



# 1 Uncertainty Quantification for Atmospheric Motion Vectors with 2 Machine Learning

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5 **Abstract.** Wind-tracking algorithms produce Atmospheric Motion Vectors (AMVs) by tracking clouds or water vapor  
 6 across spatial-temporal fields. Thorough error characterization (also known as uncertainty quantification) of wind-  
 7 tracking algorithms is critical in properly assimilating AMVs into weather forecast models and climate reanalysis  
 8 datasets. Uncertainty quantification should yield estimates of two key quantities of interest: bias, the systematic  
 9 difference between a measurement and the true value, and standard error, a measure of variability of the measurement.  
 10 The current process of specification of the errors input into inverse modelling is often cursory and commonly consists  
 11 of a mixture of model fidelity, expert knowledge, and need for expediency. The methods presented in this paper  
 12 supplement existing approaches to error specification by providing an error-characterization module that is purely  
 13 data-driven and requires few tuning parameters. This paper proposes an error-characterization method that combines  
 14 the flexibility of machine learning (random forest) with the robust error estimates of unsupervised parametric  
 15 clustering (using a Gaussian Mixture Model). Traditional techniques for uncertainty quantification through machine  
 16 learning have focused on characterizing bias, but often struggle when estimating standard error. In contrast, model-  
 17 based approaches such as k-means or Gaussian mixture modelling can provide reasonable estimates of both bias and  
 18 standard error, but they are often limited in complexity due to reliance on linear or Gaussian assumptions. In this  
 19 paper, a methodology is developed and applied to characterize error in tracked-wind using a high-resolution global  
 20 model simulation, and it is shown to adequately capture the error features of the tracked wind.

## 21 1. Introduction

22 Reliable estimates of global winds are critical to science and application areas, including global chemical transport  
 23 modeling and numerical weather prediction. One source of wind measurements consists of feature-tracking based  
 24 Atmospheric Motion Vectors (AMVs), produced by tracking time sequences of satellite-based measurements of  
 25 clouds or spatially distributed water vapor fields (Mueller et al., 2017; Posselt et al., 2019). The importance of global  
 26 measurements of 3-dimensional winds was highlighted as an urgent need in the NASA Weather Research Community  
 27 Workshop Report (Zeng et al., 2016) and was identified as a priority in the 2007 National Academy of Sciences Earth  
 28 Science and Applications from Space (ESAS 2007) Decadal Survey and again in ESAS 2017. For instance, wind is  
 29 used in the study of global CO<sub>2</sub> transport (Kawa et al., 2004), numerical weather prediction (NWP; Cassola and  
 30 Burlando, 2012), as inputs into weather and climate reanalysis studies (Swail and Cox, 2000), and for estimating  
 31 current and future wind-power outputs (Staffell and Pfenninger, 2016).

32 Thorough error characterization of wind-track algorithms is critical in properly assimilating AMVs into forecast  
 33 models. Prior literature has explored the impact of ‘poor’ error-characterization in Bayesian-based approaches to



remote sensing applications. Nguyen et al. (2019) proved analytically that when the input bias is incorrect in Bayesian methods (specifically, optimal estimation retrievals), then the posterior estimates would also be biased. Moreover, they proved that when the input standard error is ‘correct’ (that is, it is as close to the unknown truth as possible), then the resulting Bayesian estimate is ‘efficient’; it has the smallest possible error. Additionally, multiple active and passive technologies are being developed to measure 3D winds, such as Doppler wind lidar (DWL) and radar and infrared/microwave sensors that derive AMVs using feature-tracking of consecutive images. Therefore, an accurate and robust uncertainty quantification methodology will allow for more accurate assessments of mission impacts, and the eventual propagation of data uncertainties for these instruments.

Velden and Bedka (2009) and Salonen et al. (2015) have shown that height assignment contributes a large component of uncertainty in AMVs tracked from cloud movement and from sequences of infrared satellite radiance images. However, height assignment is not the dominant portion of the error in AMVs obtained from water vapor profiling instruments (e.g., infrared and microwave sounders). As such, this study will focus on errors in the AMV estimates at a given height. Previous work has demonstrated several different approaches for characterizing AMV vector error. One common approach is to employ quality indicator thresholds, as described by Holmund et al (2001), which compare changes in AMV estimates between sequential timesteps and neighboring pixels, as well differences with model predictions, to produce a quality indicator to which a discrete uncertainty is assigned. The Expected Error approach, developed by Le Marshal et al. (2004), builds a statistical model using linear regression against AMV-radiosonde values to correct AMV observation error.

In this study, we detail a data-driven tool for building an AMV uncertainty model using observing system simulation experiment (OSSE) data. We build on the work by Posselt et al. (2019) in which a water vapor feature-tracking AMV algorithm was applied to a high-resolution numerical simulation, thus providing a global set of AMV estimates which can be compared to the reference winds produced by the simulation. In this case, a synthetic “true” state is available with which AMVs can be compared and errors are quantified, and it is shown that tracking errors in AMV estimates are state dependent. Our approach will use a conjunction of machine learning (random forest) and unsupervised parametric clustering (Gaussian mixture models) to build a model for the uncertainty structures found by Posselt et al. (2019). The realism and robustness of the resulting uncertainty estimates depend on the realism and representativeness of the reference dataset. This work builds upon the work of Bormann et al. (2014) and Hernandez-Carrascal and Bormann (2014), who showed that wind tracking could be divided into distinct geophysical regimes by clustering by cloud conditions. This study supplements that approach with the addition of machine learning, which, compared with traditional linear modeling approaches, should allow the model to capture more complex non-linear processes in the error function.

Traditional techniques for uncertainty quantification through machine learning have focused on characterizing bias but often struggle when estimating standard error. By pairing a random forest algorithm with unsupervised parametric clustering, we propose a data-driven, cluster-based approach for quantifying both bias and standard error from experimental data. According to the theory developed by Nguyen et al. (2019), these improved error characterizations



69 should then lead to improved error characteristics (e.g., lower bias, more accurate uncertainties) in subsequent analyses  
70 such as flux inversion or data assimilation.

71 The rest of the paper is organized as follows: In Section 2, we give an overview of the simulation which provides the  
72 training data for our machine learning approach and motivate and define the specific uncertainties this study aims to  
73 characterize. In Section 3, we describe the error characterization approach with the specifics of our error  
74 characterization model, including both the implementation of and motivations for employing the random forest and  
75 Gaussian mixture model. In Section 4, we provide a validation of our methods, attempting to assess the bias of our  
76 predictions. In Section 5, we discuss the implications of our error characterization approach, both on AMV estimation  
77 and data assimilation more broadly.

