

Cover letter

Dear Editors and Reviewers:

Thank you for your letter and for the Reviewers' comments concerning our manuscript entitled "Turbulence parameters measured by the Beijing Mesosphere–Stratosphere–Troposphere radar in the troposphere/lower stratosphere with three models: Comparison and analyses" (ID: amt-2021-309). Those comments are valuable and helpful for revising and improving our paper and the important guiding significance to our research. We appreciate for Editors' and Reviewers' warm work earnestly and hope that the correction will meet with approval.

We have studied each comment carefully and have made corrections as far as possible. We recalculated the turbulence parameters and reorganized the overall manuscript, by taking $c_1=0.27$ for the D-H model and using one-way and two way half power half width of radar beam correctly. These changes do not affect the structure of the article. Revised portions are marked in red in the paper. For details of the revised manuscript, please refer to the document "Author's track-changes" and uploaded manuscript. The main corrections in the paper and the responses to the Reviewers' comments are as flowing:

RC1 referee comment 1

Q1 I.144 explain why specular reflection is a problem for calculating turbulence parameters. Also ‘...such as in a statically...’

Response:

Thank you very much for your comments and questions, the explanation is as follows:

Most research on turbulence parameters using atmospheric radar are based on the Kolmogorov hypothesis of isotropic turbulence at the inertial sub-region scale (Batchelor, 1953; Tatarski, 1961, 1971). To detect atmospheric turbulence intensity by atmospheric radars (spectral width method), the radar echo signal should come from turbulence scattering. In fact, at some heights, such as near the tropopause region, the radar echo can be affected by specular reflection. The specular (Fresnel) reflection is not the same as that for refractive index perturbations due to turbulence. The layer can be treated as a planar mirror with a small reflection for incident field. Therefore, if the vertical gradient of refractive index does not move vertically and does not change the shape of stratification either, the spectrum width will be affected by specular reflection, making the spectral width method inapplicable.

The modified content revised version(clean) for Q1 is as follows:

Line No:146-149 (Section2.1, Page No:6 of 26)

“However, the vertical beam is more susceptible to specular reflection, especially in the tropopause region, which rather than isotropic scattering due to isotropic turbulence affects the spectrum (e.g., Fukao et al., 1994; Tsuda et al., 1986; Birner, 2006). The spectral width method is based on the isotropic scattering, which has the hypothesis that the radial wind speed variance (Doppler spectral width) detected by the radar is equal to the turbulence intensity.”

Q2 Table 1 – explain what is meant by coherent and incoherent averaging (these terms don’t mean much to readers unfamiliar with the MST radar technique)

Response:

Thank you very much for your suggestion.

The modified content revised version(clean) for Q2 is as follows:

Coherent integration (uniting the signal phase at the identical range bin)

Incoherent integration (averaging processing in the frequency domain)

Q3 I.156 How long a period is the data collected for a single observation (that is compared to the sonde)? 50 s?

In medium mode, it takes about 5min for five beams to complete once data acquisition. For example, 2012.05.08, 15:40 (UTC). It starts at 35'4" and ends at 40'10". So it cost 5'6" for five beams.

Q4 I.186 broadening not widening

Response:

Thank you for your reminding. We have replaced "Wind shear widening" with "Wind shear broadening."

The modified content revised version(clean) for Q4 is as follows:

Line No:191 (Section2.3.1, Page No:8 of 26)

Q5 I.206 Equation 7 is wrong – it should read $\dots r \cos \gamma + \theta^2 / 24 \dots$. Also, having introduced the notation $\theta_{1/2}$ earlier, this expression should be used in this equation to avoid any confusion in the mind of the reader

Response:

Thank you very much for your comments and suggestions.

We have used the following expressions in the new version:

θ is used as potential temperature

$\theta_{1/2}^{(1)}$ is used as one way half power half width

$\theta_{1/2}^{(2)}$ is used as two way half power half width

$\frac{\partial u}{\partial z}$ is used as vertical gradient of horizontal wind

Q6 I.212 Deghan and Hocking's model is actually quite different to Nastrom's – it doesn't just contain one extra term for example. They derived it from their own 3-D simulations so you should explain more clearly how the two models differ.

Response:

Thank you for your reminder and suggestions.

Dehghan and Hocking (2011) gave a new calculation model (referred to as the D-H model) based on their own independent 3-D model as their reference, when Nastrom (1997) also introduced a 3-D model.

For Nastrom's (1997) model, the assumptions are as following:

The model made the following assumptions:

- (i) The model is two dimensional, and assumes only an xz vertical plane.
- (ii) The polar diagram is assumed to have sharp edges at $\pm n$, where n is the one-way half-power width of the beam.
- (iii) The polar diagram is assumed to have constant gain within the region between n and $+n$, and changes abruptly to zero gain further from the beam center.
- (iv) All calculations are performed with the one-way beam only.
- (v) The pulse is assumed to be a square function.
- (vi) The returned power is calculated by assuming the pulse is centered at a fixed range R_0 , and does not consider that the returned power is a convolution between the scattering function and the pulse.
- (vii) No consideration is made for dependence of backscattered power on range within the pulse.
- (viii) The receiver is assumed to have an infinite beam-width.
- (ix) It is assumed that the beam-width is unchanged as the beam moves to off-zenith angles

Dehghan and Hocking (2011) allowed the various terms and cross-terms that they expected to have power-law formats, with the coefficients and power laws being left as variables. And they point out three important scales—the vertical projection of the pulse, the vertical distance from the lower point of the beam to the upper point, and the vertical projection of the distance for U to C. The three items can be covered by the main basic vertical length units, Δr and $\zeta = 2r\theta_{1/2}^{(1)}\sin\chi$. And they give the three-dimensional analytical expression for $\sigma_{s\&b}^2$.

