler-velocity estimate on the signal-to-noise ratio and parameters of the signal quantization.

The third approach is based on the Gaussian model of the radial-velocity estimate. It was obtained using methods of the theory of probability and mathematical statistics in the analysis of a Gaussian narrow-band random process. It was formulated first for radar sensing problems [5–7] and reformulated in [8, 9] for problems of lidar sensing.

It was shown in [10] that a Doppler lidar signal is a nonstationary non-Gaussian random process in coherent detection of optical fields scattered by a large number of particles.

Therefore, the influence of the non-Gaussian statistics of the signal on the behavior of the radial-velocity estimate remains open. In this work, based on the theory of probability and mathematical statistics, an approach to studying the estimate for the Doppler shift of a non-Gaussian narrow-band random process is proposed. Applying this approach to solving problems of Doppler lidar sensing resulted in a non-Gaussian model of the estimate for the radial velocity of turbulent flows. The second part of the work presents main determinations of the Doppler-shift estimate for the noise suppression method. In the third part, applying the perturbation method in the analysis of the estimate for the Doppler shift of a non-Gaussian signal of a wind lidar is discussed. The behavior of the estimate of radial wind velocity in the case of homogeneous and isotropic turbulence is considered in the fourth part of this work. Results of numerical simulation are discussed in the fifth part. In the fifth part, limit cases of the obtained results are presented; they are also compared with the local, nonlocal, and Gaussian models of the radial-velocity estimate. In the end, the main conclusions are presented.

2. ESTIMATE FOR THE DOPPLER SHIFT

The estimate of the Doppler shift is defined as the first spectral moment

$$\hat{f}_d = \frac{1}{\hat{S}_0}\sum_{k=-m}^{m} f_k \hat{S}(f_k), \tag{1}$$

where $\hat{S}(f_k)$ is the estimate of the spectrum and $\hat{S}_0 = \sum_{k=-m}^{m} \hat{S}(f_k)$ is the normalization coefficient.

The noise-suppression method [5–7] is based on simple subtraction of noise power spectrum $N(f_l)$ from the estimate of power spectrum $\hat{s}(f_l)$ of the signal itself. The expression for the estimate of the spectrum for this method has the form

$$\hat{S}(f_l) = \hat{s}(f_l) - N(f_l), \tag{2}$$

$$\hat{s}(f_l) = \frac{T_s}{M}\left|\sum_{k=-m}^{m} j(t_k)\exp\{-2\pi i f_l T_s k\}\right|^2, \tag{3}$$

$$N(f) = NT_s, \tag{4}$$

where $j(t_k)$ is the Doppler wind lidar signal, $f_l = \dfrac{l}{M_s T_s}$, $-m \le l \le m$, T_s is the quantization interval, $M_s = 2m + 1$ is the number of quantization intervals, $N = \dfrac{1}{T}\sum_{l=-m}^{m} N(f_l)$ is the noise power, and $T = M_s T_s$. The Doppler wind lidar signal is a sum of two statistically independent parts $j(t_k) = j_s(t_k) + j_n(t_k)$, where $j_s(t)$ and $j_n(t)$ are the aerosol and noise components of the signal. The aerosol component of signal $j_s(t)$ is a non-Gaussian random process with Gaussian conditional statistical characteristics and the noise component of signal $j_n(t)$ is a Gaussian white noise [10, 11].

3. THE PERTURBATION METHOD

The estimate of the Doppler shift is a ratio of two random values; the application of the perturbation method for studying this estimate is based on the iteration procedure [5–7]

$$\hat{S}(f) = S(f) + \Delta\hat{S}(f) + \ldots, \tag{5}$$

$$\hat{S}_0 = S_0 + \Delta\hat{S}_0 + \ldots, \tag{6}$$

where $S(f)$ and S_0 are the power spectrum and normalization coefficient in the zero-order perturbation theory and $\Delta\hat{S}(f)$ and $\Delta\hat{S}_0$ are perturbing additives in the first-order perturbation theory.

