General comment:

My general impression is that the new version of the manuscript reads better. However, it still needs some improvements before it can be accepted for publication. Although I previously suggested some changes, it seems that the authors need a stronger and more explicit supervision. I found a couple major problems in this version that I already pointed out in my previous review and the authors didn't seem to address them satisfactorily.

Specific comments:

1. In my second review, I recommended that the authors read the paper carefully again and check every single sentence for consistency. The authors' response was:
   "**We carefully revised the manuscript according to the reviewer's comments."**
   When I started reading the new version of the manuscript, the first thing that struck me was the abstract, in which you say:"… at **three** ARM fixed-location atmospheric observatories and from **three** ARM mobile observatories deployed around the world for various field campaigns, which cover from Tropics to Polar regions and over both ocean and land surfaces." and that information is then repeated in the Introduction.

   According to the above, you have six sources of data in total. Just to remind you one of the points from my previous review:
   L487: "at six ARM observatories located around the world" – This is confusing. Once you say nine (Tab. 1, Fig. 1), then you say six. Again, I strongly encourage the authors to read their paper very carefully and check every single sentence for consistency.
   Response: We thank the reviewer for pointing out the typo. The reviewer is correct that it should be nine.

   Clearly, it was an error not a typo and that error was then repeated in the abstract and the Introduction and is still there. It is difficult for me to understand what the authors mean by "we carefully revised the manuscript" since I find the same simple errors again.

   "which cover from Tropics to Polar regions" – cover what? Use lowercase.

   I suggest to remove the number of stations from the abstract and describe them in a more general way, for instance as a data set from different climate zones probing a variety of PBL regimes.
2. Title: should it be "with" or "and"? The current form suggests that radiosonde data is good and ceilometer data needs to be evaluated.

3. L30: Not true. PBL height does not characterize the structure of the lowest few kilometers of the atmosphere by any means. It only indicates where the top of that PBL structure is located.

4. L36-37: "by an inversion layer of potential temperature" – this sentence is unclear. Is it a temperature inversion? Why "inversion layer"? For convective PBLs potential temperature actually does not have an inversion at the top as its stratification changes from neutral to stable, right? Did you mean real temperature, for which temperature inversion can make more sense?

5. There are at least 9 different methods of estimating PBL height. Von Engeln and Teixeira (2013) mention many of them. In large-eddy simulations, it is common to apply gradient methods (for temperature or moisture) or indeed look at turbulence properties. Some examples worth citing:

   Bopape, M.-J.M.; Plant, R.S.; Coceal, O. Resolution Dependence of Turbulent Structures in Convective Boundary Layer Simulations. *Atmosphere* **2020**, *11*, 986. https://doi.org/10.3390/atmos11090986

   J. Kurowski, M., P. Malinowski, S. and W. Grabowski, W. (2009), A numerical investigation of entrainment and transport within a stratocumulus-topped boundary layer. Q.J.R. Meteorol. Soc., 135: 77-92. https://doi.org/10.1002/qj.354

6. L84: ", the robustness…" – this should either be a new sentence or you should add "and" in between

7. L87: "long term" – this is too vague and means different things for different people; be more specific: multi-year?
8. L97: "low-level cloud-free" – what are low level cloud free conditions here? What does that low level refer to? Does it mean there are clouds at some upper levels?
9. L131-132: "three commonly used methods developed by Heffter (1980; referred to as the Heffter method hereafter)" – does it mean you name the three methods as the Hefter method?
10. That description of 3 methods is still slightly chaotic.
    Please summarize the text between L130 and L155 with an additional table: Method name, algorithm (e.g., Richardson number, potential temperature gradient), reference, etc. which should help understand your methodology.

11. L145: when you say between fifth and second levels of sounding data it means nothing to the reader. Explain what levels 5 and 2 mean. Is it at a fixed height which is the same at all stations under all conditions?
    In principle, if you use a threshold for potential temperature difference, it means you assume that there must exist a minimum gradient. So that method also relies on temperature gradient.
12. L160: We need to discuss this definition of Ri one more time. According to your description: "Ri at a given altitude can be calculated from sounding data"

$$Ri_b = \left(\frac{gz}{\theta_{v0}}\right)\left(\frac{\theta_{vz}-\theta_{v0}}{u_z^2+v_z^2}\right)$$

    As I explained in my previous reviews, this particular definition refers to the entire layer from 0 (surface) up to some height z. This is not Ri at a given altitude and you can only do it for a layer of finite height. If you want to look at a layer of thickness dz, then it should rather be:

$$R_B = \frac{(g/T_v)\Delta\theta_v\Delta z}{(\Delta U)^2+(\Delta V)^2}$$

    Because your definition replaces all the gradients with the values at the height z, it implicitly assumes that those values diminish at z=0 (which is true for u and v, but not for theta that uses a difference between level 0 and z in your equation. How exactly is Ri calculated in your data?

If you still claim that Ri in your equation (1) is given at the height z, then we have a problem with understanding basic equations used in your study. I tried to explain it in my previous reviews, but was so far unsuccessful.
It is critically important that you understand and explain the way you use Ri in your study correctly as it is the foundation of your analysis.

13. L265: “because Liu-Liang method uses different methods” – confusing
14. Fig. 11: Make axes labels and ticks larger.