To simplify the formula, the values for constant variables are given by the fitting. The final model formula is called model "c" by themselves, and the difference reduces to 5.7%. And Further improvement to mode C may be achieved by some modest adjustments at the critical point. They took the correction term involving incorporation of a set of Gaussian and hyperbolic tangent corrections, and the mean error reduced to 4.1%, relative to a full model.

The modified content revised version(clean) for Q6 is as follows:

Line No:229-231 (Section2.3.3, Page No:10 of 26)

“Dehghan and Hocking (2011) gave a new calculation model (referred to as the D-H model) based on their own independent 3-D model as their reference, while Nastrom (1997) also introduced a 3-D model. The simplified equation is as follows:”

Q7

General question – Dehghan and Hocking (paragraph leading to their eqn 10) argue that the factor 3 in the first term on the RHS of equation 7 is actually wrong and should be replaced by $4\ln 2$ (2.77) if σ is the velocity variance, which is where their κ comes from. Could you discuss this point and test what difference it makes to your work?

1.363 Why do you use $c_1 = 0.45$ for the DH model when Dehghan and Hocking themselves use 0.27? (just before their eqn 3)? Saying that you use 0.45 ‘to facilitate comparison’ does not make sense.

Response:

As considered by the reviewer, the value of c_1 is very important, because it is significant for the calculation of turbulence parameters. But if the value of c_1 is constant, then only the values of ε are affected, not the characteristic of ε varying with height.

The distribution of σ_t^2 according to the N-2D model and D-H model is very consistent. And the trends with height of σ_t^2 are similar for the three models, when σ_t^2 calculated by the H model is smaller than that of the N-2D/ D-H model at all heights. The beam and shear broadening $\sigma_{b\&s}^2$ calculated by the H model are distributed discretely and are larger than the calculation results of the other two models at each height.

Obviously, the value of c_1 is a critical issue that needs more research, but is not the focus of this paper. So the previous version of this paper considers the effect of spectral width, but this study does not discuss the effect of c_1 in-depth, so only two values of c_1 are selected to simplify things.

But as considered by the reviewers, the value of c_1 is poorly considered in the previous version. So, our new version takes c_1 as 0.27, as suggested by D-H.

As mentioned above, the specific influence of c_1 cannot be well explained in this work, but it is also the work to be carried out in the future.

The modified content revised version(clean) for Q7 is as follows:

We recalculated the turbulence parameters and reorganized the overall manuscript, by taking $c_1=0.27$ for D-H model

RC2 referee comment 2

In this study, three formulations for estimating broadening components of radar spectral widths proposed in previous studies are compared using three-year observations by a VHF radar at Xianghe Station, China. The debroadening algorithm is necessary for accurate estimation of turbulent energy dissipation rates because overestimation of broadening components will lead to negative energy dissipation rates. This study may include interesting results based on thorough comparison of broadening components and resulting energy dissipation rates, and I feel that the overall logic of the study is well organized. However, there seems to be an error in the calculation of the broadening component. Specifically, in the small vertical shear region, the three formulations should yield almost same results, but there is a significant difference between them. I suspect that the authors misunderstand the one-way and two-way beam width. I recommend the authors to recalculate them and reorganized the overall manuscript.

General comments

Q1. If there is no vertical shear ($\left|\frac{\partial u}{\partial z}\right| = \sqrt{(\partial u_x/\partial z)^2 + (\partial v_y/\partial z)^2}$), the three formulations are almost same:

$$\text{Hocking (1985): } \sigma_b = \frac{\theta_{1/2}^{(2)} \cdot u}{\sqrt{2 \ln 2}} = \frac{\theta_{1/2}^{(1)} \cdot u}{2\sqrt{\ln 2}}, \quad \sigma_s = 0$$

$$\text{Nastrom (1997): } \sigma_{s\&b}^2 = \frac{\theta_{1/2}^{(1)2}}{3} u^2 \cos^2 \chi,$$

$$\text{Dehghan and Hocking (2011): } \sigma_{s\&b}^2 = \frac{\theta_{1/2}^{(1)2}}{k} u^2 \cos \chi,$$

where $\theta_{1/2}^{(1)}$ and $\theta_{1/2}^{(2)}$ are one-way and two way half power half width of radar beam, respectively; u is horizontal background wind; χ is the zenith angle of the beam (15°); and $4 \ln 2 \approx 2.77$. Note that the formulations of Hocking (1985) and Dehghan and Hocking (2011) are identical in no shear condition except for $\cos \chi$, which is approximately 0.93 (Table 1 in the manuscript). Figure 4 shows the vertical profiles of mean horizontal wind speed and vertical shear. It is found that mean vertical shear is very small around an altitude of 12.5 km, and thus the broadening components estimated from three formulations are expected to have similar values at the altitude. However, from Figure 5, $\sigma_{s\&b}^2$ from Hocking (1985)'s formulation is much larger than those from Nastrom (1997) and Dehghan and Hocking (2011). Other figures (e.g., Figure 2c, 3, 6, etc.) also show similar results. I am sure that something is wrong with the calculation of broadening components. One possibility is that the authors are not aware of the difference between one-way and two-way half power

half width ($\theta_{1/2}^{(1)} = \sqrt{2}\theta_{1/2}^{(2)}$)

Response:

Thank you very much for your professional and specific comments and questions. We recalculate turbulence parameter and reorganized the overall manuscript, by using the one-way and two-way beam width correctly.

