

1 **Field Testing Two Flux Footprint Models**

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7 **Abstract**

8 A field study was undertaken to investigate the accuracy of two micrometeorological flux footprint models for calculating the gas
9 emission rate from a synthetic 10 x 10 m surface area source, based on the vertical flux of gas measured at fetches of 15 to 50 m
10 downwind of the source. Calculations were made with an easy to use tool based on the Kormann-Meixner analytical model and
11 with a more sophisticated Lagrangian stochastic dispersion model. A total of 59 testable 10 minute observation periods were
12 measured over nine days. On average, both models underestimated the actual release rate by approximately 30%, mostly due to
13 large underestimates at the larger fetches. The accuracy of the model calculations had large period-to-period variability, and no
14 statistical differences were observed between the two models in terms of overall accuracy.

15 **1 Introduction**

16 Micrometeorological techniques such as eddy covariance and flux-gradient measure a vertical flux of gas in the atmosphere, which
17 can be used to deduce the flux from an underlying surface area of interest. If the underlying surface is expansive and horizontally
18 homogenous, the measured atmospheric flux and the surface flux can be considered equivalent (Dyer, 1963). However, if the area
19 of interest has a limited spatial extent, or is located some distance from the atmospheric measurement, the relationship between the
20 two fluxes can be complex, as the measured flux may be capturing a dynamic mixture of surface fluxes from both inside and
21 outside the area of interest. In these cases, flux footprint modelling can be used to quantify the relationship between the measured
22 atmospheric flux and the surface flux from the area of interest.

23 The analytical flux footprint model of Kormann and Meixner (2001), hereafter referred to as the KM model, is widely used to
24 evaluate and interpret flux measurements taken over spatially limited surface sources. The KM model relies on a simplified
25 representation of atmospheric transport (Schmid, 2002) to create an easily computable footprint. It has been used to help quantify
26 ammonia fluxes from fertilized plots (Spirig et al., 2010), interpret methane fluxes from heterogeneous peatland areas (Budishchev
27 et al., 2014), and to reject periods where the footprint extends outside the source of interest (Stevens et al., 2012). Other footprint
28 models use a more realistic treatment of atmospheric transport (e.g., Kljun et al., 2002; Sogachev and Lloyd, 2004). Using a state
29 of the art Lagrangian stochastic (LS) footprint model, Wilson (2015) found a clear separation between the footprints computed
30 with the LS and KM models, depending on atmospheric stability and the distance from the measurement location. While more
31 rigorous footprint models are clearly more defensible, the simpler KM model has the advantage of rapid analysis and the existence
32 of software tools that make its application more accessible to non-specialists (Nefel et al., 2008).

33 This field study compares the accuracy of the KM footprint model with a more rigorous LS model. The motivation for this study
34 was the question of whether the accuracy of the LS model was sufficiently better than the KM model so as to justify a more
35 complex LS application. In this experiment we released gas at a known rate from a small synthetic area source and measured the
36 vertical gas flux at a downwind location using the eddy covariance technique. The KM and LS models were then used to calculate
37 the source emission rate from the measured atmospheric flux. The accuracy of those calculations is examined in this report.

38 **2 Methods**

39 **2.1 Gas Release**

40 The experiment took place on an extensive, flat agricultural field at the University of Alberta's Breton Research Farm, in Alberta,
41 Canada (53° 07' N, 114° 28' W). Measurements were made after autumn harvest, and the surface was rye (*Secale cereale* L.)
42 stubble with an average height of 3 cm. No obstructions to the wind were present within 250 m of the measurement site.

43 A synthetic source of carbon dioxide (CO₂) gas was constructed using 10 lengths of ½" (12.7 mm) diameter PVC pipe, each 10 m
44 long. The 10 pipes were loosely positioned to create a nominal 10 x 10 m square source area. Compressed CO₂ gas (99.9 % purity)
45 passed through a mass flow controller (Aalborg Instruments and Controls, Inc. Orangeburg, NY, USA) to a manifold (17 L) having
46 outlets for each of the 10 pipes. Gas outlets of 1/16" (1.6 mm) diameter were placed every 50 cm along each pipe. We assumed
47 equal flow rates from each outlet due to the high head loss across each outlet relative to the manifold pressure (e.g., Flesch et al.,
48 2004).

