<u>Answers to Dr. Jana Mendrok</u>

This study introduces a method for the retrieval of snow microphysical properties (ice water content, mean size, shape) from polarimetric and dual-wavelength radar observations. The method as introduced here is based on a bunch of rather rude assumptions, in particular the shape model (homogeneous spheroids) and the mass-size relation. In my view, the study lacks to show the effect that these assumptions have on the retrieval results and to properly state the limitations of the method.

Thank you for your comment about our manuscript. As already mentioned in the paper, but maybe it needed to be more emphasized (and it is now), this work serves as a feasibility study exploring the combination of two spatially separated radars to derive microphysics information about the detected – in the radar beams cross-section – atmospheric hydrometeors. Our major focus is to ensure that we are able to obtain high quality dual-wavelength ratio measurements and as a further extension of this work, to use the radar measurements combined with a simple ice particle model and assumptions for the particle size distribution (PSD) and mass-size relation (m(Dmax)) of the ice particles, to develop an ice microphysics retrieval. Using spheroid as a particle model, we are then able to test our a-priori assumptions for the ice particles as its simplicity allows for easy calculations of the particles mass (estimated from the mass-size relation) and thus, effective density. Therefore, we are able to investigate the effect of our guesses for the particles shape, size and mass on the retrieved parameters (apparent shape as well as median size and ice water content of the ice particles PSD), using the soft spheroid model to represent the detected ice hydrometeors.

After a more up-to-date literature review about the limitations of the soft spheroid approximation we now included a more detailed argumentation why we still decided to stick to this rather simple model:

> *"Although more complex ice particle and scattering models are available, this work will use the soft spheroid approximation out of the following reasons: (1) In this work we aim to provide a feasibility study to combine two spatially separated radars to better constrain the ice crystal shape in microphysical retrievals using simultaneous DWR and ZDR observations from an oblique angle. Besides instrument coordination, the actual measurements and the assessment of measurement errors, the ice crystal and scattering model are just one component. Due to its simple and versatile setup, this work will utilize the soft spheroid approximation to study the benefit of additional ZDR measurements and the role of the observation geometry. (2) More importantly, to our knowledge, the more accurate SSRGA described by Hogan and Westbrook (2014) does not (yet) provide polarimetric variables used in this study, namely the ZDR. (3) In anticipation of a prognostic aspect ratio of ice crystals in bulk microphysical models (e.g., the adaptive habit prediction; Harrington et al., 2013), we aim to keep a minimal set of degrees of freedom to remain comparable with these modelling efforts. (4) Using ice spheroids we are able to vary parameters such as median size, aspect ratio and ice water content independently, which serve as degrees of freedom of the ice spheroids, and calculate their optical properties without much computational cost as in other scattering algorithms (e.g., DDA) that are used in more realistic ice crystal shapes simulations. Moreover, using spheroids we can better understand the ambiguities between these simple, aforementioned degrees of freedom."*

The resulting limitations are now also mentioned more prominently throughout the discussion and we included the following paragraph to mention the limited scope of this work already in the introduction:

> *"Due to this simplification, this study will focus on the feasibility to combine DWR and ZDR from spatially separated radar instruments into a common retrieval framework. Due to the missing internal structure of soft spheroids, the known underestimation of the radar backscatter and generally lower ZDR for larger snowflakes will limit this study to ice aggregates with sizes in the millimeter regime. This will include the onset of ice aggregation within clouds above the melting layer (ML) but will exclude heavy snowfall close to the ground. Anyhow, this region is rarely included in the measurement region with an overlap between the two scanning radar instruments."*

To detail and justify my concerns:

The authors write (L588) "we have to assume a suitable m(Dmax) relationship" and I cannot agree more. The authors obviously define "suitable" from the agreement of their scattering calculations with statistics of observations of DWR and ZDR. The scattering calculations are based on a range of assumptions, including e.g the size distribution, the mass-size relation, the spheroidal particle model, the shape of the spheroids.

The paper gives a first glimpse that the results of the ice retrieval are definitely affected by the different assumptions that we use in our ice scattering simulations performed by T-Matrix (e.g., Mishchenko and Travis (1994); Mishchenko et al. (1996) and more). In particular, Fig. 8 as well as Sect. 4.3.1 *Unknown mass-size relationship* show differences in the retrieved parameters when the well-known Brown and Francis (Brown and Francis, 1995; hereafter BF95) and aggregates (Yang et al., 2000) m(Dmax) are used. Moreover, the different shape (oblate or prolate) assumption also affects the ice retrieval's results (Sect. 4.1). However, to be more specific on how and how much the a-priori simulation assumptions about m(Dmax), particles oblate or prolate shape, shape parameter μ of particle size distribution (PSD), wobbling of ice particles etc. can alter the ice retrieval's results, we are already working on a second manuscript which will serve as a sensitivity analysis on the aforementioned assumptions. To be more specific, in the current version of our manuscript we have now stressed out that *the retrieved parameters would be …. under these assumptions…*

Out of these, mass-size relation is rather well constrained by observations of a range of different measurement techniques (including 3D imaging of falling snow "in the wild" (Leinonen et al., 2021)) and its range of variation is comparably small. Moreover, it is a crucial microphysical parameter in weather and climate models.