We re-analyzed the data and found that there were actually two problems:

1. One-way and two-way half power and half width are incorrectly understood.
2. Even if that mean vertical shear is very small (around an altitude of 12.5 km), $\sigma_{s\&b}^2$ calculated by H model is larger than the other two.

We will answer these two questions one by one.

Question1. One-way and two-way half power and half width are incorrectly understood.

We reviewed the code and found that H model used $\theta_{1/2}^{(1)}$ instead of $\theta_{1/2}^{(2)}$, so we rewrote the codes. And the comparison between before and after versions is shown in the figure below (Fig.1,2). $\theta_{1/2}^{(1)}$ is used as one way half power half width, $\theta_{1/2}^{(2)}$ is used as two way half power half width, and $\theta_{1/2}^{(1)} = \sqrt{2}\theta_{1/2}^{(2)}$.

The result shows that, compared with the previous version, $\sigma_{s\&b}^2$ calculated by the H model is smaller in the latest version. And the values ($\sigma_{s\&b}^2$, σ_t^2 , ε , Kz) calculated by the three models are closer to each other.

But, the values of $\sigma_{s\&b}^2$ calculated by the H model is still larger than the other two models. Therefore, we conducted simulation experiments to verify the sensitivity of the three models to the vertical shear and horizontal wind speed. That's another question “2. Even if that mean vertical shear is very small (around an altitude of 12.5 km), $\sigma_{s\&b}^2$ calculated by H model is larger than the other two.”

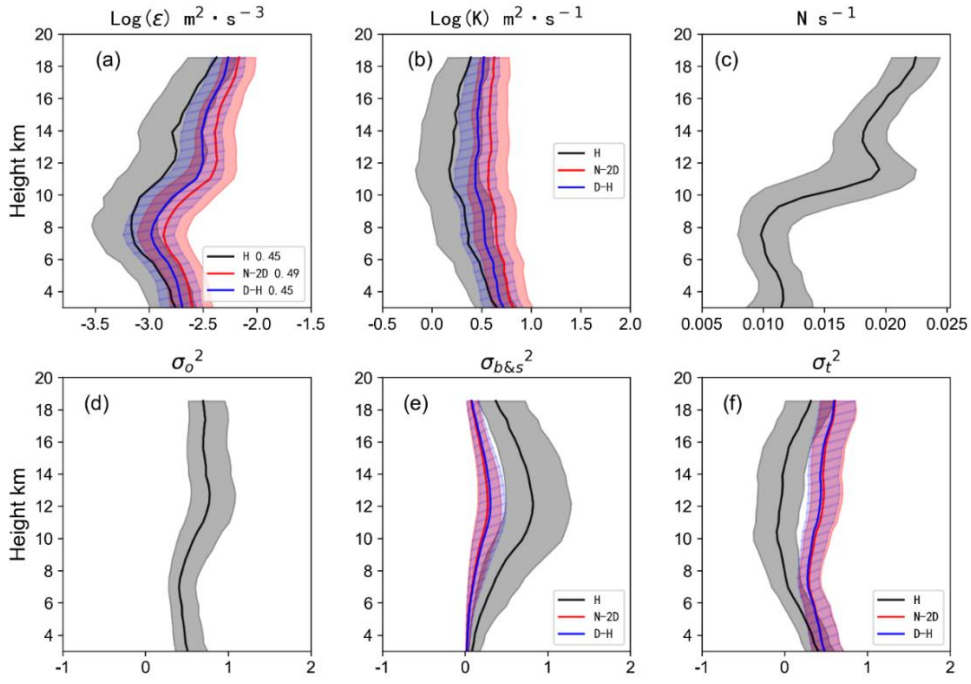


Figure 1 Previous version

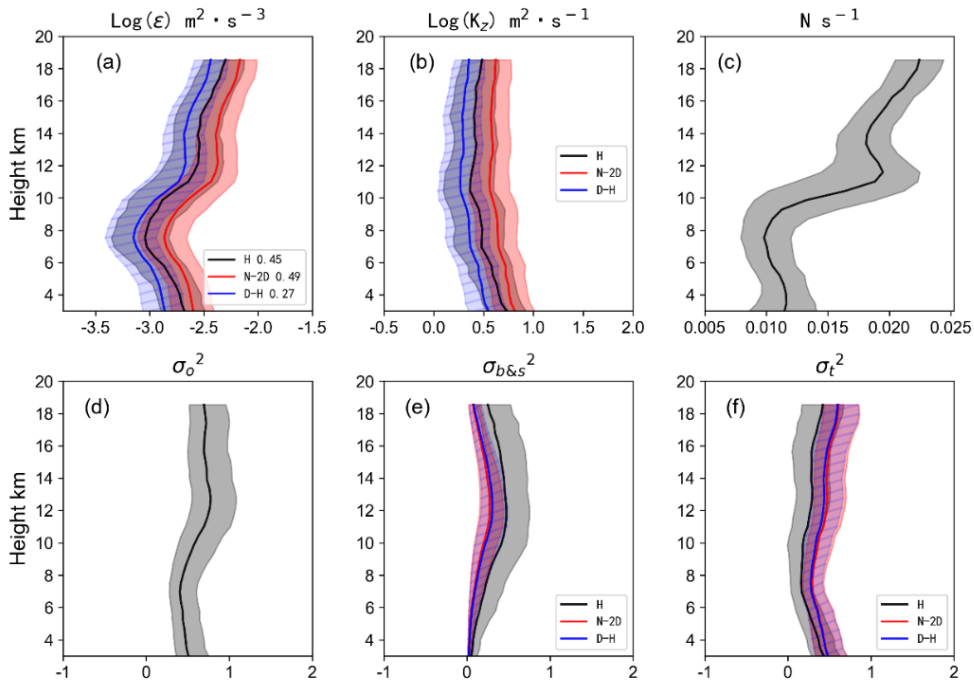


Figure 2 Recalculated version (The latest version)

2. Even if that mean vertical shear is very small (around an altitude of 12.5 km), $\sigma_{s\&b}^2$ calculated by H model is larger than the other two.