## 78 2. Experimental Set-up

### 79 2.1 Simulation and Feature-Tracking Algorithm

80 While our methodology in principle could be used to quantify uncertainties in any measurements used in data  
81 assimilation, in this paper we devote special emphasis to the use case of wind-tracking algorithms. In particular, we  
82 trained our model on the simulated data used by Posselt et al. (2019), in which they applied an AMV algorithm to  
83 outputs from the NASA Goddard Space Flight Center (GSFC) Global Modeling and Assimilation Office (GMAO)  
84 GEOS-5 Nature Run (G5NR; Putman et al. 2014). The Nature Run is a global dataset with  $\sim 7$  km horizontal grid  
85 spacing that includes, among other quantities, three-dimensional fields of wind, water vapor concentration, clouds,  
86 and temperature. The AMV algorithm is applied on four pressure levels (300hPa, 500hPa, 700hPa, and 850hPa) at 6-  
87 hourly intervals, using three consecutive global water vapor fields spaced one hour apart, and for a 60-day period from  
88 07/01/2006 to 08/30/2006. The water-vapor fields from GEOS5 were input to a local-area pattern matching algorithm  
89 that approximates wind speed and direction from movement of the matched patterns. The algorithm searches a pre-  
90 set number of nearby pixels to minimize the sum-of-absolute-differences between aggregated water vapor values  
91 across the pixels. Posselt et al. (2019) describes the sensitivity of the tracking algorithm and the dependency of the  
92 tracked winds on atmospheric states in detail.

93 It is important to note that the AMV algorithm tracks water vapor on fixed pressure levels. In practice, these would be  
94 provided by satellite measurements, whereas in this paper we use simulated water vapor from the GEOS-5 Nature  
95 Run. The height assignment of the AMVs is assumed to be perfectly known (or, at the very least, the pressure level  
96 uncertainty is captured by the satellite measurement uncertainty rather than the AMV estimate). As such, we focus  
97 solely on observational AMV error and not on height assignment error.

98 A snapshot of the dataset at 700hPa is given in Figure 1, where we display the true water vapor from Nature Run (top  
99 left panel), the true wind speed from Nature Run (top right panel), the tracked wind from the AMV-tracking algorithm  
100 (bottom right panel), and the difference between the true and tracked wind (bottom left panel). Note that the wind-  
101 tracking algorithm tends to have trouble in region where the true water vapor content is close to zero. It is clear that



while the wind-tracking algorithm tends to perform well in most regions (we can classify these regions are areas where the algorithm is skilled), in some regions the algorithm is unable to reliably make a reasonable estimate of the wind speed (unskilled). We will examine these skilled and unskilled regimes (and their corresponding contributing factors) in the section 3.

## 2.2 Importance of Uncertainty Representation in Data Assimilation

Proper error characterization for any measurement, including AMVs, is important in data assimilation. Data assimilation often uses a regularized matrix inverse method based on Bayes' theorem, which, when all probability distributions in Bayes' relationship are assumed to be Gaussian, reduces to minimizing a least-squares (quadratic) cost function Eq (1):

$$J = (\mathbf{x} - \mathbf{x}_b)\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b) + ((\hat{\mathbf{y}} - \mathbf{a}) - \mathbf{H}[\mathbf{x}])^T \mathbf{R}^{-1}((\hat{\mathbf{y}} - \mathbf{a}) - \mathbf{H}[\mathbf{x}]) \quad (1)$$

where  $\mathbf{x}$  represents the analysis value,  $\mathbf{x}_b$  represents the background field (first guess),  $\mathbf{B}$  represents the background error covariance,  $\mathbf{y}$  represents the observation, and  $\mathbf{H}$  represents the forward operator that translates model space into observation space. This translation may consist of spatial and/or temporal interpolation if  $\mathbf{x}$  and  $\mathbf{y}$  are the same variable (e.g., if the observation of temperature comes from a radiosonde), or may be far more complicated (e.g., a radiative transfer model in the case of satellite observations).  $\mathbf{R}$  represents the observation error covariance, and  $\mathbf{a}$  represents the accuracy, or bias, in the observations. The right-most part of Eq. (1) can be interpreted as a sum of the contribution of information from the data ( $\mathbf{y} - \mathbf{H}[\mathbf{x}] - \mathbf{a}$ ) and the contribution from the prior ( $\mathbf{x} - \mathbf{x}_b$ ), which are weighted by their respective covariance matrices. In our analysis, the AMVs obtained from the wind-tracking algorithm is used as 'data' in subsequent analysis. That is, the tracked wind data  $\hat{\mathbf{y}}$  is a biased and noisy estimator of the true wind  $\mathbf{y}$ , and might be assumed to follow the model Eq. (2):

$$\hat{\mathbf{y}} = \mathbf{y} + \epsilon \quad (2)$$

where  $\epsilon$  is an error term, commonly assumed to be Gaussian with mean  $\mathbf{a}$  and covariance matrix  $\mathbf{R}$  (i.e.,  $\epsilon \sim N(\mathbf{a}, \mathbf{R})$ ), which are the same two terms that appear in Equation (1). As such, for data assimilation to function, it is essential to correctly specify the bias vector  $\mathbf{a}$  and the standard error matrix  $\mathbf{R}^{-1}$ . Incorrect characterizations of either of these components could have adverse consequences on the resulting data assimilation analyses with respect to bias and/or the standard error (Nguyen et al., 2019).

## 3 Methodology

### 3.1 Generalized Error Characterization Model

An overview of our approach is outlined in Figure 2. Given a set of training predictors  $\mathbf{X}$ , training responses  $\hat{\mathbf{Y}}$ , and the true response  $\mathbf{Y}$ , our approach begins with two independent steps. In one step, a Gaussian mixture model is trained



on the set of  $X$ ,  $\hat{Y}$ , and  $Y$ . This clustering algorithm identifies geophysical regimes where the nonlinear relationships between the three variables differ. In the other step, a random forest is used to model  $Y$  based on  $X$  and  $\hat{Y}$ . This step produces an estimate of the true response (we call this  $\tilde{Y}$ ) using only the training predictors and response. We then employ the Gaussian mixture model to estimate the clusters which the set of  $X$ ,  $\hat{Y}$ , and  $\tilde{Y}$  pertain to. Subsequently, we compute the error characteristics of each cluster of  $X$ ,  $\hat{Y}$ , and  $\tilde{Y}$  in the training dataset. Therefore, given a new point consisting solely of  $X$  and  $\hat{Y}$ , we can assign it to a specific cluster and ascribe to it a set of error characteristics. This forms the basis for our error characterization model.

What follows in this paper is an implementation of the error characterization model obtained for a subsample of the GEOS-5 Nature Run at a fixed height of 700hPa. In particular, we trained the error characterization on a random sample from the first 1.5 months of the Nature Run, and show the results obtained when applying it to a test sample drawn from the subsequent 0.5 months of the Nature Run.

### 3.2 Error Regime

When examining the relationship between AMVs and simulated true winds in Figure 3, it is clear that there are two distinct ‘error-regimes’ present in the dataset. The majority of AMV estimates can be categorized as ‘skilled’, wherein their estimate lies clearly along a one-to-one line with the simulated true wind. However, there is also clearly an ‘unskilled’ regime, for which the AMV estimate is very close to zero when there are actually high or mid-level true wind values present. Our goal is to provide unique error characterizations for each error regime, because the error dynamics are different within each regime. Furthermore, when we analyze this error and its relationship to water vapor, we see that ‘unskilled’ regime correlates highly with areas of low water vapor in Figure 4. This matches the error patterns discussed in Posselt et al. (2019).

