This manuscript presents derivation of formulations for analytic treatment of error covariance matrix of spectral dual-polarization observations. This is an important study and of interest to the radar meteorology community. I would like to congratulate the authors on the heroic job of deriving theoretical formulations. I believe that it should be published in AMT after comments are addressed.

The main problem of this manuscript, it is not easy to read. Just to give you a few examples:

- In equation (9) you use Bhh, Zdr, rho_hv and Phi_dp. Why do you use Bhh and not Zhh, which would be more commonly used? Of course, Bhh is not Zhh, it is a spectral Zhh. But the same applies to Zdr and the rest of variables. It would be good if you would try to use more widely used notations.
- On line 89, page 3. You state "For each transmitted pulse (the term pulse is used throughout the study, although for radars with frequency modulated continuous wave signals a chirp would have been implied)". Then in Section 5 you return to using chirp and introduce chirp sequence. It took me some effort to adjust to that transition. I would suggest that you either use pulses or chirps.

The other concern is whether results of this study will ever be used. Because the formulations are rather complex, there is a good chance that they will never be adopted. Is there an approximation that can be used and that would work for most applications? If yes, could you make a recommendation. Of course, the other option could be a follow up study, demonstrating practical applications.

Abstract: "This study presents the first-ever complete characterization of random errors in dual-polarimetric spectral observations of meteorological targets by cloud radars."

While this statement is true, at least to my knowledge, it seems to me that the underlying assumptions are not very, if at all, different from what are used to describe normal dual-polarization observations. Could you please elaborate what are the main differences?
I would argue that the derived expressions presented in this study are representing a subset of conditions for which expressions in (Doviak and Zrnic) or (Chandrasekar and Bringi) are derived. One big simplification, which I believe is valid for spectra observations at least to some extent, used in this study is that averaging is performed using independent spectra, i.e. Ns represents number of independent averages. That means that one does not need to consider the correlation function, which is a must for time-domain observations. I think a more complete discussion on this should be included in the article. You are giving a short discussion on line 105. I would suggest that you compare and contrast the assumptions used in this study and more classical studies.

"One of the main conclusions of the study is that the convenient representation of spectral polarimetric measurements including differential reflectivity ZDR, correlation coefficient ρHV , and differential phase ΦDP is not suited for the proper characterization of the error covariance matrix."

I believe you need to explain this point more explicitly. In the summary the discussion is rather short:

”It is illustrated that elements of Σc have considerable differences from those estimated from the measurements. The differences are related to the first-order Taylor approximation which does not take into account non-linearities. In contrast, Σb agrees well with the observations. The correlation between calculated elements of Σb with those estimated from the observations exceeds 0.965.”

I did not understand if the problem is due to the first-order Taylor approximation used or something else. In many applications an approximation would perfectly well. What would you recommend using to get an estimate of the uncertainty? The relations you are deriving may be too complex for most applications.

”The joint PDF for polarimetric observations obtained for a single pulse can be found in Middleton (1996, chapter 9.2). Single-pulse measurements, however, are rarely used in the radar meteorology because of the low sensitivity and higher requirement for storage space. The observed radar spectra, almost always, result from the averaging of a number of return pulses. Unfortunately, a solution for the case of averaging over a number of pulses is not yet available in literature.”

I am not sure if the authors are completely correct. Please see the reference below, where analytical forms of pdf of estimates dual-pol variables are derived. While the derived pdfs are given for SAR observations, the underlying assumptions are the same.

Jong-Sen Lee, K. W. Hoppel, S. A. Mango and A. R. Miller, "Intensity and phase statistics of multilook polarimetric and interferometric SAR imagery," in IEEE Transactions on Geoscience and Remote Sensing, vol. 32, no. 5, pp. 1017-1028, Sept. 1994, doi: 10.1109/36.312890.

Line 165: Please explain what Ns actually means. I expect that you imply that Ns stands for number of independent spectra used to compute an average. This means that spectra are computed from non overlapping time sequencies. Is this correct?

Section 5. “Measurements were made during a rain event on 21 June 2021 at 7:44 UTC. I/Q measurements provide high data rate of about 900 MB min−1. Therefore, about 3 min of I/Q measurements were collected for the analysis. Since different chirp types 275 have different properties, in the following only I/Q data collected with the first chirp type are used. Taking into account that the first chirp type has 37 range bins, in total 2.2 × 103 chirp sequences (15.9 × 106 chirps) are available in each polarimetric channel.”

A number of questions arises. What did you measure, rain, cloud, etc? What was SNR? What is the impact of noise (I suggest that you state explicitly what is the impact of noise on your formulations)? You need to give a better description of the observations.