Review of amt-2021-361: Towards the use of conservative thermodynamic variables in data assimilation: preliminary results using ground-based microwave radiometer measurements, Marquet et al.

The paper demonstrates the analyses of model profiles with the assimilation of ground-based microwave radiometer (MWR) brightness temperatures with a 1D-Var scheme and the use of two conservative thermodynamic variables as control variables in a case study. The presented work shows the potential of using the described conservative thermodynamic variables for 1D-Var MWR retrievals, and the benefit of the obtained, less weather-dependent, model error covariance matrices. It also hints at the benefit of MWR data for data assimilation in the studied fog case and is therefore a valuable step towards the use of MWR data in operational NWP systems.
The methodology is well explained, the experiment is carefully analysed and results are clearly presented. The assessment is done both in observation and model space. The manuscript is well written, and the figures are illustrative of the relevant results.
I support the publication of this article after the authors addressed the comments and suggestions listed below.

General comments

2.1 The moist-air entropy potential temperature: The section explains the most important points in the derivation of the moist-air entropy potential temperature and provides an extensive list of references for further information. Although it is beyond the scope of this manuscript to go in full details here, I found the description of the conservation laws (l. 74ff) a little short and difficult to follow for people not familiar with these variables. Some sentences are also quite long and complicated. I would very much welcome a reformulation of this paragraph.

4.3 Vertical profiles of analysis increments and 5 Conclusions: Do you have the possibility to make a stronger link between the increments and potential improvements in the model profiles? Are the increments located at the right place and going in the desired direction? You provide a good indication for temperature increments with respect to MWR measurements in l. 271ff. A comparison to soundings, as you mention in l. 321, could be especially valuable. Would it be possible to provide a preliminary assessment?

Specific comments

- **Title:** I suggest changing the title to 'Towards the use of conservative thermodynamic variables in data assimilation: a case study using ground-based microwave radiometer measurements' to make the scope of the manuscript clearer.
- **2.2 The 1D-Var formalism:** Very clear and concise description of 1D-Var, pleasant to read! It would maybe be good to include a reference to a more detailed description of 1D-Var theory.
- **2.3 The conversion operator:** To help make it easier to understand, it would be helpful to add in this section that this conversion from $((\theta_s)_a,\, q_t)$ to $(T,\, q_v,\, q_l)$ is needed to feed RTTOV after the analysis.

- **4.1 The background error cross correlations:** I am not familiar with the variable $(\theta_s)_a$. Could you make a little clearer why you expect a link between $(\theta_s)_a$ and q_t in clear-sky atmosphere (l. 234)? If this should be evident from section 2.1 it would maybe be good to highlight it more.
- **4.2 1D-Var analysis fit to observations:** In order to better compare the REF and EXP analyses in the regions of concern, it might be interesting to compute and compare RMSE and bias only within the blue rectangular boxes.
- **5 Conclusions:**
  - **l. 300ff** Maybe this paragraph would better suit to section 2.1, where the variables are introduced and described.
  - **l. 326ff** Do you expect results of TB to vary a lot after including q_i in the conversion operator? Where there ice clouds in the presented case (l. 327)?

Technical corrections

Text:

l. 5: '[...] that are currently highly dependent on weather conditions [...]' $\rightarrow$ '[...] that are highly dependent on weather conditions when using classical variables, [...]'

l. 28: 'scheme' $\rightarrow$ 'schemes'

l. 38: '[...] that data assimilation systems used to be [...]' $\rightarrow$ '[...] that most data assimilation systems are [...]'

l. 67: Use $s_{d0}(T_0, p_0)$ in the formula, as in l. 69.

l. 71: State that the explanation for Λ_r follows later in the section.

l. 73: '[...] sublimation.' $\rightarrow$ '[...] sublimation, respectively.'

l. 106: Remove the point at the end of the title.

l. 143: 'set-up' $\rightarrow$ 'setup'

l. 147: '[...] such a HATPRO [...]' $\rightarrow$ '[...] such as a HATPRO [...]'

l. 155: 'lowest' instead of 'first few'

l. 192: I propose to mention RTTOV-gb here after $\mathcal{H}$ because it was already mentioned earlier in the text.

l. 195: comma after '(LWP)'

l. 201: No new paragraph before l. 201

l. 236-237: $\mathbf{B}_\mathbf{z}(\theta_s, q_t) \rightarrow \mathbf{B}_\mathbf{z}((\theta_s)_a, q_t)$?

l. 276: 'similarity' $\rightarrow$ 'similarities'

l. 284-287: Suggestion: 'Most of the liquid water is created in low clouds. Additionally, increments of q_l above 600 hPa are larger and more extended vertically and in time in EXP, where condensation occurs over a thicker atmospheric layer between 500 hPa and 300 hPa after 12 UTC. In the REF experiment, the creation of liquid water above 500 hPa only reaches values of 0.3 g/kg sporadically, for example at 21UTC. In the EXP setup, [...]'

l. 289: '[...] keeps unchanged the vertical structure of the q_l profile [...]' $\rightarrow$ '[...] keeps the vertical structure of the q_l profile unchanged [...]'

l. 289-290: 'Liquid water is added where it already existed [...]' $\rightarrow$ 'In REF, liquid water is only added where it already exists [...]'

l. 293: '[...] show similar structures to [...]' $\rightarrow$ '[...] show structures similar to [...]'

l. 296: 'value' instead of 'interest'?

l. 297: 'for assimilating' $\rightarrow$ 'to assimilate'

l. 299: 'over South-Western France' $\rightarrow$ 'in South-Western France'

l. 303: '(in e.g. fronts, [....])'

l. 305: 'significance' instead of 'interest'?

l. 324: 'increment' $\rightarrow$ 'increments'

Figures:

- **Figure 1:** Would it be possible to provide this figure with slightly larger font size?
- **Figure 2:**
  - Do you mean 2 K instead of 0.2 K in the caption?
  - 'Relative Humidity' without capital letters
  - Could you add a description of the arrows (2(b), 2(d)) in the caption?
- **Figure 5:**
  - Please make the numbering of the channels consistent between figure (1-13) and caption (0-12). Would it maybe be possible to even enumerate the channels in the figure after the MWR channel numbers (1-2,4-14 as channel 3 was removed, if I am correct)?
  - Provide units to the color bar.
  - I find it a little difficult to compare the values between panels (a) and (b)/(c) and really point out the magnitude of the reduction of deviations. Would it be possible to use the same range in the color bar of all three panels? Or alternatively at least point out the different scaling in the caption for clarity.
- **Figure 6:** I suggest centering the color map to fit 0 to the change of color between blue and red.