### 3.3 Gaussian Mixture Model

These distinct regimes present an opportunity to employ machine learning. Bormann et al. (2014) and Hernandez-Carrascal and Bormann (2014) demonstrated that cluster (also called regime) analysis is a successful approach for wind-tracking error characterization, and so we aim to train a clustering algorithm that is capable of determining whether any individual AMV estimate belongs in the ‘skilled’ or ‘un-skilled’ cluster. In particular, we use a clustering algorithm that can take advantage of the underlying geophysical dynamics, since we see the relationship between the error-regimes and water vapor content. To this end, we employ a Gaussian mixture model, a clustering algorithm based on estimating a training set as a mixture of multiple Gaussian distributions. A mathematical overview follows:

1. Define each location containing simulated true winds, water vapor, and AMV estimates as a random variable  $x_i$
2. Define  $\theta$  as the population that consists of all  $x_i$  in the training dataset
3. Model the distribution of the population  $P(\theta)$  as:



$$P(\theta) = \sum_j^K \pi_j N(\mu_j, \Sigma_j) \quad (3)$$

Where  $N(\mu_j, \Sigma_j)$  is the normal distribution with mean  $\mu_j$  and covariance  $\Sigma_j$ ,

$K$  is the number of clusters, and  $\pi_j$  is the mixture proportion.

3. An Expectation–Maximization Algorithm determines  $\pi_j, \mu_j, \Sigma_j$  for  $K$  clusters

4. Density estimation gives us  $P(x_i \in k_j) = p_{ij}$

5. Maximum  $p_{ij}$  is the assigned cluster for point  $x_i$

The mixture model clustering is based on the R package ‘Mclust’ developed by Fraley et al. (2012), which builds upon the theoretical work of Fraley and Raftery (2002) for model-based clustering and density estimation. The process uses an Expectation-Maximization algorithm to cluster the dataset, estimating a variable number of distinct multivariate Gaussian distributions from a sample dataset. Training the Gaussian mixture model on this dataset provides a clustering function which outputs a unique cluster for any data point with the same number of variables.

In one dimension, a Gaussian mixture model looks like the distributions depicted in Figure 5: instead of modelling a population as a single distribution (Gaussian or otherwise), the GMM algorithm fits multiple Gaussian distributions to a population. A key aspect is that this algorithm has the capability of assigning a new point to the most likely distribution. For example, in the 1-D figure, a normalized AMV estimate with a value of 10 would be more likely to originate from the broad cluster ‘2’ than the narrow cluster ‘4’. In this case, we model the population as a Gaussian mixture model in five-dimensional space, which consists of two simulated true wind vector components ( $u$  and  $v$ ), two AMV estimates of these wind components ( $\hat{u}$  and  $\hat{v}$ ), and the simulated water vapor values, all of which have been standardized. Each cluster has a 5-dimensional mean vector for the center and a  $5 \times 5$  covariance matrix defining their multivariate Gaussian shape. The estimation of a covariance matrix allows for the characterization of the relationships between the different dimensions within each cluster, and as such the gaussian mixture model approach provides greater potential for understanding the geophysical basis of error regimes than other unsupervised clustering approaches.

In Figure 6, we applied the Gaussian mixture model to true  $u$  and  $v$  wind data using 9 clusters. Although Figure 4 indicates that the data tends to separate roughly into ‘skilled’ and ‘unskilled’ regimes, we opted to choose 9 clusters in the Gaussian mixture model after several sensitivity tests across all pressure levels found 9 to be the minimum number of clusters needed to ensure the separation into these separate regimes, as well as allowing for further stratification of sub-regimes within the skilled and unskilled regimes

By re-analyzing the AMV estimate in relation to the simulated true winds, separated into the cluster that each point has been assigned to (Figure 6), we find that the clustering approach successfully separates the AMV estimates according to their ‘skillfulness’. Essentially, we repeat Figure 3 but divide the AMV estimates by cluster. We see that,



for example, clusters 4, 5, and 7 clearly represent cases in which the feature-tracking algorithm provides an accurate estimate of the true winds, with very low variance around the one-to-one line (i.e., low estimate errors). Clusters 1, 2, 3, and 9 are somewhat noisier than the low-variance clusters, with error characteristics similar to those of the entirety of the dataset. Clusters 6 and 8, on the other hand, are clearly unskilled in different ways. Cluster 6 is a noisy regime, which captures much of the more extreme differences between the AMV estimates and the simulated true winds. Cluster 8, on the other hand, represents the low AMV estimate, high true wind regime. This cluster is returning AMVs with values of zero where the true wind is clearly non-zero because of the very low water vapor present. We see that the clustering algorithm succeeds in capturing physically interpretable clusters without having any knowledge of the underlying physical dynamics. We further see the stratification of the regimes when analyzing the absolute AMV error in relation to the water vapor content (Figure 7). We see that clusters that have similar behaviors in the error pattern (such as 1, 2, and 3) represent different regimes of water vapor content.

### 3.5 Random Forest

The clustering algorithm requires the true wind vector component values ( $u$  and  $v$ ) in order to classify the AMV error. When applying the algorithm in practice to tracked AMV wind from real observations, the true winds are unknown. Therefore, we develop a proxy for the true winds using only the AMV estimates and the simulated water vapor itself. This is an instance in which the application of machine learning is desirable, since machine learning excels at learning high-dimensional non-linear relationships from large training datasets. In this case, we specifically use random forest to create an algorithm which predicts the true wind values as a function of the tracked wind values and water vapor.

Random forest is a machine learning regression algorithm which, as detailed by Breiman (2001), employs an ensemble of decision trees to model a nonlinear relationship between a response and a set of predictors from a training dataset. Here, we chose random forest specifically because it possesses certain robustness properties that are more appropriate for our applications than other machine learning methods. For instance, random forest will not predict values that are outside the minimum and maximum range of the input dataset, whereas other methods such as neural networks can certainly exceed the training range, sometimes considerably so. Random forest, due to the sampling procedure employed during training, also tends to be robust to overtraining in addition to requiring fewer tuning parameters compared with methods such as neural networks.

We trained a random forest with 50 trees on a separate set of tracked winds and water vapor values to predict true winds using the ‘randomForest’ package in R. While the random forest estimate as a whole does not perform much better than the AMV values in estimating the true wind (2.89 RMSE for random forest vs 2.91 RMSE for AMVs), as shown in Figure 8, it does not display the same discrete regimentation as the AMV estimates in Figure 3. Relative to the AMV estimates, the error in each of the random forest estimates is closer to the mean of error of the entire dataset. As such, the random forest estimates can act as a proxy for true wind values in our clustering algorithm — they remove the regimentation which is a critical distinction between the AMV estimates and the true wind values.