The correct use of one way half power half width $\theta_{1/2}^{(1)}$ and two way half power half width $\theta_{1/2}^{(2)}$ reduce the differences of $\sigma_{s\&b}^2$ between the three models, especially around an altitude of 12.5 km (mean vertical shear is very small).

But in fact, the result is not as expected, the broadening components estimated from three formulations don't have similar values at the altitude (mean vertical shear is very small, around an altitude of 12.5 km). The values calculated by the H model are still larger than the other two models.

We conducted simulation experiments and further analyzed the cause. The reasons are as follows:

(a) Simulation experiments:

When $\frac{\partial u}{\partial z}$ is strictly equal to 0, the broadening components are slight different between the three models. But when $\frac{\partial u}{\partial z}$ is not strictly equal to 0, the H model will be significantly larger than the other two.

As shown in the figure, when take $\frac{\partial u}{\partial z} = \pm 0.01 \text{ s}^{-1}$ (Fig.3 c, d), $\sigma_{s\&b}^2$ estimated from the H model is large than the other two models. One of the exceptions is that $\sigma_{s\&b}^2$ estimated from H model is similar to that from D-H model when $\frac{\partial u}{\partial z} = -0.01 \text{ s}^{-1}$.

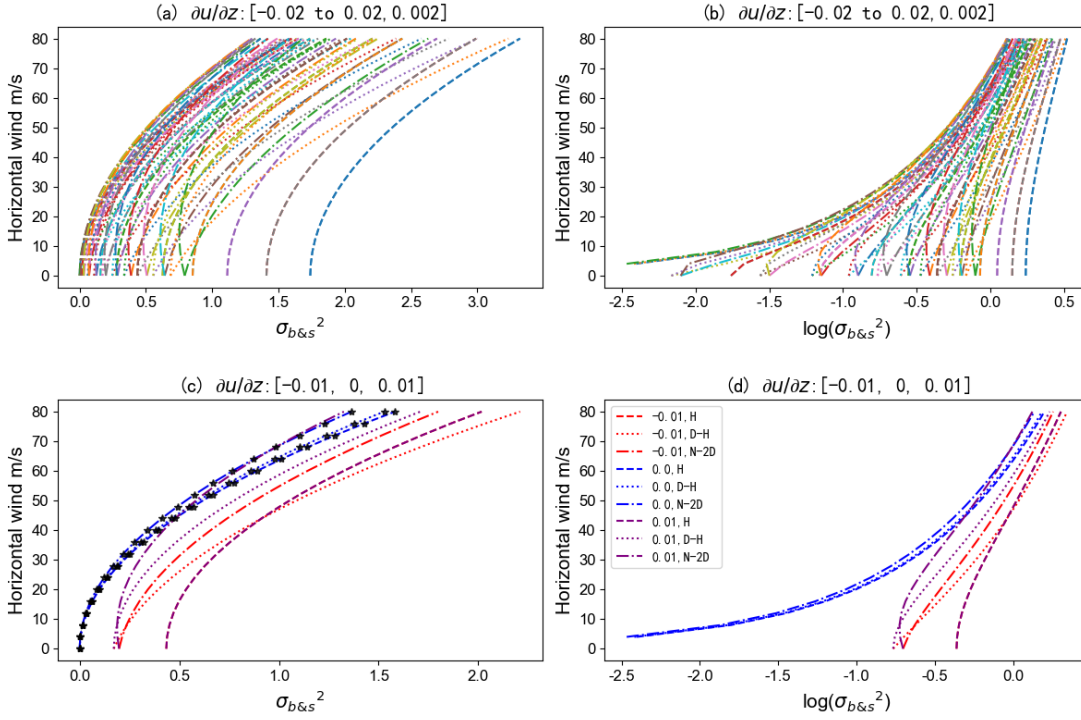


Figure 3 The $\sigma_{s\&b}^2$ estimated from the three models relate to the vertical shear and horizontal wind speed. (a)(b) $\frac{\partial u}{\partial z}$ is in the range of -0.02 to 0.02 s^{-1} , per 0.002 s^{-1} . (c)(d) $\frac{\partial u}{\partial z}$ has just 3 numbers, -0.01 s^{-1} , 0 s^{-1} , 0.01 s^{-1} . (a)(c) are $\sigma_{s\&b}^2$, (b)(d) are $\log(\sigma_{s\&b}^2)$. (c)(d) red lines are $\frac{\partial u}{\partial z} = -0.01$ s^{-1} , blue lines are $\frac{\partial u}{\partial z} = 0$ s^{-1} , purple lines are $\frac{\partial u}{\partial z} = 0.01$ s^{-1} . (c) The stars are $\frac{\partial u}{\partial z} = 0$ s^{-1} . The broadening components estimated from H model is just related to $|\frac{\partial u}{\partial z}|$, so there are just 5 lines instead of 6 lines.