Thank you for pointing out this concern. As mentioned above, our study is not focused on falling snow close to the ground. While the mass-size relationship of falling snow might be rather well constrained by observations with multi-angle snowflake cameras (MASC) on the ground, the mass size relationship is one of the least constrained and strongly varying ice crystals properties higher up within clouds. The large variability of in-cloud mass-size relation has been found in various in situ studies (e.g., Heymsfield et al., 2010; Xu et al., 2016) and has been stressed in numerous studies as one of the largest sources of uncertainties in ice microphysical retrievals (e.g., Deng et al., 2013; Delanoë et al., 2014; Ham et al., 2017).

On the other hand, the spheroidal particle model is a highly artificial model that is known to benot well suited to represent scattering properties of particles of low effective density like snow aggregates. This regards microwave scattering properties in general (e.g. Eriksson, 2015; Eriksson, 2018), but polarimetric properties in particular (eg. Schrom and Kumjian, 2018). Aspect ratio, in addition, is a highly simplistic parameter to describe the shape of typically irregularly shaped particles. That is, it is questionable how well aspect ratios observed from irregularly shaped particles can constrain reasonable values for homogeneous soft particles (like spheroids or even plates).

As already mentioned, our intention is to use a simple and easy-handled ice particle model so that we can seamlessly change our assumptions for ice particle populations and check their effect on the ice microphysics retrieval algorithm. Using a more realistic but more complicated ice particle model could lead to additional assumptions in our study. The use of the spheroid model to represent ice particles has been debated in many studies in the past. Therefore, we have now included a part presenting related literature (Sect. 1.2 *Representation of ice atmospheric hydrometeors using spheroids*):

> *"Single scattering simulations are an indispensable tool to bridge the gap between microphysical properties of hydrometeors and polarimetric radar observations. In the case of ice particles, however, the calculation of scattering properties can be challenging due to their large complexity, variety in shape, structure, size and density. One of the most sophisticated*

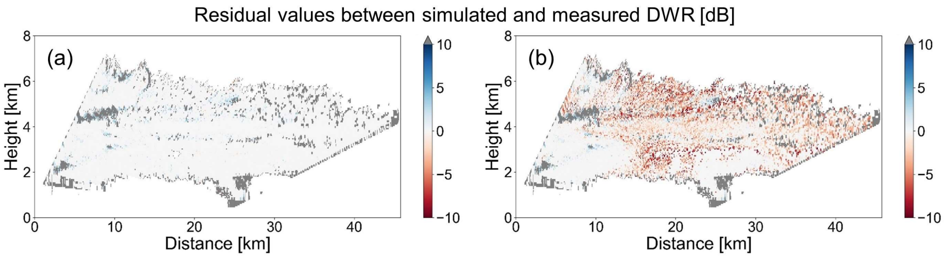
methods, the Discrete-Dipole Approximation (DDA; Draine and Flatau, 1994), can be used to calculate the scattering properties of realistic ice crystals and aggregates. However, this approximation can be computational demanding. To reduce computation cost and complexity, ice particles are often assumed to be spheres and their scattering properties are calculated using the Mie theory or they are assumed to be spheroids using the T-Matrix method (Waterman, 1965) or the Self-Similar Rayleigh-Gans Approximation (SSRGA; e.g., Hogan and Westbrook, 2014; Hogan et al., 2017; Leinonen et al., 2018a) for scattering simulations. The calculations when SSRGA is used are known to be affected by the way that ice mass is distributed throughout the particle's volume. As we aim for a simple ice particle model, we extensively used the T-Matrix method in this study, assuming the ice particles to be soft spheroids. It is a common approach in model studies that ice particles are represented by homogeneous spheroids with density equal or smaller of bulk ice. Due to its simplicity, the limitations of the spheroid approximation have been a heavily researched and debated topic in the last decade. While Tyynelä et al. (2011) showed an underestimation of the backscattering for large snowflakes, Hogan et al. (2012) suggested that horizontally aligned oblate spheroids with a sphericity of 0.6 can reliably reproduce the scattering properties of realistic ice aggregates which are smaller than the radar wavelength. The same study also concluded that for larger particles spheroids are an improvement to Mie spheres which can lead to a strong underestimation of Z_e and, in turn, strong overestimation of IWC. Leinonen et al. (2012) on the other hand showed that the spheroidal model cannot always explain the radar measurements as more sophisticated particle models do, e.g., snowflake models. Later on, Hogan and Westbrook, (2014) indicated that the soft spheroid approximation underestimates the backscattered signal of large snowflakes (1 cm size) – measured with a 94 GHz radar – up to 40 and 100 times for vertical and horizontal incidence, respectively. In contrast, the simple spheroidal particle model could successfully explain measurements of slant-45° linear depolarization ratio, SLDR, as well as SLDR patterns on the elevation angles (Matrosov, 2015) during the Storm Peak Laboratory Cloud Property Validation Experiment (StormVEx). In Liao et al. (2016) it was found that randomly oriented oblate ice spheroids could reproduce scattering properties in Ku- and Ka-band similar to these from scattering databases when large particles were assumed to have a density of 0.2 g cm^{-3} and a maximum size up to 6 mm. Although Schrom and Kumjian (2018) showed that some ice crystal shapes as branched planar particles could be better represented by plate crystals than spheroids, the simple spheroidal model has been used in recent studies to represent ice aggregates as in Jiang et al. (2019) or to retrieve shape from LDR as in Matrosov (2020). In all these studies, it is recognized that the spheroidal model requires less assumed parameters compared to more complex particle models."