49 The vertical CO₂ flux downwind of the synthetic source was measured using the eddy covariance (EC) technique. The
50 instrumentation included a fast-response CO₂/H₂O analyser (Li-7500DS, Licor Biosciences, Lincoln, NE, USA) and a sonic
51 anemometer (CSAT-3, Campbell Scientific, Logan, UT, USA) co-located at a height of 1.97 m above ground. The 10 Hz
52 concentration and wind measurements were processed using the EddyPro® open source software (version 6.2.1 LI-COR
53 Biosciences, Lincoln, NE, USA) to obtain 10 minute (min) average fluxes of CO₂. The flux calculation applied a double coordinate
54 wind rotation, Webb-Pearman-Leuning correction terms for density fluctuations (Webb et al., 1980), and spectral corrections for
55 inadequate high and low frequency response of the sensors (Moncrieff et al., 1997, 2004). Quality checks for steady state conditions
56 and integral turbulence characteristics were used to exclude error-prone periods (Foken and Wichura, 1996).

57 Gas releases took place over nine days, with the center of the synthetic source positioned (Fig. 1) at one of three nominal distances
58 from the EC system (fetches of 15, 30, and 50 m). Placement of the source relative to the EC system depended on the expected
59 wind direction. Because CO₂ is naturally emitted from the landscape it was important that the synthetic CO₂ release rate be
60 sufficiently high so as to create a measured atmospheric flux that was many times larger than the natural landscape flux. Nicolini
61 et al. (2017) found a CO₂ release rate of 22 L min⁻¹ was sufficient to distinguish the release signal from background levels. Our
62 situation was helped in that the experiment took place during the dormant autumn season when landscape CO₂ fluxes were small.
63 Gas was released at rates between 30 and 90 L min⁻¹, with larger rates used for the larger fetches. Prior to any release interval, and
64 immediately after each hour of gas release, a 30 min period of background CO₂ flux was measured. These background fluxes
65 (which were consistently small) were subtracted from the EC measured fluxes prior to undertaking the footprint analyses.

66 Our study consisted of more than 300 10 min flux measurement periods, and included periods of gas release, background flux
67 measurements, and transitions when gas was released but a steady state plume may not have been established over the field site
68 (we assumed this occurred 10 min after gas was turned on). There was a total of 125 valid gas release periods. From this total we
69 excluded 66 periods from our analysis based on two broad factors:

- 70 • 19 periods were excluded for having wind conditions associated with unreliability in the EC measurements or the
71 dispersion model calculations: light winds with a friction velocity $u^* < 0.05 \text{ m s}^{-1}$, or an inferred roughness length $z_0 >$
72 0.25 m . A low u^* filtering criterion is often used in EC analyses (e.g., Rannik et al., 2004) and in dispersion model
73 calculations (e.g., Flesch et al., 2014). The z_0 filtering criterion indicates an unrealistic wind profile given the bare soil
74 conditions of our site, and the likelihood of inaccurate dispersion model calculations given that wind profile.
- 75 • 47 periods were excluded when the EC measurement location was not obviously in the source plume. This included
76 periods when the measured CO₂ flux was less than zero, when the wind direction deviated more than 30 degrees from the
77 line between the EC site and the source center, or when the LS footprint model (described below) indicated the plume
78 may not have reached the EC measurement site (i.e., fewer than 1,000 of 1,000,000 backward trajectories released from
79 the EC site reached the source).

80 These quality control criteria eliminated over half of the gas release periods, leaving 59 periods for the footprint analysis. The
81 final data are provided in the supplemental material accompanying this report.

82 2.2 Flux Footprint Models

83 2.2.1 Kormann and Meixner (KM) Model

84 The KM model is based on an analytical solution to the steady-state advection-diffusion equation, assuming simplified power-law
85 profiles for windspeed and eddy diffusivity, and a crosswind diffusion component (Kormann and Meixner, 2001). We used the
86 ART Footprint tool software (Nefel et al., 2008) based on the KM model to calculate the synthetic source emission rate (Q_{KM} , g
87 $C\ m^{-2}\ s^{-1}$) from the measured EC flux. The calculation uses the spatial outline of the source polygon, the EC measurement height
88 (z_{EC}), the horizontal wind speed at height z_{EC} , the friction velocity (u^*), the standard deviation of the lateral wind velocity (σ_v), and
89 the Obukhov length (L). The wind variables were measured with a 3-D sonic anemometer (part of the EC system). In this study,
90 the ratio of the KM-calculated synthetic emission rate to the actual release rate (Q_{KM}/Q) is the metric for model testing. A perfectly
91 accurate calculation gives $Q_{KM}/Q = 1$.