(b) Seasonal characteristics of $\frac{\partial u}{\partial z}$ in Beijing

1. In fact, the height of maximum horizontal wind speed has obvious seasonal variation in Beijing. The positive and negative values have an impact on the statistical results of $\frac{\partial u}{\partial z}$. This leads to mean vertical shear being very small at 12.5 km in Beijing. The median, upper and lower quartile profiles of $|\frac{\partial u}{\partial z}|$ are shown in Fig.4.
2. There are always strong vertical shear for several kilometers above ($\frac{\partial u}{\partial z} < 0$) or below ($\frac{\partial u}{\partial z} > 0$) the height of maximum horizontal wind speed.

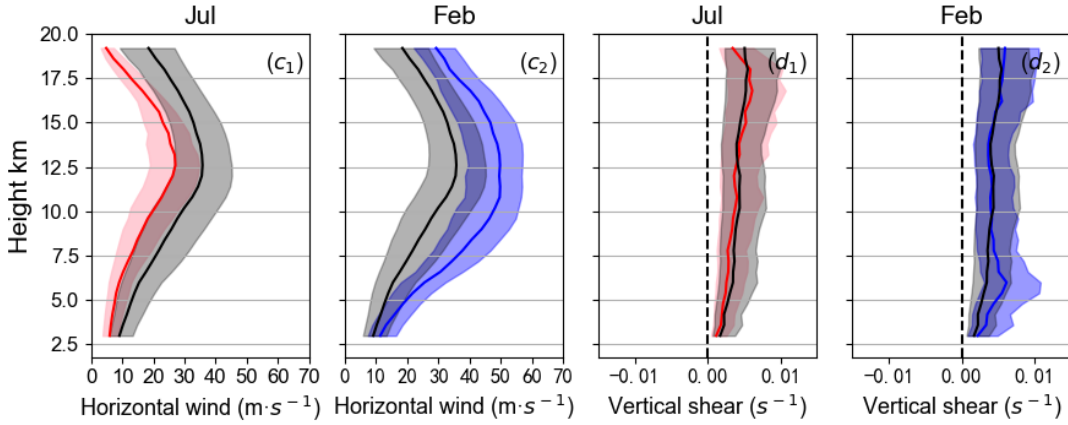


Figure 4 The median, upper and lower quartile profiles of $|\frac{\partial u}{\partial z}|$. It is the same as Fig4 in the manuscript, but for absolute value of $\frac{\partial u}{\partial z}$.

To sum up, although that the mean vertical shear is very small around an altitude of 12.5 km in Beijing, the horizontal wind vertical shear at this height is not strictly equal to 0 for many individual profiles. As the Simulation experiments show, when $\frac{\partial u}{\partial z}$ is not strictly equal to 0, the turbulence spectrum width of H model will be larger than that of the other two models. Thus it is reasonable that not all of them, the broadening components estimated from three formulations, have similar values at the altitude (mean vertical shear is very small).

The modified content revised version(clean) for Q1 is as follows:

We recalculate turbulence parameter and reorganized the overall manuscript, by using the one-way and two-way beam width correctly.

Q2: P.16, L.349-352: I wonder why the difference of median energy dissipation rates between including negative values and excluding them is so small. In general, mean energy dissipation rates without negative values included will be quite large compared to that calculated from both positive and negative values. For example, Kurosaki et al. (1996) showed that median Kz calculated from both positive and negative spectral widths are significantly different from that from only positive spectral width. Dehghan and Hocking (2011) showed a similar example for a radar with two-way beamwidth comparable to that used in this study. One of the exceptions is Kohma et al. (2019), who used an algorithm developed by Nishimura et al. (2020) to estimate the beam broadening component accurately. As a result, the difference of medians between with and without

negative energy dissipation rates is small.

Response:

Thank you very much for your comments and questions. We use $\log(\varepsilon)/\log(Kz)$ for statistics and give the average of the median of each height. Such expressions may mislead the reader.

Therefore, we make statistics on ε/Kz and $\log(\varepsilon)/\log(Kz)$ after recalculating the turbulence parameters. And the statistics are as follows (Table1):

Table 1 The ratios of the median mean ε and Kz calculated by the H model, N-2D model and D-H model (including/excluding negative values)

	including/excluding negative values	H (C1=0.45)	N-2D(C1=0.49)	D-H (C1=0.27)
ε	including negative values	0.0019	0.0034	0.0018
	excluding negative values	0.0029	0.0034	0.0020
Kz	including negative values	2.39	4.37	2.24
	excluding negative values	3.66	4.37	2.47
Log (ε)	including negative values	-2.7	-2.5	-2.8
	excluding negative values	-2.6	-2.5	-2.8
Log(Kz)	including negative values	0.48	0.64	0.35
	excluding negative values	0.56	0.64	0.39

The profiles of ε , Kz , $\log(\varepsilon)$ and $\log(Kz)$ are shown in Figures 5 to 8.

The result shows that the difference between including negative values and excluding them is closely related to the proportion of N-TKE. Compared with the H model, the differences between including negative values and excluding them are very small for the N-2D model and D-H model, which is because the H model has a higher proportion of N-TKE.

The modified content revised version(clean) for Q2is as follows:

Line No:373-382 (Section3.3, Page No:17-18 of 26)

“The turbulence spectrum width contains negative values, as do ε and Kz . The difference between including negative values and excluding them is closely related to the proportion of N-TKE. Compared with H model, the difference between including negative values and excluding them is very small for N-2D model and D-H model, which is due to the fact that the H model has a higher proportion of N-TKE. The ratios of the median mean ε calculated by the H model, N-2D model and D-H model (including/excluding negative values) are 0.019/0.029, 0.0034/0.0034 and 0.0018/0.0020, respectively. The ratio of Kz is 2.39/3.66, 4.37/4.37 and 2.24/2.47, respectively. Several studies

showed that the mean energy dissipation rates without negative values included will be quite large compared to that calculated from both positive and negative values (Kurosaki et al. 1996; Dehghan and Hocking, 2011). One of the exceptions is Kohma et al. (2019), who used an algorithm developed by Nishimura et al. (2020) to estimate the beam broadening component accurately. As a result, the difference of medians between with and without negative energy dissipation rates is small.”