To summarize, mass-size relation is a well-constrained parameter, while aspect ratio is not and is a rather artificial parameter. Hence, I wonder (and question), why the authors chose to use a mass-size relation that is unreasonable for snow aggregates but insist on keeping the aspect ratio within "reasonable" bounds.

We value your critique. As mentioned above we do not imply that our approach will yield reasonable results for large snowflakes. This work was designed as a feasibility study to combine simultaneous DWR and ZDR measurements within a common retrieval framework. While people have used these joint observables to constrain or rule out specific ice crystal shapes during their discussions in the past, to our knowledge, this study is the first attempt to combine both observables in a microphysical retrieval. To that end we employed two strongly different mass-size relationships to test and to understand the interrelationship between DWR and ZDR for a very simple and intuitive ice crystal model.

As I understand, the results of the retrieval method presented here strongly depend on the choice of mass-size relation. This poses the question how reliable retrieved microphysical properties (primarily IWC) are when selecting a relation that is far off the range of commonly accepted values. As stated above, I miss a comprehensive estimate of the uncertainties (or, actually, errors) that this unreasonable assumption results in – not within the (forward modelling-retrieval) system here, which is selfconsistent, but in more realistic retrievals (if independently measured IWC are not available, e.g. by analyzing the retrieved IWC based on a range of for different m(Dmax) in the retrieval).

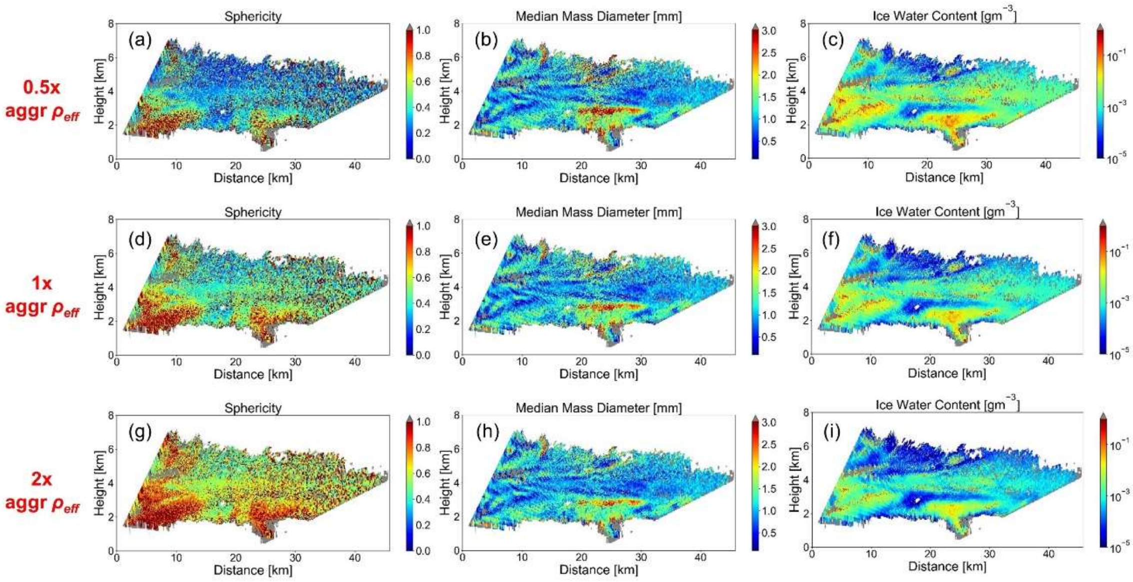
Using the same assumptions and testing two different m(Dmax) we find that ice spheroids with mass-size relation corresponding to aggregates from Yang et al. (2000) can explain better our radar observations against BF95, especially for larger ice particles and thus, higher DWR values, e.g., our current Fig. 16, also attached below. Panels (a) and (b) present residual values between the measured and the simulated DWR when ice spheroids are oblates, follow exponential PSD and have a mass-size relation corresponding to aggregates and BF95, respectively.