For a signal with non-Gaussian statistics, the choice of the zero-order perturbation theory is nontrivial. It was shown in [12] that choosing absolute values of the average power spectrum and average normalization coefficient, which are absolute statistical characteristics, as the zero approximation leads to nonuniform approximation of perturbation-theory series. As a result of such a choice, the variance of Doppler-shift fluctuations increases without bound for small dimensions of the scattering volume, which contradicts the physical sense.

By renormalizing perturbation-theory series [12], one can avoid problems that are connected with nonuniform approximation and give a correct physical interpretation of the obtained results. In this work, the choice of the zero-order perturbation theory is justified based on the analysis of statistical characteristics of a Doppler lidar signal.

In [10], absolute and conditional statistical properties of a pulse Doppler lidar signal in a turbulent atmosphere were studied. The coordinates of particles at the initial time instant, as well as their velocities and number, were assumed to be random parameters. Absolute statistical characteristics were determined by averaging over all random parameters: position of particles in the scattering volume, velocity, and number of particles. Conditional statistical characteristics are a result of averaging the quantities over all random parameters but fluctuations of the turbulent-flow velocity.

Analysis of the statistical characteristics demonstrated [10] that a Doppler lidar signal upon coherent detection of optical fields scattered by a large number of particles is a nonstationary non-Gaussian random process with Gaussian conditional statistical characteristics. This behavior of the conditional statistics means that we can use perturbation-theory methods that were developed for a Gaussian random process in [5–7] if the conditional statistical characteristics are chosen as the zero approximation. Therefore, the zero order of the iteration procedure for the considered nonstationary non-Gaussian random process has the form

$$S(f) = \overline{\hat{S}(f)}, \quad S_0 = \overline{\hat{S}_0}, \tag{7}$$

where $\overline{}$ is the operator of averaging over all random parameters except for wind velocity fluctuations.

It is seen from expression (7) that spectrum $S(f)$ and normalization coefficient S_0 are, on the one hand, conditional statistical characteristics describing statistical properties of $\hat{S}(f)$ and $\hat{S}_0$ and, on the other hand, quantities that depend on the turbulent-flow velocity in a complicated fashion; passing to the average spectrum and average normalization coefficient requires additional averaging over fluctuations of the turbulent-flow velocity. Thus, the quantities $S(f) = \overline{\hat{S}(f)}$ and $S_0 = \overline{\hat{S}_0}$ can be interpreted as partially averaged random values, which subsequently gives us grounds to call them a partially averaged spectrum and partially averaged normalization coefficient.

Using main principles of [5–7] on constructing the iteration procedure and formulas (1) and (5)–(7), one can show that the expression for the estimate of the shift of the Doppler wind lidar signal in the first order perturbation theory has the form

$$\hat{f}_d = f_d + f'_g = \sum_{k=-m}^{m} \frac{S(f_k)}{S_0} f_k + \sum_{k=-m}^{m} \frac{\Delta\hat{S}(f_k + f_d)}{S_0} f_k, \quad (8)$$

where

$$f_d = \sum_{k=-m}^{m} \frac{S(f_k)}{S_0} f_k \quad \text{and} \quad f'_g = \sum_{k=-m}^{m} \frac{\Delta\hat{S}(f_k + f_d)}{S_0} f_k$$

are the zero and first orders of the perturbation theory, respectively.

In [5–7], where the case of the Gaussian statistics was considered, a second-order correlation function of the Gaussian useful signal was used. In this study, in contrast to [5–7], the analysis of the estimate for the shift of a Doppler wind lidar signal (8) is based on the conditional correlation second-order useful signal function obtained in [10],

$$\begin{aligned}&\overline{j_s\left(t + \frac{\tau}{2}\right) j_s^*\left(t - \frac{\tau}{2}\right)} \\ &= S\exp\left\{-\frac{\tau^2}{4\tau_0^2}\right\} \int dR \left|p_0\left(t - 2\frac{R}{c}\right)\right|^2 e^{2ik\left(\langle u_r(R,\phi,\theta)\rangle + u'_r(R,\phi,\theta)\right)\tau},\end{aligned} \quad (9)$$

where S is the signal power, $2\tau_0$ is the pulse duration, $|p_0(R)|^2$ is the normalized Doppler lidar pattern, $\langle u_r(R,\phi,\theta)\rangle$ is the average radial wind velocity, $\langle u'_r(R,\phi,\theta)\rangle$ are fluctuations of the radial wind velocity, $t = (t_1 + t_2)/2$ and $\tau = t_1 - t_2$.

