Dear Simone Ceccherini,

Dear Editor,

I am the leading author of the Schneider et al. (2021) manuscript, on which this comment is focusing. I would like to thank Simone Ceccherini for the interest in our work and the Editor for taking into account my thoughts.

The comment of Simone Ceccherini refers exclusively to the theoretical part of our work (the appendix). The text and the equations are in general easy to follow (for dedicated experts with respective background knowledge). In the following, I refer to the line and equation numbers of the LATEXdiff document.

General comments:

I am a bit confused about a comment manuscript focusing exclusively on a paper that is still in revision and that revised work (i.e., already available improvements/clarifications/extensions of the preliminary work) is not considered.

We got detailed comments on the Schneider et al. (2021) manuscript from four different colleagues/referees. The revision of the theoretical part has already been finalized end of March and published as reply to Referee #1 on 29 March (https://doi.org/10.5194/amt-2021-31-AC1). We are currently finalizing revisions, improvements and extensions linked to the comments on using different observational and model data. We hope that the fully revised manuscript can soon be published.

I think that the initiative of Simone Ceccherini for discussing similarities (and differences) of the different data fusion methods can be useful and important. I am also grateful to Simone Ceccherini for providing a helpful referee comment on our paper (https://doi.org/10.5194/amt-2021-31-CC1). However, I am not convinced about the publication of a full manuscript that exclusively comments on work that is still in revision. In my opinion such full manuscript makes sense if it refers to finally revised work (not preliminary work). I know that we are probably taking a lot of time for our final revisions and I would like to apologize for that. However, I can only promise that we do our best in the time we have available.

In case there are constraints that require a very fast publication in AMT of this comment, I strongly suggest commenting on the revised theoretical part of our work, which has already been published in March (as the reply to Referee #1, https://doi.org/10.5194/amt-2021-31-AC1). Due to the discussion with Referee #1, we were able to better clarify when a Kalman filter approach is useful and when not. Furthermore, we put special emphasis on avoiding equations where singular (or potentially singular) matrices have to be inverted and the Simone Ceccherini comment should be adjusted accordingly (e.g. it references in line 52 to an equation that is avoided in our revised work). In the following I list the major modification that I think should be considered before a publication of the Simone Ceccherini comment.

In the revised theoretical part (Appendix A of https://doi.org/10.5194/amt-2021-31-AC1) we distinguish two situations. We start with discussing the combination of two profile retrieval products (see Appendix A2.1). There the application of a Kalman filter is not needed and the combination can be easily achieved

by using the a posteriori covariances of the two individual profile retrievals (see Eq. A11). This completely avoids the problem of a potentially singular retrieval noise matrix. This Eq. (A11) is the same as the second line of Eq. (10) of the Simone Ceccherini comment. So the comment paper could start already with Eq. (10). Then the comment shows the equivalence of Eq. 10 (i.e. our Eq. A11) with Eq. (16), the Complete Data Fusion (CDF) method. However, I do not understand the advantage of Eq. (16), if compared to Eq. (10). Actually Eq. (10) (i.e. our Eq. A11) needs less input than Eq. (16). The first equation needs the retrieval products, the a priori data, the a priori covariances, and the a posteriori covariances. The second equation needs the same input and in addition the two averaging kernels. So, why should one use Eq. (16) instead of Eq. (10)?

In Appendix A2.2 of our revised theoretical part (https://doi.org/10.5194/amt-2021-31-AC1) we discuss the combination of a profile and a column data product. This is the problem, on which our work is focusing. It is only this situation, for which we suggest the application of the Kalman filter approach. We show the large similarity of the Kalman filter approach and a combined retrieval that uses the two individual measurements. This combination of column data with profile data is not captured by the CDF (Complete Data Fusion) method as written in Eq. (16) of Simone Ceccherini's comment, because for a column observation, the profile averaging kernels and profile a posteriori covariances are not readily available (if they can be made available at all), but both are needed in the respective Eq. (16).

In general and if data with different vertical representations (fine and coarse) are going to be combined, the CDF method (Eq. 16 of Simone Cecherini's comment) can only work on the coarse vertical representation. The averaging kernel and the a posteriori covariances of the fine gridded profile can be interpolated to the coarse grid (e.g., von Clarmann and Grabowski, https://doi.org/10.5194/acp-7-397-2007), but not vice versa. This means that when combining a profile product with a column product, the CDF method can (to my understanding) only generate a combined column product. In contrast, our Kalman filter-based approach can combine profile and column data and generate a profile observation that has an improved vertical sensitivity.

Specific comments:

Related to the aforementioned last general comment: when discussing pros/cons of the different methods (end of the conclusion section), it should be mentioned, that our Kalman filter method allows for generating a profile observation with improved vertical sensitivity by combining a profile and a total column observation. This is of large importance for greenhouse gas research, where high precision total column observations are frequently used. To my understanding and according to Eq. 16 of Simone Ceccherini's comment such profile and column data combination is not possible by the CDF method.

Line 27 – 29 (level of equivalency of Warner et al. 2014 and our work): Warner et al. (2014) had the same principle idea as we, i.e. the application of a Kalman filter for combining different satellite sensor observations. However, there are also important differences. (1) Warner et al. (2014) uses horizontal fields measured by AIRS with weak vertical details as the background and focuses on improving the vertical information for this large area by using the detailed vertical information provided very locally by the observations of TES (and MLS). In their method the vertical information comes mainly from TES (or MLS, see their Figs. 6, 7 and 11) and the horizontal information from AIRS. In our method we use two sensors, which both have good horizontal coverage (we do not need an analysis in the horizontal

dimension), but different and rather synergetic vertical sensitivities. In our method we optimally combine the different vertical sensitivities and generate a combined observation that has more detailed vertical information than each of the two individual observations. (2) In Warner et al. (2014) the combination is not made in a fully optimal sense (that would be equivalent to the combined optimal retrieval products). They use a diagonal observational error covariance matrix (**R** in their Eq. 2) and a global statistics for the satellite sensors' noise errors and sensitivities (see their Figs 2 and 3 and the related text). In contrast, our method works with the individual noise errors and sensitivities of exactly the two observations that are combined, i.e. our method is rather similar to a combined optimal estimation retrieval product.

Equation (14): I was not able to get from Eq. (13) to Eq. (14). One has to write out the inverse of Eq. (13), right? However, this means that there is the term $\mathbf{F}_i^{-1}$, which however, does not exist. The impossibility of calculating $\mathbf{F}_i^{-1}$ is actually the reason for imposing the side constraint ($\mathbf{S}_a^{-1}$). Or is there another way to get from Eq. (13) to Eq. (14) that avoids the use of $\mathbf{F}_i^{-1}$?

Best regards,

Matthias Schneider