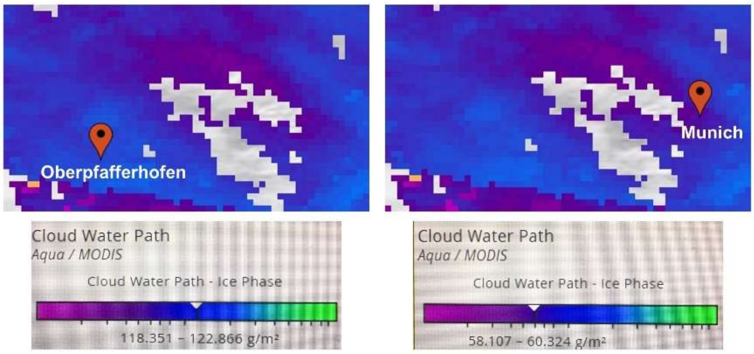
In particular, we write:

> *"Figure 16 shows the residuals between the simulated and measured DWR for aggregates (Fig. 16a) and BF95 $m(D_{max})$ (Fig. 16b). For ice spheroids that follow the $m(D_{max})$ of aggregates, the residuals are evenly distributed around 0 (mean value of +0.08 dB) suggesting that this mass-size relation can better explain our measurements in this case. In contrast, the measured DWR appeared to be higher than the simulated one for BF95 for the larger part of the cloud cross-section (reddish areas) with a mean value of –0.923 dB"*

A similar study is also conducted in our manuscript testing mass-size relations similar to that of aggregates which considers and almost constant effective density with the size (~ 0.2 g cm^{-3}). The retrieval results are shown in our current Fig. 17, also attached here.



For a better evaluation of the retrieved parameters, i.e., IWC, and as we do not have auxiliary data from January 2019 to evaluate the ice retrieval results, we used snapshots from the MODIS MYD06 product (source: https://worldview.earthdata.nasa.gov/, Platnick et al, 2017) for ice water path (IWP). In the following figure snapshots for Oberpfaffenhofen (location of POLDIRAD weather radar) and Munich (location of MIRA-35 cloud radar) are presented. Snapshots of the colorbar indicating the respective IWP value for each location are also shown.



The figure shows a gradient of the retrieved MODIS IWP which decreases from west to east. Therefore, an averaged value of IWP ~ 90 g m^{-2} was considered from MODIS for the whole radar cross-section. Using our retrieved IWC for the three cases (0.5x, 1x, 2x aggr ρ_{eff}) and integrating with height we obtain IWP ~ 46 g m^{-2}, IWP ~ 83 g m^{-2} and IWC ~ 137 g m^{-2} when the effective density is considered 0.5x, 1x, 2x times of aggr ρ_{eff}, making the 1x aggr ρ_{eff} the mass-size relation which can best explain our radar measurements for that scene. In all cases, the ice particles were assumed oblates and that they follow an exponential PSD.

The introduction to the discussion section points out two a priori assumptions on the particle properties as limitations of the method: the already discussed mass-size relation and the choice of particle model between oblate and prolate spheroids, missing to point out the much more crucial assumption of spheroids in general. This continues in the discussion of "unsuitability" of the BF95 mass-size relation, where no other reasons for the very low ZDR than the low density, seemingly exclusively resulting from the m(Dmax) assumption is discussed, which is re-iterated in the conclusions.

Thank you very much for this comment. The BF95 mass-size relation has been widely used in ice cloud studies before (e.g., Hogan et al., 2012 and many more) yielding good results in comparison to observations. Investigating how suitable the BF95 is, we notice that using BF95, which is known to predict low density for large particles when combined to ice spheroids, we cannot produce high ZDR signals but only for very small and dense particles, and not for less dense and larger ice particles with larger DWR values. Therefore, the "unsuitability" of this mass-size relation when ice spheroids are used lies in the fact that it cannot explain our ZDR measurements due to the homogeneity that the spheroid model suggests and due to the missing sharp edges of the ice aggregates, which are known to give high ZDR, when these are considered to be approximately represented by a spheroid. This explanation is now added in the current version of our manuscript.

As source for their aggregate model, the authors cite Yang et al. (2000). I find this highly misleading. Yang et al. (2000) (as well as a range of follow-up papers building on it and extending it, including Hong et al., 2009, and Ding et al., 2017) targets the explicit modeling of scattering properties of irregularly shaped particles. The irregular, non-spherical and(!) non-spheroidal(!) shape is the crucial aspect of their shape model, the core element of that research. The authors of this study, however, reduce this complexity to the minor aspect of the underlying mass-size relation (L356: "another m(Dmax) that we use is the aggregates from Yang et al. (2000)", falsely implying the equivalence of the m(Dmax) with the entire Yang et al. aggregate definition). Mass-size relation is likely the most un-aggregate-ish characteristic of the Yang aggregate model, was – probably – selected with not much care at that time (is that even originating from Yang et al. themselves, or where has it been taken from by Yang et al.?), and has been pointed out as a shortcoming of the Yang-aggregates by other authors (e.g. Eriksson, 2018).