Direct calculations of quantity f_d using formula (9) lead to the following expression for the zero-order perturbation theory:

$$f_d = \frac{2}{\lambda} \int w(R) P\{\langle u_r(R,\phi,\theta)\rangle + u'_r(R,\phi,\theta)\} dR, \quad (10)$$

where

$$w(R) = \frac{1}{M_s} \sum_{k=-m}^{k=m} \left|p_0\left(t_0 + kT_s - 2\frac{R}{c}\right)\right|^2 \quad (11)$$

is the function that defines the shape of the scattering volume.

It is seen from expressions (8) and (10) that the estimate of the Doppler shift is a sum of the regular component and two conditional fluctuation components:

$$\hat{f}_d = \langle f_d \rangle + f'_{ng} + f'_g, \quad (12)$$

where

$$\langle f_d \rangle = \frac{2}{\lambda} \int w(R) \langle u_r(R,\phi,\theta)\rangle dR$$

is the regular component,

$$f'_{ng} = \frac{2}{\lambda} \int w(R) u'_r(R,\phi,\theta) dR$$

is the conditional non-Gaussian fluctuation component, and

$$f'_g = \sum_{k=-m}^{m} f_k \frac{\Delta\hat{S}(f_k + f_d)}{S_0}$$

is the conditional Gaussian fluctuation component.

In view of the absence of a consistent theory, one can state that the definitions are insufficient for describing non-Gaussian random processes. In our case, the new concepts used for describing the estimate for the Doppler shift of a non-Gaussian narrowband random process and requiring a special explanation include the "conditional Gaussian fluctuation component" and "conditional Non-Gaussian fluctuation component."

Introducing new concepts into scientific use must reflect features of the Doppler wind lidar signal's statistics $j(t)$ and specificity of the application of the involved methods. The main feature of statistics $j(t)$ is that conditional statistical characteristics play a central role when applying the used methods. Since the origin of quantities f'_g and f'_{ng} is the result of conditional, not absolute, statistical characteristics of the signal $j(t)$, it is natural to refer to them as conditional fluctuation components of the estimate of the Doppler shift.

Iteration procedure (8) was constructed using the results of [5–7]; therefore, the general forms of expressions for the first-order perturbation theory,

$$f_g' = \sum_{k=-m}^{m} \frac{\Delta\hat{S}(f_k + f_d)}{S_0} f_k$$

coincide both in the case of analysis of Gaussian signals and in studying non-Gaussian ones. There is a difference only in the determination of fluctuations of spectrum $\Delta\hat{S}(f_k)$: in the first case, the spectrum of a Gaussian narrow-band random process is used; in the second case, the spectrum of a non-Gaussian narrow-band random process is used. Therefore, the term “conditional Gaussian fluctuation component” should be introduced for scientific use with emphasis of the fact that iteration procedure (8) is constructed using the theoretical approach [5–7] developed for a narrow-band random signal with a Gaussian statistics.

Quantity $f_{ng}' = \frac{2}{\lambda}\int w(R)u_r'(R,\phi,\theta)dR$ depends on fluctuations of radial wind velocity $u_r'(R,\phi,\theta)$. According to results of [10], non-Gaussian properties of signal $j(t)$ are caused by correlation of turbulent fluctuations of the wind-velocity field within the scattering volume the dimensions of which are determined by the function $w(R)$. Therefore, the fluctuation component of the Doppler shift of partially averaged spectrum f_{ng}' is connected with the non-Gaussian statistics of signal $j(t)$ and it can be referred to as a “conditional non-Gaussian fluctuation component.”