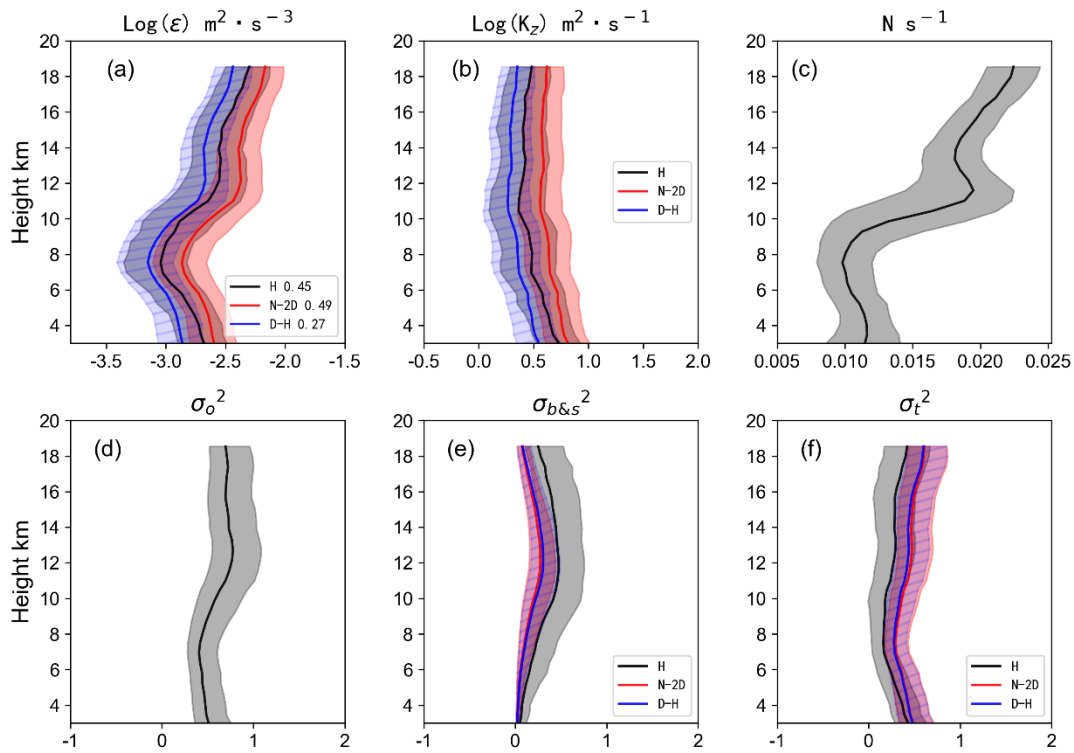


Figure 5 $\log(\epsilon)$ and $\log(Kz)$ including negative values.

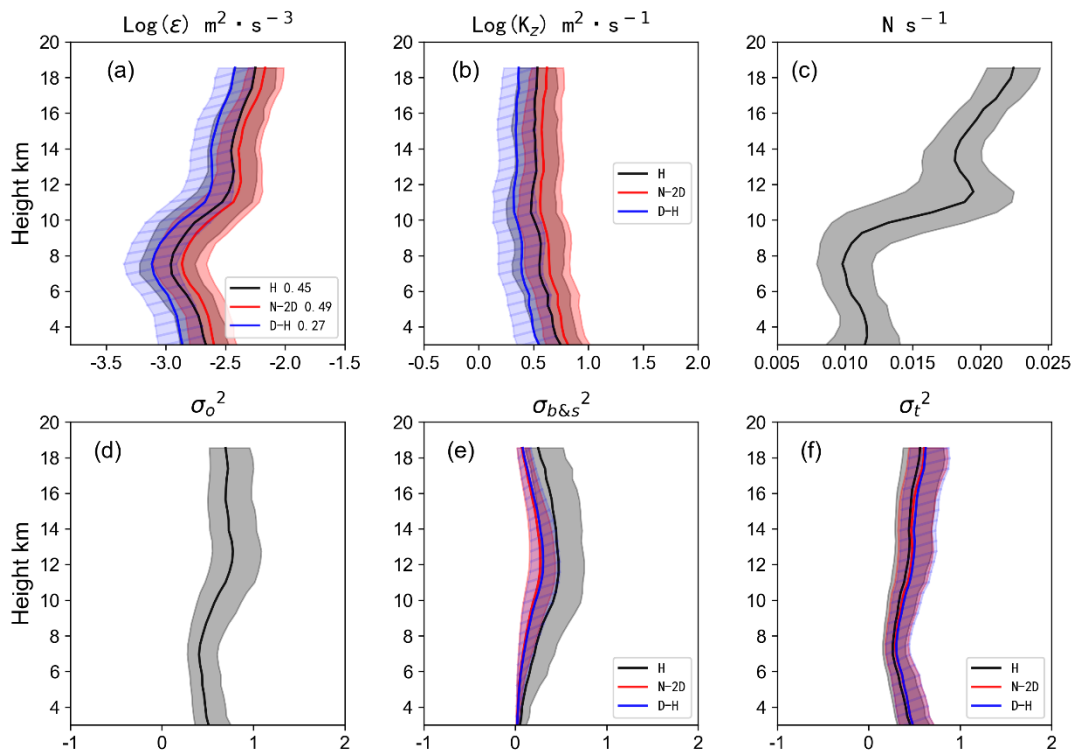


Figure 6 $\log(\epsilon)$ and $\log(Kz)$ excluding negative values.