92 2.2.2 Lagrangian Stochastic (LS) Model

93 A state of the art LS model was also used to calculate the emission rate from the synthetic source (Q_{LS} , g $C\ m^{-2}\ s^{-1}$) based on the
94 measured EC flux. The relationship between the source emission rate and the EC flux was calculated from the trajectories of
95 thousands of model “particles” travelling upwind from the EC measurement point (backward in time). We follow the calculation
96 procedure outlined in Flesch (1996) using the LS model detailed in Flesch et al. (2004). This model uses the wind velocity
97 fluctuations in the three directional components (σ_u , σ_v , σ_w), the friction velocity (u^*), the Obukhov stability length (L), the average
98 wind direction, and the surface roughness length (z_0). These properties were calculated from the 3-D sonic anemometer
99 measurements. The LS calculations were made using 1,000,000 particles for each 10 min observation interval. A perfectly accurate
100 LS model calculation gives $Q_{LS}/Q = 1$.

101 2.3 Statistical Analysis

102 The accuracies of the footprint calculations are evaluated from the ratio of the model calculated emission rate to the actual release
103 rate: Q_{KM}/Q and Q_{LS}/Q . With ratio data the geometric mean is a more meaningful measure of the central tendency than is the
104 arithmetic mean (Fleming and Wallace, 1986), and we use the geometric mean to describe our ratio data. Confidence intervals for
105 the geometric mean are calculated using the log-transformed ratio data, and then converted back to ratio units (Limpert et al.,
106 2001). The confidence intervals (CI) are asymmetrical, and we report the upper and lower limits of the intervals.

107 3 Results and Discussion

108 The synthetic emission rates calculated with both footprint models underestimate the actual emissions by roughly 30% on average.
109 The overall means of the footprint calculations, expressed as the ratio of the model calculated emission rate to the actual emission
110 rate, are $Q_{KM}/Q = 0.67$ (95% CI: 0.50, 0.89) and $Q_{LS}/Q = 0.77$ (CI: 0.60, 0.98). These means are statistically less than 1.0, but
111 not different from each other (paired t-tests with $P_s > 0.05$). The period-to-period variability in the Q/Q ratios is large, with Q_{KM}
112 $/Q$ ranging between 0.04 and 2.20 and Q_{LS}/Q between 0.06 and 4.44. Some of the variability is likely due to the small size of the
113 area source. The 10 x10 m source covers a small portion of the entire flux footprint. As opposed to larger source areas, the small
114 area should amplify the differences between the models, and increase the relative uncertainty in the footprint calculations (i.e.,
115 increasing the size of the source area means increasing the spatial integration of the footprint function in the calculations, which
116 acts to increasingly constrain the Q/Q values closer to one).

117 When examining the footprint agreements as a function of fetch (Fig. 2), we find both models are accurate at the shorter fetch of
118 15 m, as the means of Q_{KM}/Q and Q_{LS}/Q are not statistically different from 1. At the 15 m fetch the Q_{KM} calculation tends to
119 slightly overestimate the actual emission rate with $Q_{KM}/Q = 1.17$ (CI: 1.00, 1.36), while Q_{LS} tends to slightly underestimate it
120 with $Q_{LS}/Q = 0.84$ (CI: 0.68, 1.04). Based on the modelling results of Wilson (2015), we had hypothesized that there would be
121 substantial differences between the two models at the shorter fetch, with the LS model being more accurate than KM due to a better
122 representation of horizontal turbulent transport, which is particularly important for defining the footprint at short fetches. However,
123 this is not the case in this study. At the intermediate fetch of 30 m, the KM model slightly overestimates the emission rate with
124 $Q_{KM}/Q = 1.21$ (CI: 0.86, 1.71), while the LS model substantially overestimates it with $Q_{LS}/Q = 1.75$ (CI: 1.39, 2.21). At the
125 larger 50 m fetch, both models substantially underestimate the emission rate, with Q_{KM} underestimating Q by a factor of three and
126 Q_{LS} underestimating by a factor of two (on average). The underestimate of Q_{KM}/Q at the larger fetch is similar to findings by
127 Tallec et al. (2012) and Felber et al. (2015).