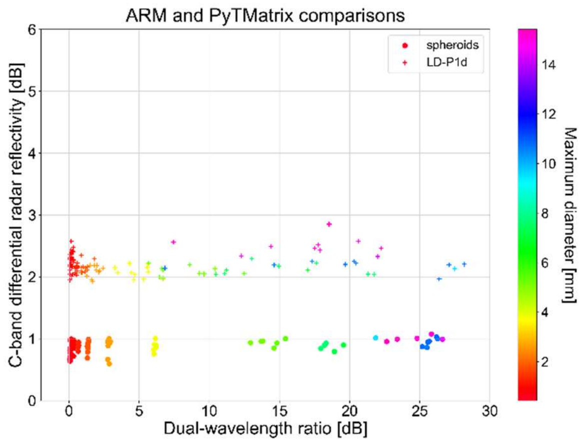
Thank you for pointing out this misleading wording. We indeed only adapted the mass-size relationship corresponding to aggregates from Yang et al. (2000) to construct corresponding soft spheroids. You are right that the aggregates were initially designed as a specific aggregation of 8 hexagonal elements (Yang and Liou, 1998) for which an m(D) was only fitted afterwards (Eq. 12 with parameters from Table 2 in Yang et al., 2000). In numerous studies, however, their microphysical (e.g., Baum et al., 2005) as well as their scattering properties (e.g., Eichler et al., 2009; Ewald et al., 2021) have turned out to be a versatile tool to explain remote sensing data from ice clouds. Moreover, after reading Eriksson et al. (2018, or 2015?), we could not find the mentioned shortcoming of Yang-aggregates. On the contrary, their triple-frequency signature ($DWR_{Ku,Ka}$ vs $DWR_{Ka,W}$) in Fig. 13 (Eriksson et al., 2018) seem to be quite capable to reproduce also radar measurements, e.g. compare Fig. 10 in Kulie et al. (2014). In this work, we chose this m(D) relationship with a higher and constant ice crystal density as a contrast to the lower-density m(D) of BF95.

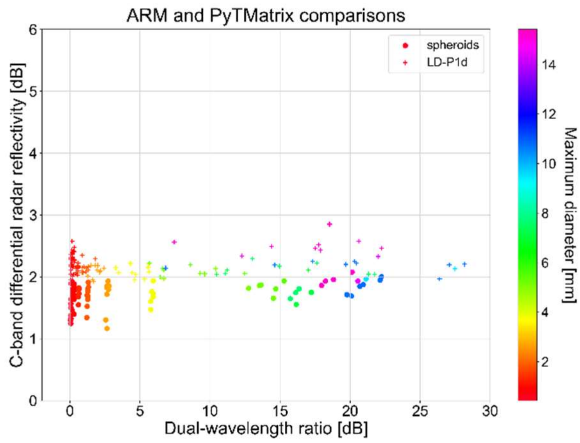
To avoid the misleading confusion between the m(D) with the entire Yang et al. aggregate definition, we changed the wording throughout the manuscript and revised the paragraph which introduce this second m(D) relationship:

> *"While the effective density of a spheroid decreases strongly with its size due to the exponent b=1.9 in BF95, we contrast this with a second $m(D_{max})$ with a higher and constant density. To that end we borrowed the $m(D_{max})$ from the irregular aggregate model from Yang et al. (2000) to create soft spheroids with an analog mass-size ratio. Originally, the construction of these aggregates was fully described in Yang and Liou (1998) as an aggregated collection of geometrical hexagonal columns. In our study, this second soft spheroid model only emulates the maximum dimension and mass of the underlying aggregates."*

To justify reference to Yang, it would be interesting to see how the scattering properties of the authors' aggregates compare to microwave properties of the actual, irregularly shaped aggregate of Yang. As far as I am aware, they are not available for preferential orientation, but both Ding et al. (2017) and Eriksson et al. (2018) provide scattering properties for this habit ("8-column aggregate") at radar wavelengths for totally randomly orientation, which should allow for a comparison of the predicted reflectivities and dual-wavelength ratios.

Thank you for this comment. After your interesting suggestion, we compared microphysical properties of our ice spheroids that follow the aggregates mass-size relation assumption from Yang et al. (2000) to aggregates from the Atmospheric Radiation Measurement (ARM) scattering database (Lu et al., 2016). In the following plots we attach some of the comparison results. Both figures show comparisons between soft spheroids (dots in the scatter plot) and low-density ARM aggregates (LD-P1d, crosses in the scatter plot).





In the left plot we used the soft spheroids analog to the m(D) of aggregates from Yang, while we used the soft spheroids from our study with double the density of m(D) from Yang for the right plot. With the doubled density, the agreement between T-matrix soft spheroids and ARM aggregates is obviously better. In the first case, with the original effective density, the soft spheroids produce 1 dB lower ZDR than these simulated with the generalized multiparticle Mie method (GMM) in ARM. We are planning to continue these kind of studies in our next paper (also including the suggested database from Ding et al. (2017) and Eriksson et al. (2018)), which is already in preparation. In case of interest, we could already include such a comparison in our manuscript Appendix.