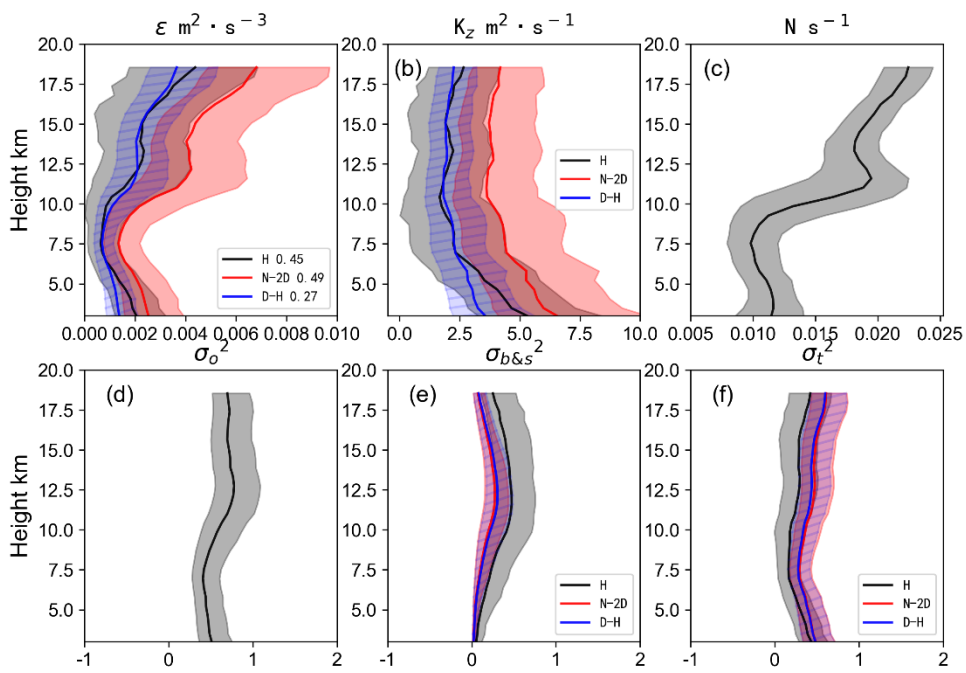


Figure 7 ε , K_z including negative values.

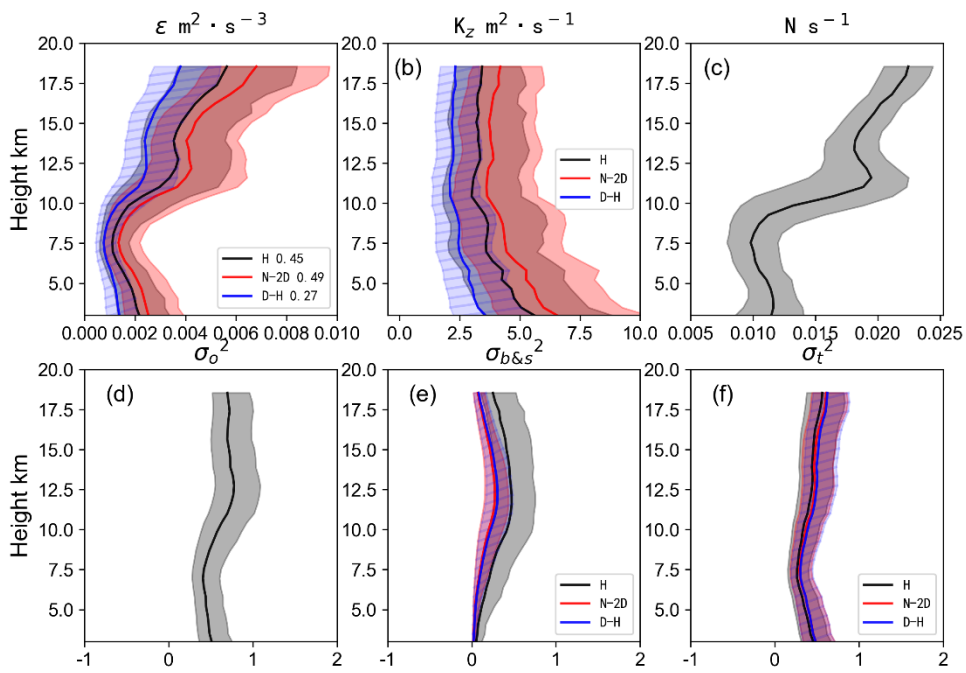


Figure 8 ε , K_z excluding negative values.

Q3:

Specific comments

About notation: The notation for several variables is problematic in the present manuscript, e.g, θ is used as potential temperature (P.3, L.90) and half-power half width (P.0, L.208), and both $|\frac{\partial u}{\partial z}|$ and $|\frac{\partial v}{\partial z}|$ are vertical gradient of horizontal wind. I recommend the authors to use consistent notations through the manuscript.

Response:

Thank you very much for your comments and suggestions.

We have used consistent notation through the manuscript in the latest version as follows:

θ is used as potential temperature

$\theta_{1/2}^{(1)}$ is used as one way half power half width

$\theta_{1/2}^{(2)}$ is used as two way half power half width

$\frac{\partial u}{\partial z}$ is used as vertical gradient of horizontal wind

Q4:

Typos etc.

Please specify the definition of vertical shear of horizontal wind:

Is it defined as $\sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}$ (u and v is zonal and meridional wind, respectively)?