### 228 3.6 Finalized Error Characterization Model

229 The foundation of the error characterization approach is to combine the random forest and clustering algorithm. We  
 230 apply the Gaussian mixture model, as trained on the true winds (in addition to the AMVs and water vapor), to each  
 231 point of water vapor, AMV estimate, and associated random forest estimate. This produces a set of clusters which,  
 232 when implemented, require no direct knowledge of the actual true state (Figure 9). We see that the algorithm manages  
 233 to separate the AMV estimates into appropriate error clusters. Once again, clusters 6 and 8 manage to capture unskilled  
 234 regimes, and clusters 4 and 5 remain extremely skillful. While there is some degradation in the performance relative  
 235 to the classification algorithm on the training set, we see in Figure 9 and Figure 10 that the error characterization still  
 236 discretizes the testing data set into meaningful error regimes.

237 By taking the mean and standard deviation of the difference between AMV estimates and true winds in each cluster,  
 238 we develop error characteristics for each cluster (Figure 11); these quantities are precisely the bias and uncertainty  
 239 that we require for the cost function  $J$  in Eq (1). We see that the unskilled clusters have very high standard errors and  
 240 they correspond roughly to the areas of unskilled regimes in Figure 3. Since each cluster now has associated error  
 241 characteristics (e.g., bias and standard deviation), it is then straightforward to assign the bias and uncertainty for any  
 242 new tracked wind observation by computing which regime it is likely to belong to.

### 243 3.7 Experimental Set up

244 In this section we will describe our experimental setup for training the data and testing its performance on a withheld  
 245 dataset. We divide the dataset into two parts: a training set consisting of the first 1.5 months of the GEOS-5 Nature  
 246 Run, and a testing set consisting of the last 0.5 month of the Nature Run. Our training/testing procedure for the  
 247 simulation data and tracked wind is as follows:

- 248 1. Divide the simulation data and tracked wind into two sets: training set of 1,000,000 points from the first 1.5  
 249 months of the Nature Run and a testing set of 1,000,000 points from the final 0.5 months of the Nature Run.
- 250 2. Using the ‘density.Mclust’ function, we train a Gaussian Mixture Model on a normalized random sample of  
 251 observations from the training dataset of true winds (u and v direction), tracked winds (u and v direction),  
 252 and water vapor with  $n=9$  clusters.
- 253 3. We train two separate random forests on a different random sample of 750,000 observations from the training  
 254 dataset. We use tracked wind (u and v direction) and water vapor to model, separately, true winds in both the  
 255 u and v directions.
- 256 4. We apply the random forests to the dataset used for the Gaussian Mixture Model. This provides a random  
 257 forest estimate for each point, which is used as a substitute for true wind values in the next step.
- 258 5. Using the ‘predict.Mclust’ function, we predict the Gaussian mixture component assignment for each point  
 259 of water vapor, tracked winds, and random forest estimate.





- 260 6. We compute the mean and standard deviation of the difference between the tracked winds and the true winds,  
 261 per direction, for each Gaussian mixture model cluster assignment. This provides a set of error characteristics  
 262 that are specific to each cluster.
- 263 7. We can apply the random forest, and then the cluster estimation, to any set of water vapor and tracked AMV  
 264 estimates. Thusly, any set of tracked AMV estimates and water vapor can be mapped to a specific cluster,  
 265 and therefore its associated error characteristics.

#### 266 4 Results and Validation

267 In this section, we compare our clustering method against a simple alternative, and we quantitatively demonstrate  
 268 improvements that result from our error characterization. Recall that in Section 3, we divided the wind-tracking  
 269 outputs into 9 regimes, which range from very skilled to unskilled. For each regime, we can quantify the uncertainty  
 270 via a 95% confidence interval, which in the Gaussian case can easily be constructed as  $[x_i - 2 \sigma_i, x_i + 2 \sigma_i]$ , where  $x_i$   
 271 the predicted mean and  $\sigma_i$  is the predicted standard deviation of the  $i$ -th cluster. To test the bias of our confidence  
 272 interval, we divide the dataset described in Section 2 into a training dataset (first 1.5 month) and a testing dataset (last  
 273 0.5 month). Having trained our model using the training dataset, we apply the methodology to the testing dataset, and  
 274 we compare the performance of the predicted confidence intervals against the actual wind error (tracked winds - true  
 275 winds). This is a type of probabilistic forecast assessment, and in this paper we assess the quality of the prediction  
 276 using a scoring rule called continuous ranked probability score, which is defined as a function of a probabilistic  
 277 forecast  $F$  (here represented by our confidence interval) and an observation  $x$  as follows:

$$278 \quad \text{CPRS}(F, x) = \int_{-\infty}^{\infty} (F(x) - \mathbb{1}(y - x))^2 dy \quad (4)$$

279 Where  $\mathbb{1}(\cdot)$  is the Heaviside step function and denotes a step function along the real line that is equal to 1 if the argument  
 280 is positive or zero, and it is equal zero if the argument is negative. The continuous rank probability score here is strictly  
 281 proper, which means that the function  $\text{CPRS}(F, x)$  attains the maximum if the data  $x$  is drawn from the same probability  
 282 distribution as the probabilistic forecast  $F$ . That is, if the data  $x$  is drawn from  $F$ , then  $\text{CRPS}(F, x) \leq \text{CRPS}(G, x)$  for  
 283 all  $G \neq F$ .

284 The alternative error characterization method that we test against is a simple marginal mean and marginal standard  
 285 deviation of the entire track - true wind datasets. This is essentially equivalent to an error characterization scheme that  
 286 utilizes one regime, and its confidence interval similarly could be constructed as  $[x - 2 \sigma, x + 2 \sigma]$ , where  $x$  and  $\sigma$  are  
 287 the marginal mean and marginal standard deviation of the residuals (i.e., tracked wind minus true winds). Here, we  
 288 use a negatively oriented version of the CRPS (i.e., Eq.(4) without the minus sign), which implies that lower is better,  
 289 to evaluate the performance of our methodology against the naive error characterization method. We plot the histogram  
 290 of the scores in Figure 12.



291 The relative behavior of the CRPS is consistent between  $u$  and  $v$  winds. The CRPS tends to have a wider distribution  
 292 when applied to the regime-based error characterization. Compared to the alternative error characterization scheme,  
 293 our methodology produces a cluster of highly accurate predictions (low CRPS scores), in addition to some cluster of  
 294 very uninformative predictions (high CRPS scores). These clusters likely correspond to the highly skilled cluster (e.g.,  
 295 Cluster 3) and the unskilled clusters (Cluster 5 and 8), respectively. Overall, the mean of the CRPS is lower for our  
 296 methodology than it is for the alternative method, indicating that as a whole our method produces a more accurate  
 297 probabilistic forecast.

