Review of 'Hierarchical Deconvolution for Incoherent Scatter Radar Data' by Ross et al.

This manuscript presents a novel approach to radar data analysis by applying recently developed mathematical techniques involving hierarchical statistical models and hyperpriors. The manuscript clearly motivates the utility of these techniques for radar data analysis in situations where different atmospheric targets are present with significantly different length-scales, and no single characteristic length scale can reasonably be assumed a priori. This situation is common in radar studies of the D- and E-region ionosphere where sporadic E layers, PMSE, or PMWE can be observed. Overall the results presented in this manuscript are a promising proof-of-concept demonstrating the utility of these hierarchical techniques. Nonetheless, the models used make certain inaccurate assumptions about radar signals that make the present work incomplete. The limitations of these assumptions require explanation and discussion of how future work could apply these techniques to more realistic signal models.

Major Issues

1. **The use of a constant and diagonal noise covariance matrix is not correct for the radar signals of interest.** This study assumes that the measurements P_m are normally distributed with covariance matrix $\sigma^2\mathbf{I}$, where σ^2 is a known constant. This assumption is only appropriate for weak signals in a particular limit, and it will generally not be appropriate for strong signals such as PMSE and PMWE. Line 136 acknowledges that the "self-noise" contribution from the target may violate this assumption in some cases without adequate additional discussion.

   The correct way to model the radar signals is to write Eq. 4 as

   $$z_m(t) = (W * \sigma)(t) + n(t)$$

   where both the target scattering amplitudes σ and the noise contributions n are independent Gaussian random processes. Assuming the noise power, $N = E\left\{|n|^2\right\}$ is independently known, Eq. 5 should be written as

   $$P_m(t) = \frac{1}{M}\sum_{\ell=1}^{M}\left|z_m^\ell\right|^2 - N.$$

   In general $P_m(t)$ is not Gaussian, but if M is sufficiently large one may invoke the central limit theorem and derive an approximate Gaussian distribution for $P_m(t)$.

   If the target signals are extremely weak compared to the receiver noise, then the covariance matrix of P_m is simply $\frac{N^2}{M}\mathbf{I}$. Therefore the model from this manuscript is correct in this weak signal limit if one identifies $\sigma^2 = \frac{N^2}{M}$. Many of the signals of interest for this work, such as sporadic E, PMSE, and PMWE will usually not satisfy this weak signal limit, and therefore the model in this manuscript is inappropriate.

   In the high signal limit, the complete expression for the relevant covariance matrix of $P_m(t)$ has all of the following difficult properties

   - It is not a constant

- It is non-diagonal for every point-spread function other than the ideal Dirac delta (self-clutter effect).
- It explicitly depends on the signal power $P = E\left\{|\sigma(t)|^2\right\}$, which is unknown a priori (self-noise effect).
- It generally depends on the pulse-to-pulse correlation function of the target as well, $R_{\ell,k} = E\left\{\sigma^\ell(t)\bar{\sigma}^k(t)\right\}$, which is also unknown a priori.

For interpulse periods of several milliseconds the pulse-to-pulse correlations can be neglected for normal E-region incoherent scatter and for sporadic E layers. For D-region incoherent scatter, PMWE, and PMSE, however, these correlations are significant, and the individual σ^ℓ from different pulses cannot be analyzed as independent measurements.

A complete formulation that correctly treats the complete covariance matrix is probably best left to future work, but the manuscript should at least discuss whether the method could conceivably accommodate more accurate treatments of the covariance matrix in the future.

2. **The manuscript does not discuss whether the estimation scheme could accommodate self-noise effects.** Equations 13, 14, and 15 are independent of the unknown P if σ^2 is assumed to be known. If self-noise effects are included, however, then the data covariance depends on the unknown powers P, and these three equations cannot be solved. The manuscript should discuss strategies for dealing with this difficulty. One possibility is to use P_m instead of P when evaluating the self-noise contributions. Another possibility is an iterative approach where $\hat{P}$ from the previous iteration is used to evaluate the self-noise contributions for the next iteration.

3. **The manuscript does not explain how the data variances are set for the examples.** Lines 247-250 describe a synthetic signal generation process that will produce realistic radar signals with self-noise and self-clutter included. As explained above these signals will be inconsistent with a constant σ^2. The real EISCAT signals will also contain self-noise and self-clutter that are inconsistent with a constant σ^2. The manuscript does not explain what value is used for σ^2 when inverting these example signals, and the results will likely depend on the choice of σ^2.

4. **The prior model for P does not constrain the solution to be positive.** The scattering power is always a positive number, and it is physically related to quantities that are positive by definition (e.g. electron density). Nonetheless, the prior model for P discussed in section 3.1 is a zero-mean Gaussian process, which implies that negative numbers are equally as likely as positive numbers, a priori. The negative numbers are unphysical. The manuscript should discuss why this prior was chosen and whether the technique could be adapted to use more physical priors in the future.

5. **The use of arbitrary units power units throughout the examples limits the reader's ability to assess the signal-to-noise regime.** While arbitrary units are acceptable, the manuscript should state the noise power level in the same arbitrary units and state the number of samples M involved. As presented it is impossible to determine the signal-to-noise ratios of the signals and how large the self-noise and self-clutter effects are likely to be.

Minor Corrections

1. Line 258 should read "explicitly control"