128 In Figure 3 we show the Q/Q ratios grouped according to atmospheric stability. The observations are separated into three groups
129 having nearly equal numbers of observations: neutral ($|L| > 60$ m), unstable ($0 > L > -60$ m), and stable ($60 > L > 0$ m). For the
130 neutral and unstable groups, the mean Q/Q from both models does not statistically differ from 1, nor does it differ between groups
131 due to the large variability in the calculations. However, in stable conditions both models are inaccurate and they substantially
132 underestimate the actual emission rate. A more detailed look at the stable cases shows the Q_{KM}/Q calculations are particularly
133 inaccurate for the 50 m fetch, with a mean of 0.14 (CI: 0.03, 0.62).

134 There are no clear patterns in terms of explaining the differences between the two footprint models based on environmental factors.
135 Whether we separate the data by fetch or by stability, the results from the two models are not statistically different from each other.
136 Wind speed, roughness length, and wind direction (deviation from a line between the EC system and the source) were also
137 considered as factors to explain the model differences, but again, no pattern was observed. The lack of model differences was
138 unexpected given the studies of Göckede et al. (2005) and Wilson (2015) showing large differences in the calculations between
139 analytical and LS models. This suggests that in our study, any systematic differences between the models were obscured by the
140 substantial period-to-period variability in the Q/Q calculations, and that the detection of model differences would require a much
141 larger observational sample size than we were able to acquire.

142 **4 Conclusions**

143 From an end-user's perspective, our results show that both the KM and LS models returned reasonably accurate flux footprint
144 estimates on average, particularly for the shorter measurement fetches. Our dataset does not consistently discriminate between the
145 performance of the two models, despite the theoretical advantages of the LS model. Based on the results of this study, we conclude
146 that the easy to use KM model can provide accurate footprint calculations that are accessible to non-specialists.

147 It is clear that the KM and LS footprint models give systematically different results (as shown in Wilson 2015); but that we were
148 unable to (statistically) observe these differences given the large period-to-period variability in the calculations and the relatively
149 small number of field observations. The small area of our synthetic source likely contributed to the large variability, and a larger
150 source may have allowed better differentiation between the models. However, period-to-period variability is the nature of footprint
151 calculations based on simplified models of atmospheric transport like the KM and LS formulations. These model calculations,
152 which at best approximate an ensemble average realization of the atmosphere, will not reflect the period-to-period fluctuations of
153 actual measurement periods.

154 **Data availability**

155 The data used in this analysis are available as supplementary material, or by request to Trevor.Coates@Canada.ca.

156 **Author contributions**

157 TC analyzed the field data and helped write the manuscript. MA design the experiment, and coordinated and collected the field
158 data. TF helped with the experimental design and data analysis, and reviewed the manuscript. GHR helped with the experimental
159 design and reviewed the manuscript.