298 Thus far we have shown that our method produces more accurate error-characterization than an alternative method  
 299 based on marginal means and variance. Now, we assess whether our methodology provides valid probabilistic  
 300 prediction; that is, we test whether the uncertainty estimates provided are consistent with the empirical distribution of  
 301 the validation data. To assess this, we construct a metric in which we normalize the difference between the true wind  
 302 and the tracked wind by the predicted variance. That is, we compute the normalized values for  $u$  and  $v$  using the  
 303 following equations:

$$304 \quad z_u = \frac{u - \hat{u}}{\sigma_u}$$

$$305 \quad z_v = \frac{v - \hat{v}}{\sigma_v} \quad (5)$$

306 Where  $u$  is the true  $u$  wind from the Nature Run data,  $\hat{u}$  is the tracked-wind, and  $\sigma_u$  is the error as assessed by our  
 307 model. The values for the  $v$ -wind are defined similarly. The residuals in Eq (5) can be considered as a variant of the  
 308  $z$ -score, and it is straightforward to see that if our error estimates are valid (i.e., accurate), then the normalized residuals  
 309 in Eq. (5) should have a standard deviation of 1. In Figure 12, we display the histogram of the normalized residuals  
 310  $z_u$  and  $z_v$ . It is clear that for both types of wind, our error characterization methodology produces highly accurate  
 311 uncertainties (std = 1.003 and 1.009 for  $u$  and  $v$ , respectively).

## 312 5 Conclusion

313 Uncertainty quantification, which is the quantification of an imperfect or incomplete state of knowledge within a  
 314 model, is an important component of data validation and scientific analysis. For wind-tracking algorithms, whose  
 315 outputs (tracked  $u$  and  $v$ ) are often used as observations in data assimilation analyses, it is necessary to accurately  
 316 characterize the bias and standard error (e.g., see Section 2.2). Nguyen et al. (2019) illustrated that incorrect  
 317 specification of these uncertainties ( $a$  and  $R$  in Eq. (1)) can adversely affect the assimilation results –  
 318 mischaracterization of bias will assimilate an incorrect tracked wind, while an erroneous standard error could  
 319 incorrectly weight the cost function.



320 In this paper, we develop an error-characterization scheme based on random forest and mixture model clustering.  
321 Here, the mixture of a parametric approach and a machine learning method allows us to combine the flexibility of  
322 machine learning with the interpretability of mixture modelling in an entirely data-driven framework. In theory, the  
323 fidelity of our method should scale with the number of training data observations, making the methodology well-  
324 suited for the massive datasets that are typical within remote sensing applications. Our error function has been applied  
325 to an AMV OSSE study using GEOS5 and its impact will be reported in a forthcoming paper.

326 We demonstrate that our methodology produces accurate error estimates (also called validity), and that it is able to  
327 identify and remove the biases within the wind-tracking algorithm's outputs. Particularly, the methodology is able to  
328 identify unskilled regimes that are physically meaningful — in our case, unskilled regimes related to regions of near-  
329 zero water vapor content. We note that our methodology is able to find this dependence between unskilled regimes  
330 and low water content without any prior knowledge or specification from the user, deducing the relationship from the  
331 underlying multivariate distribution of water vapor, true wind, and tracked wind. While we position the methodology  
332 as an error characterization tool, this property also makes it useful as an exploratory tool to aid in understanding the  
333 distribution of multivariate and potentially complex data.

334 Our algorithm consists of two parts: an emulator and a clustering algorithm. In this implementation, random forest  
335 and Gaussian mixture modelling are the approaches; in theory, these two steps could be accomplished using other  
336 algorithms belonging to the appropriate class. Future research includes replacing random forest with other machine  
337 learning methods such as neural networks or support vector machines, and investigating other methods of clustering,  
338 such as self-organizing networks. We note that the issue of bias removal in data assimilation and in remote sensing is  
339 certainly not limited to atmospheric motion vectors. The methods we have used to characterize uncertainties in AMVs  
340 are general, and can be applied to other inverse problems as well.

#### 341 **Author Contribution**

342 Teixeira and Nguyen conceived of the idea. Teixeira performed the computation. Wu provided the experimental  
343 datasets. Posselt and Su provided subject matter expertise. All discussed the results. Teixeira wrote the manuscript.  
344 All authors contributed to the subsequent draft.

345 **Competing Interest:** The Authors declare no conflict of interest.

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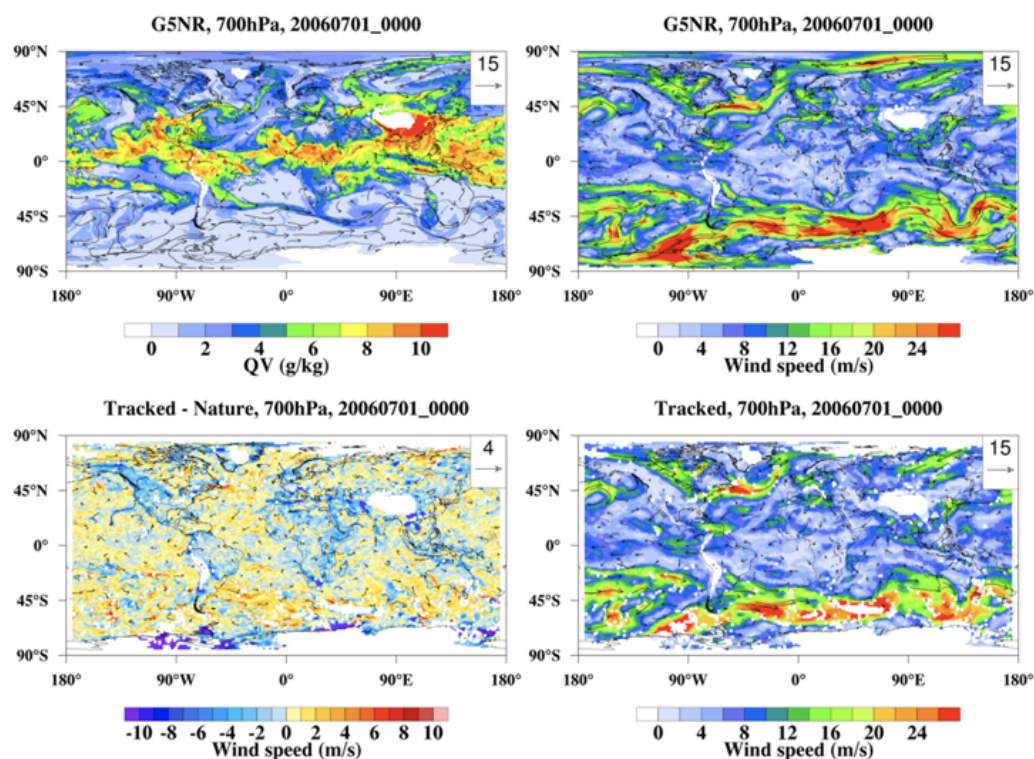
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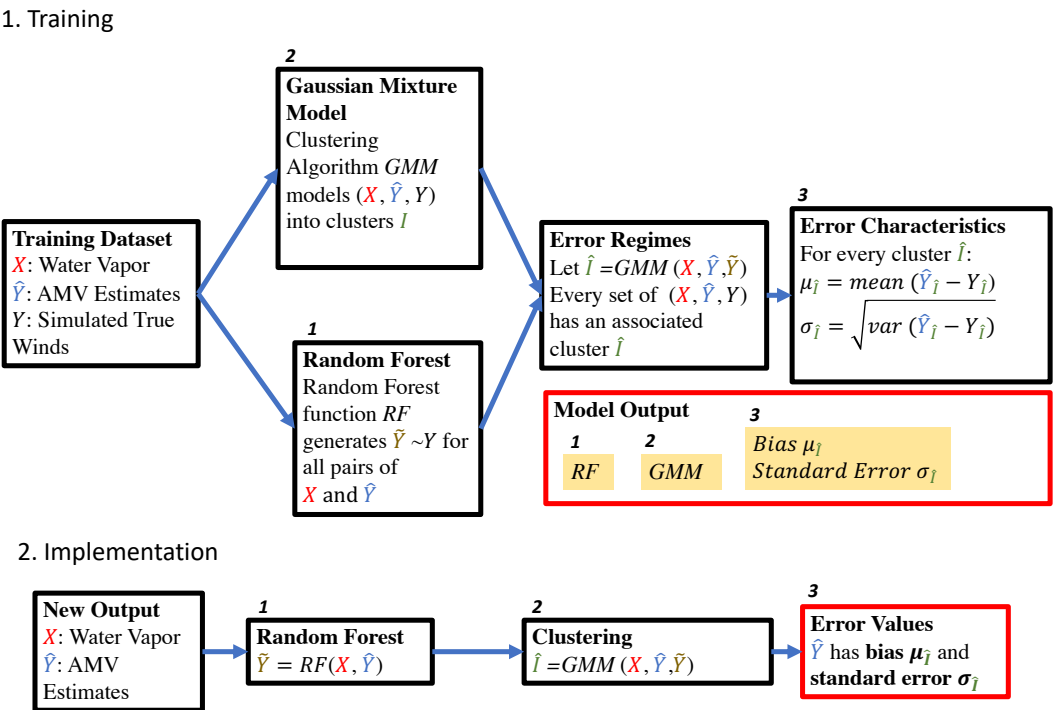
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**Figure 1:** Map of Nature Run at one timestep at 700hPa (A): Water Vapor (B): True Wind Speed (C): Difference between True Wind Speed and AMV Estimate (D): AMV Estimate.