4. HOMOGENEOUS AND ISOTROPIC TURBULENCE

In problems of Doppler sensing of wind velocity, it is convenient to characterize the estimate of Doppler shift $\hat{f}_d$ by the quantity

$$\hat{u}_r = \frac{\lambda}{2}\hat{f}_d,$$

which has the meaning of an estimate for the radial wind velocity. For a homogeneous and isotropic turbulence [13, 14], the wind-velocity field is a homogeneous field with smoothly varying average characteristics; i.e.,

$$\langle u_r(R,\phi,\theta)\rangle = \langle u_r(R_0,\phi,\theta)\rangle = u_r,$$

therefore, the expression for the estimate of the radial wind velocity can be written in the form

$$\hat{u}_r = u_r + u', \tag{13}$$

where $u' = \frac{\lambda}{2}(f_{ng}' + f_g')$ is the difference between the estimate for the radial wind velocity and its true value and R_0 is the distance from the center of the scattering volume. It is seen that the estimate of the radial wind velocity for a homogeneous and isotropic turbulence is approximately equal to its average true value,

$$\hat{u}_r \approx u_r, \tag{14}$$

and the variance

$$\sigma_{u_r}^2 = \langle(\hat{u}_r - u_r)^2\rangle = \frac{\lambda^2}{4}\langle(f_{ng}' + f_g')^2\rangle \tag{15}$$

describes the deviation of the estimate $\hat{u}_r$ from the true value u_r. The physical meaning of variance $\sigma_{u_r}^2$ is that the standard deviation $\sigma_{u_r} = \sqrt{\sigma_{u_r}^2}$ characterizes the statistical uncertainty of measurements of the average radial wind velocity. The variance calculated based on formulas (10)–(12) and (15) in the case of weak correlation has the form

$$\sigma_{u_r}^2 = \int dR_1 dR_2 w(R_1) w(R_2)\langle u_r'(R_1,\phi,\theta) u_r'(R_2,\phi,\theta)\rangle$$
$$+ \frac{\lambda\sqrt{4\langle\Delta u_r'^2\rangle - 3\langle\Delta\tilde{u}_r'^2\rangle}}{8\sqrt{\pi}M_s T_s} + \frac{2N\langle\Delta u_r'^2\rangle}{SM_s} + \frac{N^2\lambda^2}{48S^2M_sT_s^2} + \delta, \tag{16}$$

$$\langle\Delta u_r'^2\rangle = \frac{1}{8\tau_0^2k^2} + \frac{1}{2}\int_{-\infty}^{\infty} dR_1 dR_2 w(R_1) w(R_2) D_r(R_1,R_2),$$

$$\langle\Delta\tilde{u}_r'^2\rangle = \frac{1}{8k^2\tau_0^2} \tag{17}$$
$$+ \frac{1}{2}\int_{-\infty}^{\infty} dR_1 dR_2 \left|p\left(2\frac{R_1}{c}\right)\right|^2 \left|p\left(2\frac{R_2}{c}\right)\right|^2 D_r(R_1,R_2),$$

$$\delta = \frac{1}{16k^2M_sT_s\tau_0}\int_{-\infty}^{\infty} dr \frac{(r^2-1)\exp\left(-\frac{r^2}{2}\right)}{\sqrt{1+4\tau_0^2k^2D_r(d_{1/2}r)}}, \tag{18}$$

where

$$D_r(R_1,R_2) = \langle[u_r'(R_1,\phi,\theta) - u_r'(R_2,\phi,\theta)]^2\rangle.$$

The first summand of the right-hand side of (16) is the variance of conditional non-Gaussian fluctuations $\sigma_{u_r,ng}^2 = \frac{\lambda^2}{4}\langle f_{ng}'^2\rangle$ and the sum of the second, third, fourth, and fifth summands is the variance of conditional Gaussian fluctuations $\sigma_{u_r,g}^2 = \frac{\lambda^2}{4}\langle f_g'^2\rangle$. This, variance $\sigma_{u_r}^2$ is the sum of two components—variances of conditional Gaussian and non-Gaussian fluctuations; i.e., $\sigma_{u_r}^2 = \sigma_{u_r,g}^2 + \sigma_{u_r,ng}^2$.