Minor comments:

L376: Could you provide a reference for that Dm from the equation you provide is the Median mass diameter? In my understanding, when D therein is the melted mass equivalent diameter, then Dm is the mean mass diameter. Are median and mean really equal here?

The present formula indeed referred to mean mass diameter. Thank you for pointing this out. The definition has been changed now to $\int_0^{D_m} mN(D)dD = \frac{1}{2}IWC$ from e.g., Ding et al. (2020).

L384: Please specify more precisely, what Maxwell-Garnett approach was used, in particular what material forms the matrix, what the inclusions, as this can make a lot of a difference (see e.g. Eriksson, 2015).

In this work, ice spheroids were assumed to have been formed with the inclusion of air in the medium of ice. We also added a new paragraph in the introduction (Sect. 3.2.1 *Soft spheroid model: Refractive index*) describing the way that these particles are formed. In particular we write:

> *"Our soft spheroid model uses the effective medium approximation (EMA) to model the refractive index of the composite material as an ice matrix with air inclusions following the Maxwell-Garnett (MG) mixing formula given in Garnett and Larmor (1904):*
>
> $$\frac{e_{eff}-e_i}{e_{eff}+2e_i} = f_i \frac{e_i-e_m}{e_i+2e_m} \quad (4)$$
>
> *with,*
> *e_m, e_i: the permittivities of the medium and the inclusion, respectively,*
> *e_{eff}: the effective permittivity,*
> *f_i: the volume fraction of the inclusions.*
> *The complex refractive index, m, is then calculated from $m = \sqrt{e_{eff}}$. In the framework of the EMA, the electromagnetic interaction of an inhomogeneous dielectric particle (components with different refractive indices) can be approximated with one effective refractive index of a homogeneous particle (e.g., Liu et al., 2014; Mishchenko et al., 2016). In Liu et al. (2014), internal mixing was proven to best represent the scattering properties of hydrometeors. Here, the refractive index is modelled as an internal mixing of ice with air inclusions which are arranged throughout the ice particle. The same work also pointed out that the size parameter $D_{crit} = \frac{\pi d}{\lambda}$ for each of these air inclusions should not be larger than 0.4 (with d as the diameter of the inclusion). "*

References

Ding, J., Bi, L., Yang, P., Kattawar, G. W., Weng, F., Liu, Q., and Greenwald, T.: Single-scattering properties of ice particles in the microwave regime: Temperature effect on the ice refractive index with implications in remote sensing, J. Quant. Spectrosc. Ra., 190, 26–37, 2017.

Eriksson, P., Jamali, M., Mendrok, J., and Buehler, S. A.: On the microwave optical properties of randomly oriented ice hydrometeors, Atmos. Meas. Tech., 8, 1913–1933, https://doi.org/10.5194/amt-8-1913-2015, 2015.

Eriksson, P., Ekelund, R., Mendrok, J., Brath, M., Lemke, O., and Buehler, S. A.: A general database of hydrometeor single scattering properties at microwave and sub-millimetre wavelengths, Earth Syst. Sci. Data, 10, 1301–1326, https://doi.org/10.5194/essd-10-1301-2018, 2018.

Hong, G., Yang, P., Baum, B. A., Heymsfield, A. J., Weng, F., Liu, Q., Heygster, G., and Buehler, S. A.: Scattering database in the millimeter and submillimeter wave range of 100–1000 GHz for nonspherical ice particles, J. Geophys. Res., 114, D06201, https://doi.org/10.1029/2008JD010451, 2009.

Leinonen, J., Grazioli, J., and Berne, A.: Reconstruction of the mass and geometry of snowfall particles from multi-angle snowflake camera (MASC) images, Atmos. Meas. Tech., 14, 6851–6866, https://doi.org/10.5194/amt-14-6851-2021, 2021.
Schrom, R. S. and Kumjian, M. R.: Bulk-Density Representations of Branched Planar Ice Crystals: Errors in the Polarimetric Radar Variables, Journal of Applied Meteorology and Climatology, 57, 333–346, https://doi.org/10.1175/JAMC-D-17-0114.1, 2018

Data availability

Ice water paths were taken from the MODIS MYD06 product with the identifier doi:10.5067/MODIS/MOD06_L2.061 (Platnick et al, 2017)

References

Baum, B. A., Heymsfield, A. J., Yang, P., and Bedka, S. T.: Bulk Scattering Properties for the Remote Sensing of Ice Clouds. Part I: Microphysical Data and Models, 44, 1885–1895, https://doi.org/10.1175/JAM2308.1, 2005.

Brown, P. R. A. and Francis, P. N.: Improved Measurements of the Ice Water Content in Cirrus Using a Total-Water Probe, J. Atmos. Oceanic Technol., 12, 410–414, https://doi.org/10.1175/1520-0426(1995)012<0410:IMOTIW>2.0.CO;2, 1995.

Delanoë, J. M. E., Heymsfield, A. J., Protat, A., Bansemer, A., and Hogan, R. J.: Normalized particle size distribution for remote sensing application, J. Geophys. Res.: Atmos., 119, 4204–4227, https://doi.org/10.1002/2013JD020700, 2014.