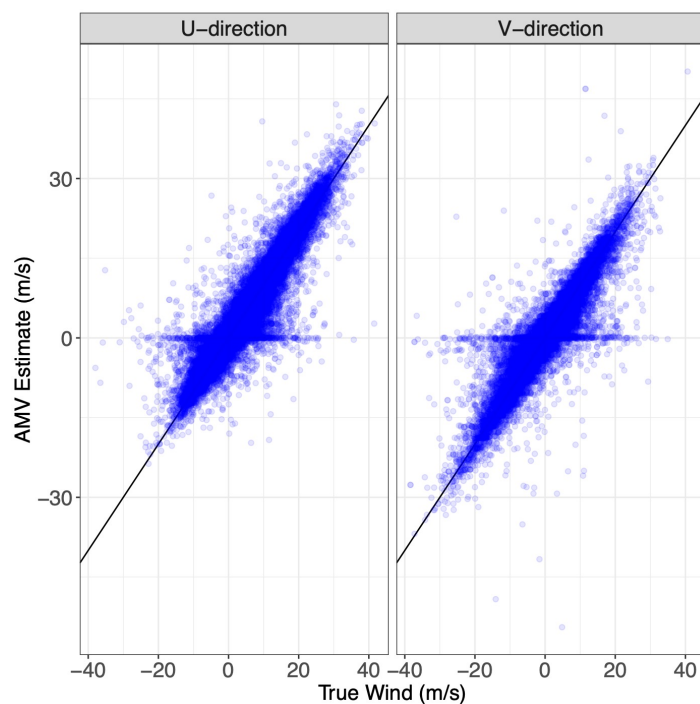
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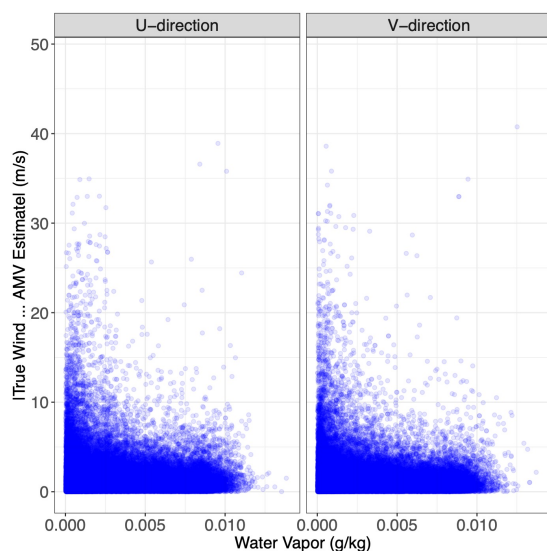
405 **Figure 2: Diagram of Training Approach and Diagram of Implementation steps.**





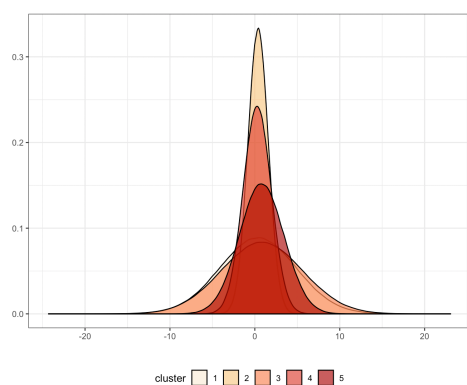
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407 **Figure 3: Scatter plot of the simulated true wind vs AMV estimates for u and v wind.**



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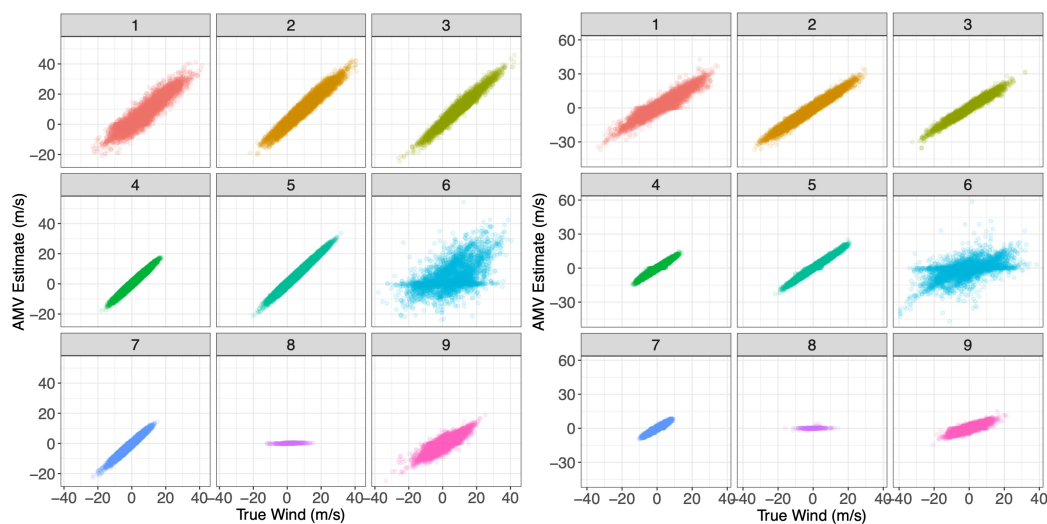
409 **Figure 4: Simulated water vapor vs the absolute value of the difference between true and tracked winds.**