5. RESULTS OF NUMERICAL SIMULATION

In [10], statistical analysis was performed based on a homogeneous 1D model of the atmospheric bound-

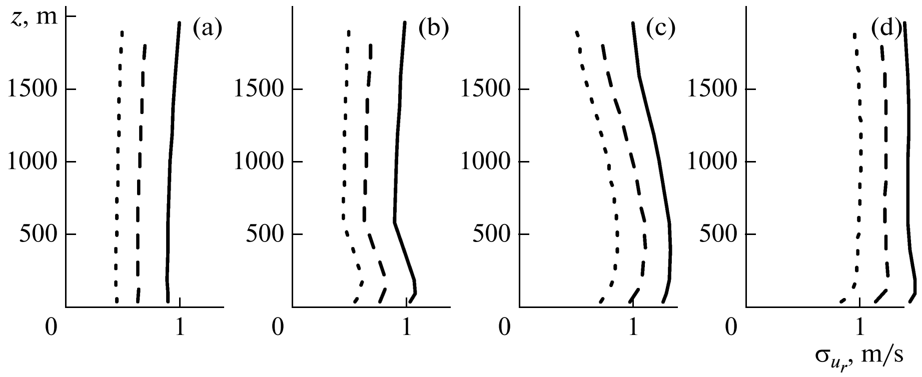

Fig. 1. Standard deviations σ_{u_r} as functions of sensing height z; number of quantization intervals $M_s = 16$ (solid curve), 32 (dashed curve), and 64 (dotted curve). UTC = (a) 6, (b) 16, (c) 21, and (d) 23 h; CDT = (a) 0, (b) 10, (c) 15, and (d) 17 h.

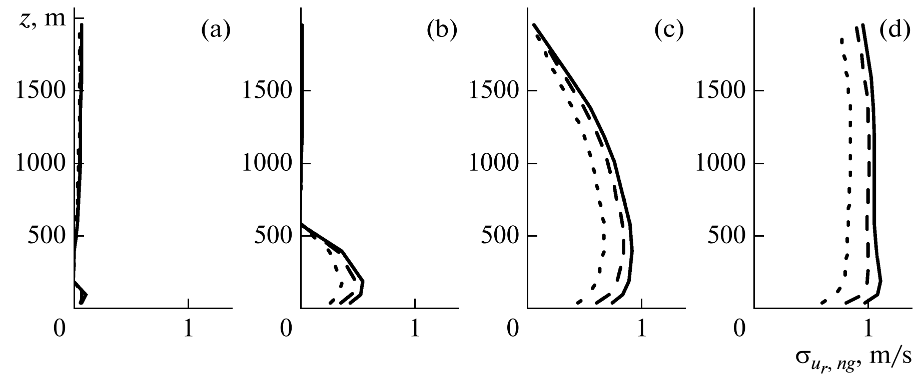

Fig. 2. Standard deviations $\sigma_{u_r,ng}$ as functions of sensing height z; number of quantization intervals $M_s = 16$ (solid curves), 32 (dashed curves), and 64 (dotted curves). UTC = (a) 6, (b) 16, (c) 21, and (d) 23 h; CDT = (a) 0, (b) 10, (c) 15, and (d) 17 h.

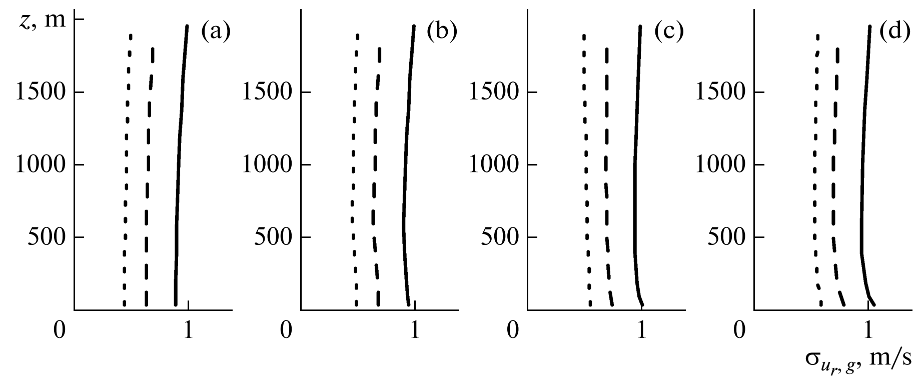

Fig. 3. Standard deviations $\sigma_{u_r,g}$ as functions of sensing height z; number of quantization intervals $M_s = 16$ (solid curves), 32 (dashed curves), and 64 (dotted curves). UTC = (a) 6, (b) 16, (c) 21, and (d) 23 h; CDT = (a) 0, (b) 10, (c) 15, and (d) 17 h.

