Response to Reviewers' Comments

We thank the reviewers for their detailed reading of the paper, for catching some errors, and for the helpful suggestions for improvement. Please see the following pages for a detailed response and a summary of our changes.

Sincerely,

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May 2, 2022

Response to Editor

As you can see from the reviews, your paper requires minor revision before it is ready for publication. It is not expected that you redo the whole method to include the high SNR cases. You might want to leave that for a future effort. However, some discussions and limitations of the current method, need to be included (see reviewer 1's suggestions).

Response: We thank the editor for the possibility to address the further comments and for the generous time extension for the revision. As suggested we have added a section 6.1 on "Limitations of the presented model" where the mentioned issues are discussed and added further information to the manuscript were necessary.

Response to Reviewer 1

This manuscript presents a novel approach to radar data analysis by applying recently developed mathematical techniques involving hierarchical statistical models and hyperpriors. The manuscript clearly motivates the utility of these techniques for radar data analysis in situations where different atmospheric targets are present with significantly different length-scales, and no single characteristic length scale can reasonably be assumed a priori. This situation is common in radar studies of the D- and E-region ionosphere where sporadic E layers, PMSE, or PMWE can be observed. Overall the results presented in this manuscript are a promising proof-of-concept demonstrating the utility of these hierarchical techniques. Nonetheless, the models used make certain inaccurate assumptions about radar signals that make the present work incomplete. The limitations of these assumptions require explanation and discussion of how future work could apply these techniques to more realistic signal models.

1. **The use of a constant and diagonal noise covariance matrix is not correct for the radar signals of interest.** *This study assumes that the measurements P_m are normally distributed with covariance matrix $\sigma^2\mathbf{I}$, where σ^2 is a known constant. This assumption is only appropriate for weak signals in a particular limit, and it will generally not be appropriate for strong signals such as PMSE and PMWE. Line 136 acknowledges that the "self-noise" contribution from the target may violate this assumption in some cases without adequate additional discussion.*

   The correct way to model the radar signals is to write Eq. 4 as

   $$z_m(t) = (W * \sigma)(t) + n(t)$$

   where both the target scattering amplitudes σ and the noise contributions n are independent Gaussian random processes. Assuming the noise power, $N = E\{|n|^2\}N$ is independently known, Eq. 5 should be written as

   $$P_m(t) = \frac{1}{M}\sum_{\ell=1}^{M} \left|z_m^\ell\right|^2 - N$$

   In general $P_m(t)$ is not Gaussian, but if M is sufficiently large one may invoke the central limit theorem and derive an approximate Gaussian distribution for $P_m(t)$.

If the target signals are extremely weak compared to the receiver noise, then the covariance matrix of P_m is simply $\frac{N^2}{N}\mathbf{I}$. Therefore the model from this manuscript is correct in this weak signal limit if one identifies $\sigma^2 = \frac{N^2}{M}$. Many of the signals of interest for this work, such as sporadic E, PMSE, and PMWE will usually not satisfy this weak signal limit, and therefore the model in this manuscript is inappropriate.

In the high signal limit, the complete expression for the relevant covariance matrix of $P_m(t)$ has all of the following difficult properties

- *It is not a constant*
- *It is non-diagonal for every point-spread function other than the ideal Dirac delta (self-clutter effect).*
- *It explicitly depends on the signal power $P = E\left\{|\sigma(t)|^2\right\}$, which is unknown apriori (self-noise effect).*
- *It generally depends on the pulse-to-pulse correlation function of the target as well, $R_{\ell,k} = E\left\{\sigma^\ell(t)\bar{\sigma}^k(t)\right\}$, which is also unknown apriori.*

For interpulse periods of several milliseconds the pulse-to-pulse correlations can be neglected for normal E-region incoherent scatter and for sporadic E layers. For D-region incoherent scatter, PMWE, and PMSE, however, these correlations are significant, and the individual σ^ℓ from different pulses cannot be analyzed as independent measurements.

A complete formulation that correctly treats the complete covariance matrix is probably best left to future work, but the manuscript should at least discuss whether the method could conceivably accommodate more accurate treatments of the covariance matrix in the future.

Response: We agree that the high SNR around the strong layers makes the measurement variances range dependent and probably also leads to measurement errors correlations. However, the simple model with a constant variance σ^2 is a practical choice for this manuscript, in which the main emphasis is in the range-dependent length-scales. The full measurement error covariance matrix could be calculated from the data if samples from each individual radar pulse were stored separately and if the radar code cycle is not excessively long. The latter limitation is because the covariance structure is different for each code in a cycle.

Changes: We have added Section 6.1. where we discuss the error covariance calculation:

In the radar signal model presented in Section 2, the incoherent scatter self-noise contribution was neglected and the measurement noise was assumed to be stationary, zero-mean, Gaussian white noise. While this is a reasonable starting point for the analysis technique development, the self-noise contribution in our data may be significant due to the presence of strong layers. The self-noise makes the noise process non-stationary and correlated, which means that one should estimate the full measurement error covariance and use it in the deconvolution process. One should thus consider possibilities to include the self-noise in the signal model and to use the improved model in the hierarchical deconvolution process.

If time resolution of the data analysis is much coarser than duration of a radar code cycle, several observations of the echoes from each code are available, and one can readily calculate the full error covariance matrix of the measurements $\mathbf{P}_m$ with the cost of increased computational complexity (Huuskonen and Lehtinen, 1996). The technique fails at the limit of very long code cycles or very high time resolutions, but this limitation is not specific to our deconvolution technique. Furthermore, the diagonal of the error covariance can be calculated also for very long code sequences, because the variances do not depend on the phase-coding.