Deng, M., Mace, G. G., Wang, Z., and Lawson, R. P.: Evaluation of Several A-Train Ice Cloud Retrieval Products with In Situ Measurements Collected during the SPARTICUS Campaign, J. Appl. Meteorol. Clim., 52, 1014–1030, https://doi.org/10.1175/JAMC-D-12-054.1, 2013.

Ding, J., Bi, L., Yang, P., Kattawar, G. W., Weng, F., Liu, Q., and Greenwald, T.: Single-scattering properties of ice particles in the microwave regime: Temperature effect on the ice refractive index with implications in remote sensing, J. Quant. Spectrosc. Ra., 190, 26–37, 2017.

Ding, S., McFarquhar, G. M., Nesbitt, S. W., Chase, R. J., Poellot, M. R., and Wang, H.: Dependence of Mass–Dimensional Relationships on Median Mass Diameter, Atmosphere, 11, 756, https://doi.org/10.3390/atmos11070756, 2020.

Draine, B. T. and Flatau, P. J.: Discrete-Dipole Approximation For Scattering Calculations, J. Opt. Soc. Am. A, JOSAA, 11, 1491–1499, https://doi.org/10.1364/JOSAA.11.001491, 1994.

Eichler, H., Ehrlich, A., Wendisch, M., Mioche, G., Gayet, J.-F., Wirth, M., Emde, C., and Minikin, A.: Influence of ice crystal shape on retrieval of cirrus optical thickness and effective radius: A case study, 114, https://doi.org/10.1029/2009JD012215, 2009.

Eriksson, P., Jamali, M., Mendrok, J., and Buehler, S. A.: On the microwave optical properties of randomly oriented ice hydrometeors, 8, 1913–1933, https://doi.org/10.5194/amt-8-1913-2015, 2015.

Eriksson, P., Ekelund, R., Mendrok, J., Brath, M., Lemke, O., and Buehler, S. A.: A general database of hydrometeor single scattering properties at microwave and sub-millimetre wavelengths, 10, 1301–1326, https://doi.org/10.5194/essd-10-1301-2018, 2018.

Ewald, F., Groß, S., Wirth, M., Delanoë, J., Fox, S., and Mayer, B.: Why we need radar, lidar, and solar radiance observations to constrain ice cloud microphysics, Atmos. Meas. Tech., 14, 5029–5047, https://doi.org/10.5194/amt-14-5029-2021, 2021.

Garnett, J. C. M. and Larmor, J.: XII. Colours in metal glasses and in metallic films, Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character, 203, 385–420, https://doi.org/10.1098/rsta.1904.0024, 1904.

Ham, S.-H., Kato, S., and Rose, F. G.: Examining impacts of mass-diameter (m-D) and area-diameter (A-D) relationships of ice particles on retrievals of effective radius and ice water content from radar and lidar measurements, 122, 3396–3420, https://doi.org/10.1002/2016JD025672, 2017.

Harrington, J. Y., Sulia, K., and Morrison, H.: A Method for Adaptive Habit Prediction in Bulk Microphysical Models. Part I: Theoretical Development, J. Atmos. Sci., 70, 349–364, https://doi.org/10.1175/JAS-D-12-040.1, 2013.

Heymsfield, A. J., Schmitt, C., Bansemer, A., and Twohy, C. H.: Improved Representation of Ice Particle Masses Based on Observations in Natural Clouds, J. Atmos. Sci., 67, 3303–3318, https://doi.org/10.1175/2010JAS3507.1, 2010.

Hogan, R. J. and Westbrook, C. D.: Equation for the Microwave Backscatter Cross Section of Aggregate Snowflakes Using the Self-Similar Rayleigh–Gans Approximation, J. Atmos. Sci., 71, 3292–3301, https://doi.org/10.1175/JAS-D-13-0347.1, 2014.

Hogan, R. J., Tian, L., Brown, P. R. A., Westbrook, C. D., Heymsfield, A. J., and Eastment, J. D.: Radar Scattering from Ice Aggregates Using the Horizontally Aligned Oblate Spheroid Approximation, J. Appl. Meteorol. Clim., 51, 655–671, https://doi.org/10.1175/JAMC-D-11-074.1, 2012.

Hogan, R. J., Honeyager, R., Tyynelä, J., and Kneifel, S.: Calculating the millimetre-wave scattering phase function of snowflakes using the self-similar Rayleigh–Gans Approximation, Q. J. R. Meteorolog. Soc., 143, 834–844, https://doi.org/10.1002/qj.2968, 2017.

Jiang, Z., Verlinde, J., Clothiaux, E. E., Aydin, K., and Schmitt, C.: Shapes and Fall Orientations of Ice Particle Aggregates, J. Atmos. Sci., 76, 1903–1916, https://doi.org/10.1175/JAS-D-18-0251.1, 2019.