ary layer and the e–l turbulence model. The turbulence characteristics were calculated for the CASES'99 site for October 6, 1999 [15]. Figures 1–3 present numerical simulation results for the behavior of daily variations of the standard deviation $\sigma_{u_r} = \sqrt{\sigma^2_{u_r}}$, standard deviation of the conditional non-Gaussian component $\sigma_{u_r,ng} = \sqrt{\sigma^2_{u_r,ng}}$, and standard deviation of the conditional Gaussian component $\sigma_{u_r,g} = \sqrt{\sigma^2_{u_r,g}}$ using models [10] for a lidar with parameters $\lambda = 2.0218$ $\tau_0 = 0.12 \times 10^{-6}$ s, $S/N \gg 1$, and $T_s = 0.02 \times 10^{-6}$ s. Daily variations of standard deviations are presented for time instants according to UTC (Universal Time Coordinated) and CDT (Central Daylight Time) standards.

As follows from Fig. 1, the statistical uncertainty in measurements of the average radial wind velocity significantly depends on the state of the atmospheric turbulence. With the increase in the turbulence energy that is observed during daylight hours, standard deviation σ_{u_r} considerably increases as compared to darkness hours.

The behaviors of quantities $\sigma_{u_r,ng}$ and $\sigma_{u_r,g}$ are different at the same time of day. It is seen from Figs. 2 and 3 that the standard deviation of the conditional Gaussian component increases with an increase in the turbulence energy. At the same time, the standard deviation of the conditional Gaussian component weakly depends on the state of atmospheric turbulence for the considered values of lidar parameters. For example, at night, when a weak turbulence is observed, $\sigma_{u_r,ng}$ is approximately equal to zero and the statistical uncertainty in measurements of the average radial wind velocity is determined by the standard deviation of the conditional Gaussian component $\sigma_{u_r,g}$. In the second half of the day, when the turbulence energy increases, $\sigma_{u_r,ng}$ and $\sigma_{u_r,g}$ are on the same order of magnitude and their contribution to the statistical uncertainty in measurements of the average radial wind velocity is similar.

6. LIMIT CASES OF THE OBTAINED RESULTS

Let us consider three limit cases of formulas (9)–(12) and (16) and compare them with ground expressions corresponding to the local and nonlocal models, as well as to the Gaussian model of the radial-velocity estimate.

The first limit case corresponds to small dimensions of the scattering volume. Let us assume that these dimensions are much less than the turbulence scale l, i.e., $w(R_m) = \delta(R_m - R)$. We also assume that the expansion of the perturbation-theory series can include only the zero order, which means that the conditional Gaussian fluctuations are equal to zero, $f_g' = 0$. In this case, it follows from formulas (9)–(12) and (16) that the expressions for $\hat{f}_d$ and $\sigma^2_{u_r}$ take the form

$$\hat{f}_d = \frac{2}{\lambda}(u_r + u_r'), \tag{19}$$

$$\sigma^2_{u_r} = \frac{2}{3}e, \tag{20}$$

where $u_r'(R,\phi,\theta) = u_r'$ are fluctuations of the radial wind velocity at the point that corresponds to the center of the scattering volume and e is the kinetic energy of turbulence. It is seen that Eq. (19) describes the Doppler effect and formula (20) is a simple consequence of this effect. Therefore, for small dimensions of the scattering volume and $f_g' = 0$, formulas (9)–(12) and (16) pass over into basic equations of the local model.

In the second case, it is assumed that only the contribution of conditional Gaussian fluctuations is small ($f_g' = 0$); then, it follows from (9)–(12) and (16) that

$$\hat{f}_d = \frac{2}{\lambda}\int dR_m w(R_m) u_r(R_m,\phi,\theta), \tag{21}$$

$$\sigma^2_{u_r} = \int dR_m dR_n w(R_m) w(R_n) \langle u_r'(R_m,\varphi,\theta) u_r'(R_n,\phi,\theta)\rangle. \tag{22}$$

It is seen from Eqs. (21) and (22) that these asymptotics take into account the averaging of the wind-velocity field over the space of the scattering volume and the formulas coincide with formulas of the nonlocal model [3, 4].

