Response Letter to the reviewers' comments on manuscript

"A Bayesian parametric approach to the retrieval of the atmospheric number size distribution from lidar data"

In the following, the reviewers' comments are reported in italic, and the Authors' responses are marked with an "A." and given in plain text.

Reviewer 1

The revised manuscript is improved and a lot of the reviewer questions are answered.
But there are still a few open important points:

A. We thank the reviewer for this comment.

- Only one CRI was used for the simulations: 1.49+0.019, which is known from other References that it is a "good" one, i.e., the degree of ill-posedness is small.
A: "… We did test the method with several different values of CRI, but we are only using a single value in the manuscript to make the results easier to interpret. … the retrieval becomes more difficult as the imaginary part of the CRI grows. …."

In my opinion this is a very critical point. The authors should at least show one more example with a more difficult, but of course realistic in nature, CRI and should give a remark to the limitation.

A. We thank the reviewer for this comment. In fact, the cases defined in the text "quasi-real" refer to SD determined by AERONET in three different sites, and were processed with the values of the refractive index determined by AERONET: therefore CRIs are different from the refractive index used for the simulations. But this was visible only in Table 8 and not explicitly reported and discussed in the text. We have therefore modified the text. Furthermore, in order to provide a term of comparison of the reconstruction of the same SDs with different CRI, we have added the results of the reconstruction of the same "quasi-real" cases with a rather extreme CRI value of

1.57 + i0.043. This led to the addition of 3 figures (the (c) panels in Fig 7-9), a new table and the respective comment.

- Abstract: We show that the proposed algorithm provides satisfactory results even when the assumed number of modes is different from the true number of modes, and substantially excellent results when the right number of modes is selected.
- Line 355: The preliminary results presented in this paper indicate that the proposed method can effectively retrieve uni– and bi–modal distributions from extinction coefficients measured at two wavelengths and backscattering coefficients measured at three wavelengths.

In contrast: Figure 6. Reconstructions obtained by running the inversion algorithm with a bimodal distribution, when data are generated by a bimodal distribution.
In contrast: Figure 8. Results obtained by applying our proposed method, with a unimodal (left) and with a bimodal (right) distribution, to the experimental dataset recorded on Etna.

In my opinion the two sentences (abstract and line 355) are far too optimistic!

A. We modified both sentences, that are now less optimistic.

- Line 230: ….where r_min and r_max assume in our case the values 0.01µm and 20µm, respectively.
- Line 336: In the bimodal SD reconstruction, Fig. 6 and Table 6, the "fine" mode is always reconstructed better than the "coarse" mode, this effect is connected to the wavelengths used for determining the optical parameters of the lidar measures.

This is correct. But my main concern is the value of r_max=20µm. This could suggest that the algorithm is working even very well for such large particles. I do not believe this. In all examples or figures, respectively, one can see that r_max is only about 11µm which is already very large in comparison to the used wavelengths and only about the half of 20. Are there any examples made by the authors with a second mode between 10 and 20µm? This would be interesting for the community!

A. The value of r_max = 20mm refers only to the range used for the discretization of the radius values in the reconstruction; the maximum value of the modal radius of the SD that the method reconstructs is always less than 7µm in all the analyzed cases. We have modified the text to clarify this point.

Reviewer 2

Authors answered correctly to many of my questions and the manuscript has been improved.

A. We thank the reviewer for this comment.

However, I still have two concerns that need to be better discussed in the manuscript.
i) it is unrealistic to have Lidar measurements with less than 5 % errors in retrieved optical parameters. Better discussion is needed about the limitations in the retrieval with the proposed scheme.

A. As reported by the referee, the error on the optical parameters plays a critical role in the quality of the reconstruction and the value of 5% on a single measurement is quite optimistic, especially in the measurement of the extinction coefficient. However, if we consider the average value of the optical parameters (2 alpha and 3 beta) corresponding to an atmospheric layer with a thickness of 1Km, with an error of 10-15% (see References [1,2] below) on a single point with a resolution of the order of 100m, the statistical error is less than 5%.

ii) retrieval of aerosol refractive index is the key in current challenges in aerosol science, more even that size distribution. That is the weaker point of the proposed scheme. So further discussion is still needed in the manuscript

A. We added a paragraph in the Discussion section where we discuss the limitation related to the assumption of known CRI:

   "A second limitation concerns the subjectivity in the choice of the CRI, which was assumed to be known in the present study. Our analysis on quasi-real data showed how the quality of the retrieval may depend on the value of the CRI, and partly deteriorates when the imaginary part grows, particularly for larger modes. This is a known issue with lidar data, that can possibly be solved in a Bayesian framework by devising better priors. In addition, a full Bayesian model including the CRI among the unkonwns can be devised, however, with an increased number of unknowns it will be necessary to exploit more prior information to reduce the degree of ill-posedness."

[1] Mattis, Ina, et al. "Dual‐wavelength Raman lidar observations of the extinction‐to‐backscatter ratio of Saharan dust." Geophysical Research Letters 29.9 (2002): 20-1.

[2] GAW Aerosol Lidar Observation Network. "GAW Report No. 178." https://library.wmo.int/doc_num.php?explnum_id=9387