Kulie, M. S., Hiley, M. J., Bennartz, R., Kneifel, S., and Tanelli, S.: Triple-Frequency Radar Reflectivity Signatures of Snow: Observations and Comparisons with Theoretical Ice Particle Scattering Models, 53, 1080–1098, https://doi.org/10.1175/JAMC-D-13-066.1, 2014.

Leinonen, J., Kneifel, S., Moisseev, D., Tyynelä, J., Tanelli, S., and Nousiainen, T.: Evidence of nonspheroidal behavior in millimeter-wavelength radar observations of snowfall, J. Geophys. Res.: Atmos., 117, https://doi.org/10.1029/2012JD017680, 2012.

Leinonen, J., Kneifel, S., and Hogan, R. J.: Evaluation of the Rayleigh–Gans approximation for microwave scattering by rimed snowflakes, Q. J. R. Meteorolog. Soc., 144, 77–88, https://doi.org/10.1002/qj.3093, 2018.

Liao, L., Meneghini, R., Tokay, A., and Bliven, L. F.: Retrieval of Snow Properties for Ku- and Ka-Band Dual-Frequency Radar, J. Appl. Meteorol. Clim., 55, 1845–1858, https://doi.org/10.1175/JAMC-D-15-0355.1, 2016.

Liu, C., Lee Panetta, R., and Yang, P.: Inhomogeneity structure and the applicability of effective medium approximations in calculating light scattering by inhomogeneous particles, J. Quant. Spectrosc. Radiat. Transfer, 146, 331–348, https://doi.org/10.1016/j.jqsrt.2014.03.018, 2014.

Lu, Y., Jiang, Z., Aydin, K., Verlinde, J., Clothiaux, E. E., and Botta, G.: A polarimetric scattering database for non-spherical ice particles at microwave wavelengths, 9, 5119–5134, https://doi.org/10.5194/amt-9-5119-2016, 2016.

Matrosov, S. Y.: Evaluations of the Spheroidal Particle Model for Describing Cloud Radar Depolarization Ratios of Ice Hydrometeors, J. Atmos. Oceanic Technol., 32, 865–879, https://doi.org/10.1175/JTECH-D-14-00115.1, 2015.

Matrosov, S. Y.: Ice Hydrometeor Shape Estimations Using Polarimetric Operational and Research Radar Measurements, 11, 97, https://doi.org/10.3390/atmos11010097, 2020.

Mishchenko, M. I. and Travis, L. D.: T-matrix computations of light scattering by large spheroidal particles, Opt. Commun., 109, 16–21, https://doi.org/10.1016/0030-4018(94)90731-5, 1994.

Mishchenko, M. I., Travis, L. D., and Mackowski, D. W.: T-matrix computations of light scattering by nonspherical particles: A review, Journal of Quantitative Spectroscopy and Radiative Transfer, 55, 535–575, https://doi.org/10.1016/0022-4073(96)00002-7, 1996.

Mishchenko, M. I., Dlugach, J. M., and Liu, L.: Applicability of the effective-medium approximation to heterogeneous aerosol particles, J. Quant. Spectrosc. Radiat. Transfer, 178, 284–294, https://doi.org/10.1016/j.jqsrt.2015.12.028, 2016.

Platnick, S., S. Ackerman, M. King, G. Wind, K. Meyer, P. Menzel, R. Frey, R. Holz, B. Baum, and P. Yang, 2017. MODIS atmosphere L2 cloud product (06_L2), NASA MODIS Adaptive Processing System, Goddard Space Flight Center, [doi:10.5067/MODIS/MOD06_L2.061; doi:10.5067/MODIS/MYD06_L2.061]

Schrom, R. S. and Kumjian, M. R.: Bulk-Density Representations of Branched Planar Ice Crystals: Errors in the Polarimetric Radar Variables, J. Appl. Meteorol. Clim., 57, 333–346, https://doi.org/10.1175/JAMC-D-17-0114.1, 2018.

Tyynelä, J., Leinonen, J., Moisseev, D., and Nousiainen, T.: Radar Backscattering from Snowflakes: Comparison of Fractal, Aggregate, and Soft Spheroid Models, J. Atmos. Oceanic Technol., 28, 1365–1372, https://doi.org/10.1175/JTECH-D-11-00004.1, 2011.

Waterman, P. C.: Matrix formulation of electromagnetic scattering, Proc. IEEE, 53, 805–812, https://doi.org/10.1109/PROC.1965.4058, 1965.

Yang, P. and Liou, K.: Single-scattering properties of complex ice crystals in terrestrial atmosphere, Contributions to Atmospheric Physics, 1998.

Yang, P., Liou, K. N., Wyser, K., and Mitchell, D.: Parameterization of the scattering and absorption properties of individual ice crystals, J. Geophys. Res.: Atmos., 105, 4699–4718, https://doi.org/10.1029/1999JD900755, 2000.

Xu, Z. and Mace, G. G.: Ice Particle Mass--Dimensional Relationship Retrieval and Uncertainty Evaluation Using the Optimal Estimation Methodology Applied to the MACPEX Data, JAMC, 56(3), 767—788, 2016.