2. **The manuscript does not discuss whether the estimation scheme could accommodate self- noise effects.** *Equations 13, 14, and 15 are independent of the unknown P if σ^2 is assumed to be known. If self-noise effects are included, however, then the data covariance depends on the unknown powers P, and these three equations cannot be solved. The manuscript should discuss strategies for dealing with this difficulty. One possibility is to use P_m instead of P when evaluating the self-noise contributions. Another possibility is an iterative approach where $\hat{P}$ from the previous iteration is used to evaluate the self-noise contributions for the next iteration.*

Response: In theory, the measurement error covariance matrix could be used in the inversion via Cholesky factorization, but implementing this is left for a future work.

Changes: Section 6.1.:

The full measurement error covariance matrix, denoted by $\mathbf{R}$, can be incorporated into the deconvolution model. We can utilise the Cholesky factor $\mathbf{S}$ of $\mathbf{R}$, i.e., $\mathbf{S}\mathbf{S}^T = \mathbf{R}$, such that $\mathbf{S}^{-1}\mathbf{P}_m = \mathbf{S}^{-1}\mathbf{A}\mathbf{P} + \mathbf{S}^{-1}\boldsymbol{\varepsilon}$.

This whitens the error vector, making its components independently distributed. After this, the original algorithm can be used by setting the theory matrix to $\mathbf{A}^* = \mathbf{S}^{-1}\mathbf{A}$ and $\sigma^2 = 1$.

3. **The manuscript does not explain how the data variances are set for the examples.** *Lines 247-250 describe a synthetic signal generation process that will produce realistic radar signals with self-noise and self-clutter included. As explained above these signals will be inconsistent with a constant σ^2 . The real EISCAT signals will also contain self-noise and self-clutter that are inconsistent with a constant σ^2 . The manuscript does not explain what value is used for σ^2 when inverting these example signals, and the results will likely depend on the choice of σ^2 .*

   Response: We have added the values of the standard deviations (in the same arbitrary units that are used in the figures). We have included also the thermal background noise levels, because the standard deviations used in the inversion were larger than the thermal background to accommodate for the self-noise from the strong layers.

   Changes: Line 240: a noise variance σ^2 and
   Line 269: The results depend also on the standard deviation of the measurement error, which was set to $\sigma = 0.1$ (in the arbitrary units used in Fig. 3). The value is larger than standard deviation of the background noise in the averaged data (0.04) to accommodate for the self-noise from the strong scattering layer.
   Line 317: Standard deviation of the measurement error was set to $\sigma = 0.1$ (in the units used in Fig. 6). The value is an order of magnitude larger than the thermal background noise in the time-averaged data (0.01) to accommodate for the significant self-noise from the strong layer.
   Line 355: Measurement error standard deviation was set to $\sigma = 0.06$ (in the units used in Fig. 8). The value is again considerably larger than the thermal background level (0.005) to accommodate from the self-noise from the very strong layer.

4. **The prior model for P does not constrain the solution to be positive.** *The scattering power is always a positive number, and it is physically related to quantities that are positive by definition (e.g. electron density). Nonetheless, the prior model for P discussed in section 3.1 is a zero-mean Gaussian process, which implies that negative numbers are equally as likely as positive numbers,*

a priori. The negative numbers are unphysical. The manuscript should discuss why this prior was chosen and whether the technique could be adapted to use more physical priors in the future.

Response: While the prior model as such promotes values which can be negative, and thus non-physical, the estimators produced are dominated by the likelihood which typically guarantees positivity of the estimators. In the case the estimators would be negative, then the algorithm can be considered to produce non-physical estimators, and we can use this information as an indicator to pinpoint the cases where, e.g., the data is somehow corrupt. Naturally we could force the prior to be non-negative with standard tricks, like logarithmic transformation of the unknown. However, this would induce non-linearities and further complicate the computations, and thus increasing the computation times significantly. Thus, even though of the possible negativity of the prior process, in practice, this is computationally faster, and provides clear estimators for all the properly measured cases.

Changes: We added the following test to Section 3.1: "We note that the prior model is a zero-mean process with negative, and thus non-physical values. We could force the prior to be non-negative with standard tricks, like logarithmic transformation of the unknown. However, this would induce non-linearities and thus increase computation times. In addition, as the likelihood typically guarantees positivity of the estimators, so one could consider non-physical estimators as indicator pinpointing the cases where, e.g., the data is somehow corrupt. "

5. **The use of arbitrary units power units throughout the examples limits the reader's ability to assess the signal-to-noise regime.** *While arbitrary units are acceptable, the manuscript should state the noise power level in the same arbitrary units and state the number of samples M involved. As presented it is impossible to determine the signal-to-noise ratios of the signals and how large the self-noise and self-clutter effects are likely to be.*

   Response: We have added the number of averaged pulses and standard deviation of the background noise for each of the examples.

   Changes: Line 253: average power profiles were calculated over 665 subsequent transmitted pulses, which leads to 1 s time resolution.

Line 313: average profiles of the backscattered power were calculated with 0.9 s (665 pulses) time resolution.
Line 353: The profile is an average over 128 subsequent pulses (0.2 s in time).
Changes related to noise power levels are included in our response to comment 3.

Minor Corrections

1. *Line 258 should read "explicitly control"*

Response: We believe that the comment was about line 58, which said "without the need to explicit control" instead of 258.

Changes: Line 58: explicitly control