Response:

Thank you very much for your suggestions. As you have considered, $\frac{\partial u}{\partial z}$ is a question worth considering.

We assume that, i. The wind profile is linear. ii. Within a sampling volume of two range bins, regardless of the influence of wind direction. So values of $\frac{\partial u}{\partial z}$ can take into account the change in scalar values.

However, the horizontal wind we get is a vector, and there must be changes in wind directions.

In these three models, $\frac{\partial u}{\partial z}$ is used to calculate the horizontal wind speed at each point in the sample volume and $\frac{\partial u}{\partial z}$ is assumed to be constant.

But all three models are simplified, so we cannot be sure that a scalar or vector representation is more reasonable.

One point is inevitable, $\frac{\partial u}{\partial z}$ has positive and negative.

Scalar:

$$\partial u / \partial z_s = \frac{\partial \sqrt{u_x^2 + v_y^2}}{\partial z}$$

About vector, we think the following formula is more suitable:

$$\partial u / \partial z_v = \begin{cases} (-1) \cdot \sqrt{\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial v_y}{\partial z}\right)^2}, & \frac{\partial \sqrt{u_x^2 + v_y^2}}{\partial z} < 0 \\ 1 \cdot \sqrt{\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial v_y}{\partial z}\right)^2}, & \frac{\partial \sqrt{u_x^2 + v_y^2}}{\partial z} > 0 \end{cases}$$

where u_x and v_y is zonal and meridional wind, respectively.

So we use the observational data from Beijing MST (2012 to 2014) to analyze the differences between $\partial u / \partial z_s$ and $\partial u / \partial z_v$ as shown in Fig.9. The result shows there is little difference between them, the mean absolute value of $(\partial u / \partial z_s - \partial u / \partial z_v)$ is 0.00047. And we take $\partial u / \partial z \equiv \partial u / \partial z_v$ in this study.

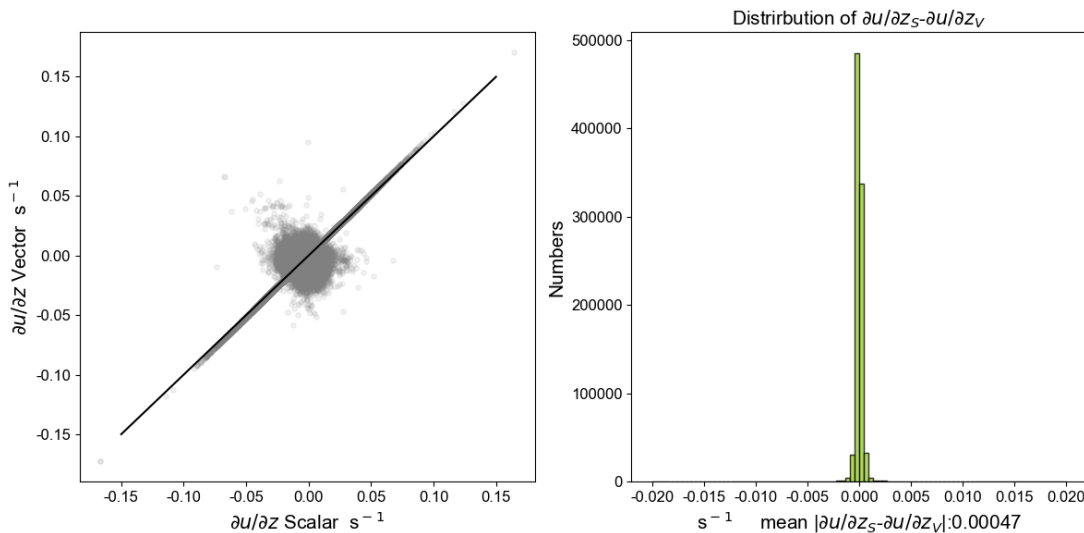


Figure 9 Difference of $\frac{\partial u}{\partial z}$ between scalar and vector

The modified content revised version(clean) for Q4 is as follows:

Line No:196-208 (Section2.3.1, Page No:8-9 of 26)]

“In fact, the horizontal wind we get is a vector, and there must be changes in wind directions. In these three models, $\frac{\partial u}{\partial z}$ is used to calculate the horizontal wind speed at each point in the sample volume and is assumed to be constant. But all three models are simplified, so we cannot be sure that a scalar or vector representation is more reasonable. One point is inevitable, $\frac{\partial u}{\partial z}$ has positive and negative values.

About scalar, the vertical shear of horizontal wind $\partial u/\partial z_s$ is as following:

$$\partial u/\partial z_s = \frac{\partial \sqrt{u_x^2 + v_y^2}}{\partial z}, \quad (6)$$

About vector, we think the following formula is more suitable:

$$\partial u/\partial z_v = \begin{cases} (-1) \cdot \sqrt{\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial v_y}{\partial z}\right)^2}, & \frac{\partial \sqrt{u_x^2 + v_y^2}}{\partial z} < 0 \\ 1 \cdot \sqrt{\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial v_y}{\partial z}\right)^2}, & \frac{\partial \sqrt{u_x^2 + v_y^2}}{\partial z} > 0 \end{cases}, \quad (7)$$

where u_x and v_y is zonal and meridional wind, respectively.

So we use the observational data from Beijing MST (2012 to 2014) to analyze the differences between $\partial u/\partial z_s$ and $\partial u/\partial z_v$. The result shows there is little difference between them, and the mean absolute value of $\partial u/\partial z_s - \partial u/\partial z_v$ is 0.00047. We take $\partial u/\partial z \equiv \partial u/\partial z_v$ in this study.”