160 **Competing interests**

161 The authors declare that they have no conflict of interest.

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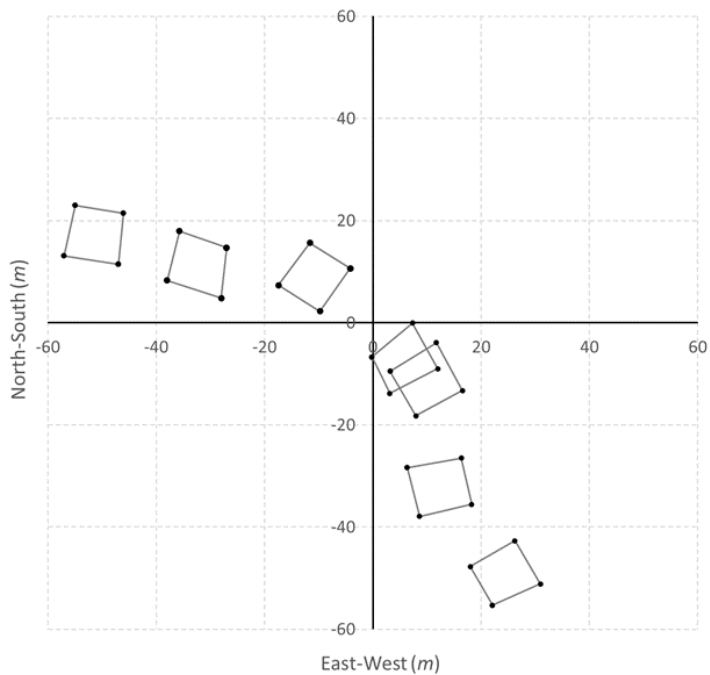
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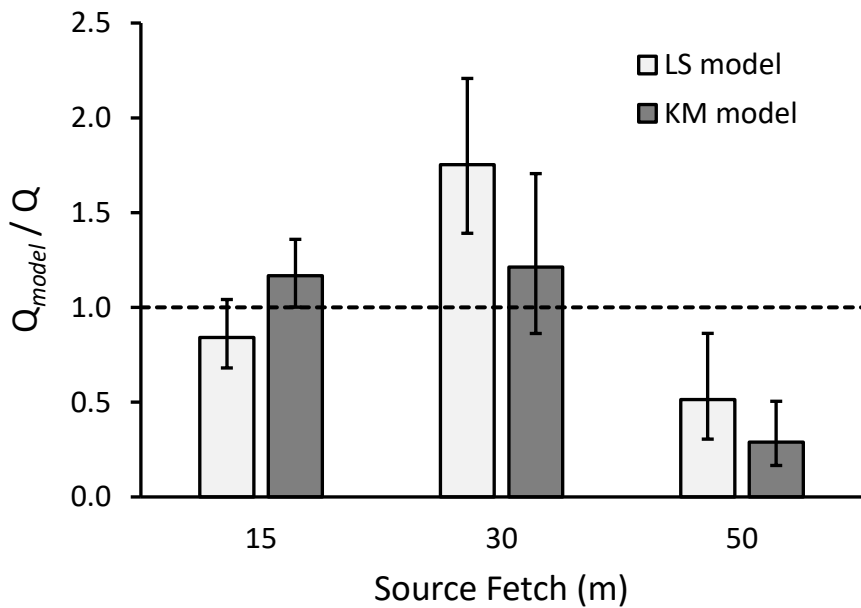
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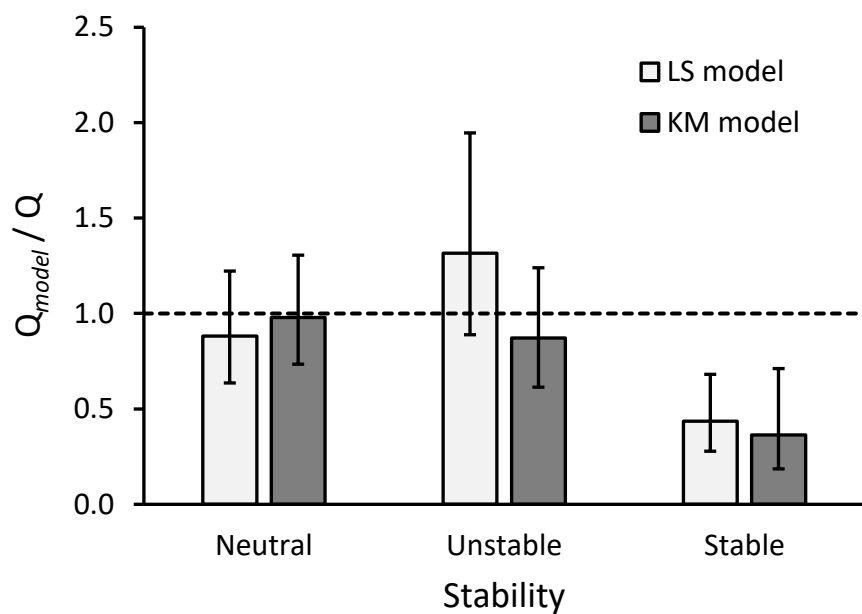
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223 **Figure 1: Map of the synthetic source locations used in the study (polygons). The eddy covariance system was located at position (0,0).**

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227 **Figure 2: Agreement ratio of the footprint model calculated emission rate (Q_{model}) to actual release rate (Q), grouped by source fetch.**
228 **Calculations are from the LS and KM models. The columns show the geometric mean, and the error bars show the 95% confidence**
229 **interval of the mean. The horizontal dashed line represents a Q_{model} / Q ratio of one, or a perfect model calculation.**

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Figure 3: Agreement ratio of the footprint model calculated emission rate (Q_{model}) to actual release rate (Q), grouped by atmospheric stability: neutral ($|L| > 60$ m), unstable ($0 > L > -60$), and stable ($60 > L > 0$). Calculations are from the LS and KM models. The columns show the geometric mean, and the error bars show the 95% confidence interval of the mean. The horizontal dashed line represents a Q_{model} / Q ratio of one, or a perfect model calculation.