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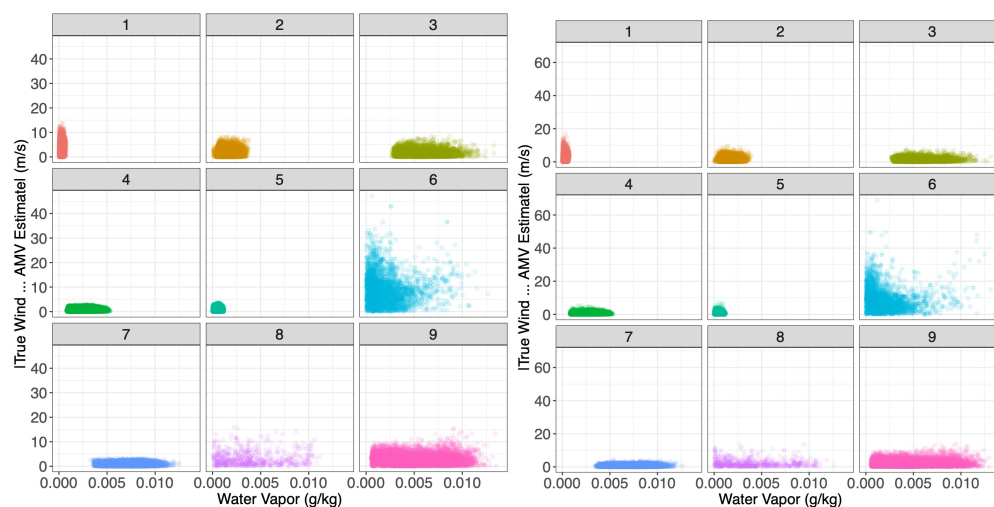
411 **Figure 5: Example of Gaussian Mixture Model in one dimension. Density Figures for the U-Direction AMV**  
 412 **Estimate dimension of fitted Gaussian mixture.**

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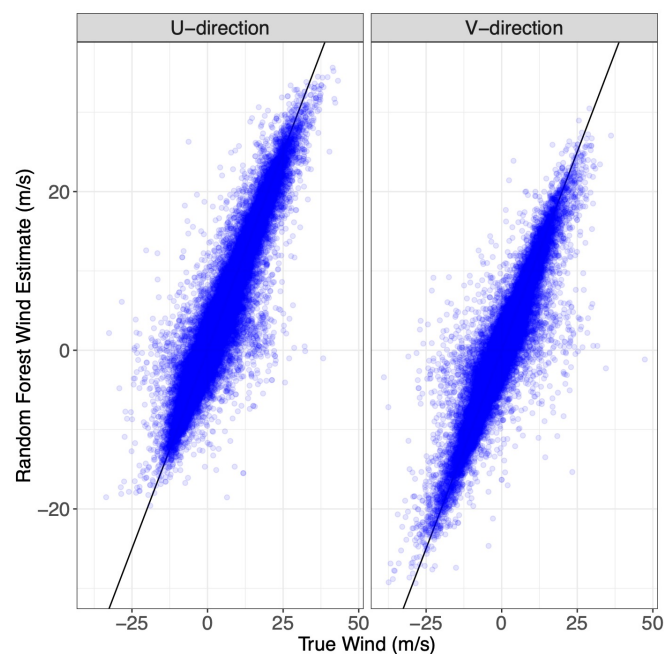
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415 **Figure 6: Scatterplot of simulated true wind vs AMV Estimates, each sub-panel corresponding to the specific**  
 416 **Gaussian mixture component to which each point has been assigned. (A): U-Direction Wind (B): V-Direction**  
 417 **Wind.**



418

419 **Figure 7: Scatterplot of Water Vapor vs Absolute Tracked Wind Error, each sub-panel corresponding to the**  
 420 **specific Gaussian mixture component to which each point has been assigned. (A): U-Direction Wind (B): V-**  
 421 **Direction Wind.**



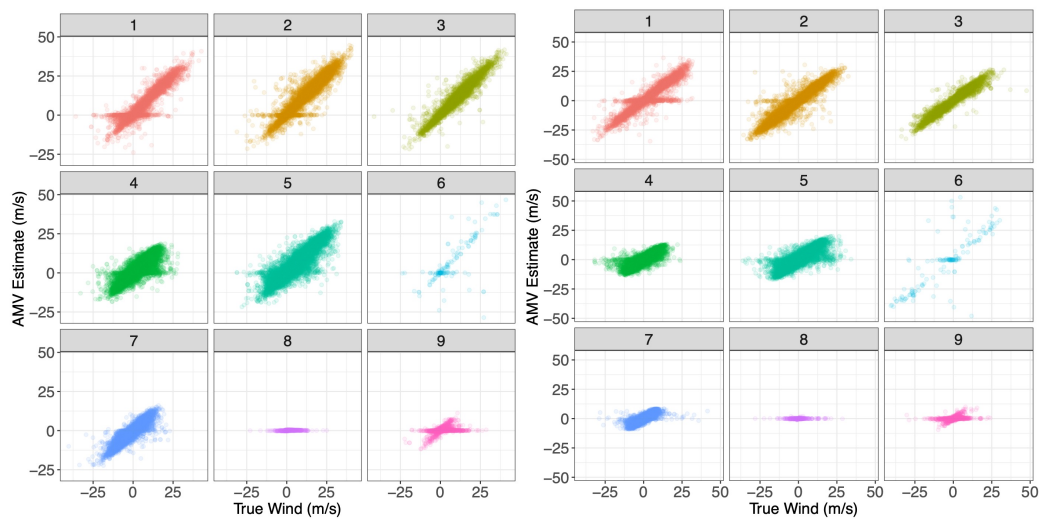
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423 **Figure 8: Scatterplot of true wind estimate vs random forest produced estimate. (A): U Direction (B): V**  
 424 **Direction**