When considering the third case, we assume that the dimensions of the scattering volume are large; i.e., they are much larger than turbulence scale l. In this case, the Doppler lidar signal becomes a Gaussian narrow-band random process [10] and Eqs. (9)–(13) and (16) take the form

$$\hat{f}_d = \langle f_d\rangle + f_g', \quad f_g' = \frac{1}{S_0}\sum_{k=-m}^{m} f_k \Delta\hat{S}(f_k + f_{g,d}), \tag{23}$$

$$\sigma^2_{u_r} = \frac{\lambda\sqrt{\langle \Delta u_r'^2\rangle}}{8\sqrt{\pi} M_s T_s} + \frac{2N\langle \Delta u_r'^2\rangle}{SM_s} + \frac{N^2\lambda^2}{48 S^2 M_s T_s^2}, \tag{24}$$

where $\langle \Delta u_r'^2\rangle = \frac{1}{8\tau_0^2 k^2} + \frac{2}{3}e$. Comparison of formula (23) for large scales of the scattering volume with a similar expression obtained for a signal with a Gaussian statistics shows that this formula is an alternative notation of a series of the perturbation theory for the estimate of the Doppler shift of the signal. The difference of formula (23) from similar equations [5–9] is that (23) is based on partially averaged quantities $S(f) = \overline{\hat{S}(f)}$ and $S_0 = \overline{\hat{S}_0}$. However, this difference manifests itself only in an alternative way of constructing the perturbation theory; for a Gaussian statistics of the signal, it leads to the same results. For example, expression (24), which is a corollary of formula (23), is a well-known equation obtained under the assumption that the statistics of the signal is Gaussian.

Thus, the presented comparisons show that Eqs. (9)–(13) and (16) in limit cases coincide with investigations results obtained using different theoretical approaches [3–9].

7. CONCLUSIONS

Based on the performed study, one can come to the following conclusions. A theoretical approach to studying the behavior of the estimate for the radial velocity of turbulent flows in the atmosphere for coherent detection of scattered optical radiation was proposed. The approach is based on a non-Gaussian model of the estimate for the radial velocity; the model was obtained in this work based on results of statistical analysis of a pulse Doppler lidar signal [10]. The estimate of the radial velocity was analyzed using perturbation-theory methods developed in the theory of probability and mathematical statistics. It was shown that the estimate of the Doppler shift in the first-order perturbation theory is a sum of the regular component and two conditional fluctuation components—Gaussian and non-Gaussian ones. The zero order of the perturbation theory is determined by a sum of the regular component and conditional non-Gaussian components; the perturbing additive in the first order is a conditional Gaussian component.

In the case of a homogeneous and isotropic turbulence, the estimate of the radial wind velocity is approximately equal to its true value. The difference between the estimate of the radial wind velocity and its true value is a random value and is determined by the behavior of conditional Gaussian and non-Gaussian components. The variance $\sigma_{u_r}^2$, which is a sum of variances of conditional Gaussian and non-Gaussian fluctuations, significantly depends on the state of atmospheric turbulence and increases with an increase in the turbulence energy.

The standard deviation of the conditional non-Gaussian component $\sigma_{u_r,ng} = \sqrt{\sigma_{u_r,ng}^2}$ increases with an increase in the turbulence energy, and that of the conditional Gaussian component $\sigma_{u_r,g} = \sqrt{\sigma_{u_r,g}^2}$ weakly depends on the state of atmospheric turbulence for the considered values of lidar parameters. Under weak turbulence, $\sigma_{u_r,ng}$ is approximately equal to zero and $\sigma_{u_r} \approx \sigma_{u_r,g}$. The contributions of the conditional Gaussian and non-Gaussian components to σ_{u_r} are similar if the turbulence energy is high.

Comparison of the proposed theoretical approach with the results of [3–9] shows that basic equations of the non-Gaussian model coincide with formulas of the local, nonlocal, and Gaussian models in limit cases.

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