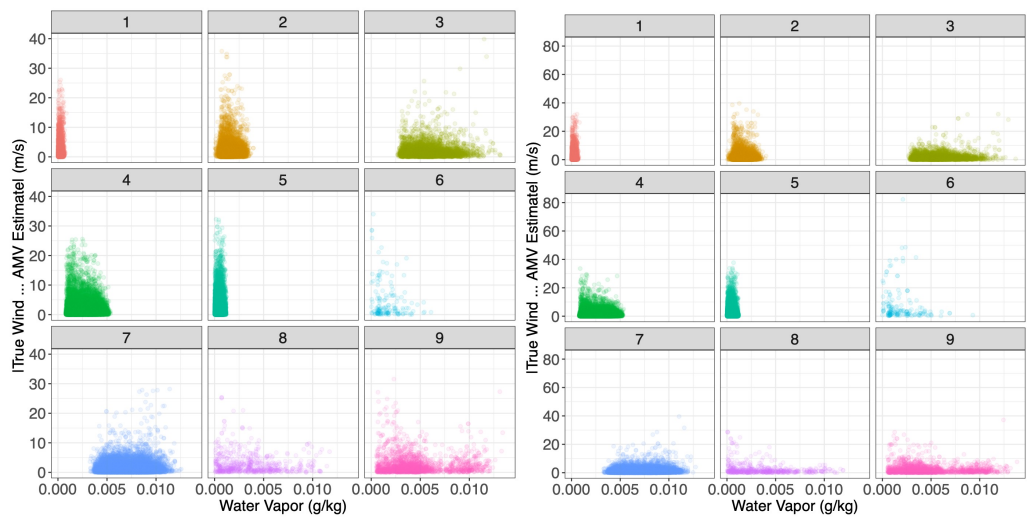
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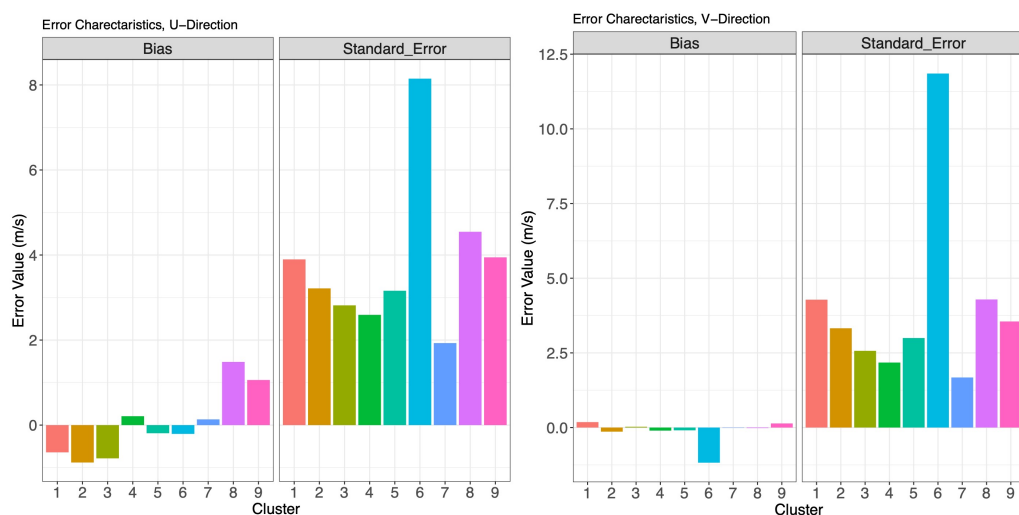


427 **Figure 9: Scatterplot of simulated true wind vs AMV Estimates, each sub-panel corresponding to the specific**  
428 **Gaussian mixture component to which each point has been assigned when the true wind value has been**  
429 **substituted by the random estimate. (A): U-Direction Wind (B): V-Direction Wind**

430

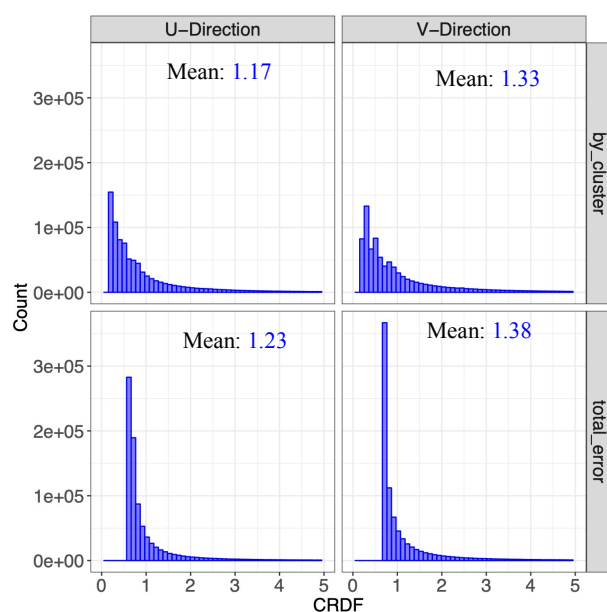


431 **Figure 10: Water Vapor vs Absolute Tracked Wind Error, each sub-panel corresponding to the specific**  
432 **Gaussian mixture component each point has been assigned when the true wind value has been substituted by**  
433 **the random estimate. (A): U-Direction Wind (B): V-Direction Wind**



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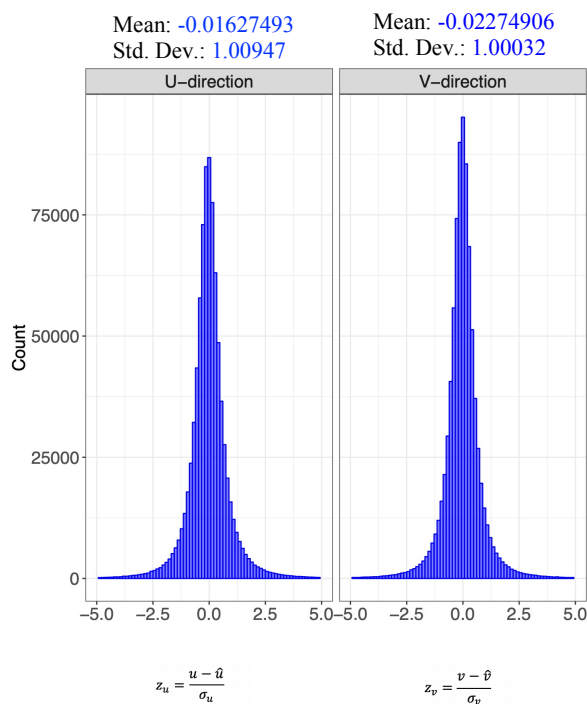
435 **Figure 11: (A): Bias (Left Panel) and Standard Error (Right Panel) for each Gaussian mixture cluster in**  
 436 **figure 6, U direction. (B): Same as (A) for V-direction**



437

438 **Figure 12: CRSP applied to different error approaches. (A): Cluster Errors for U Winds (B): Total Errors**  
 439 **for U Winds (C): Cluster Errors for V Winds (D): Total Errors for V Winds.**

440



**Figure 13: U and V winds normalized using Error Clusters**