Dear Reviewer #1,

First of all, we would like to thank you for the positive and very interesting comments that certainly are very helpful to improve the manuscript. Please, find the detailed answers below. The original text of your comments is highlighted in blue color.

Manuscript discusses estimation of errors of aerosol and surface parameters retrieved from remote sensing observation by using covariance matrix of retrieved parameters. This approach is an alternative to more commonly used sensitivity analysis approach which is based on perturbing synthetic measurements and radiative transfer model parameters by assumed biases and then inverting perturbed measurements to observe deviation of retrieved parameters from those used in generation of synthetic observation. The advantage of sensitivity approach is its intuitive simplicity and clarity. Its disadvantage, however, is a long time of error estimation which prevents using of this approach in real time. From other hand, covariance matrix of retrieved parameters is calculated after each inversion which allows error estimation in real time. The diagonal elements of covariance matrix provide standard deviation of retrieved parameters and non-diagonal elements are proportional to correlation coefficients between them. In spite of their rare use, the non-diagonal elements of covariance matrix have a valuable information about retrieval tendencies which can be used for better understanding and improving of information content of measurements.

The main focus of the manuscript is development, testing, validation and applying covariance matrix approach to real observations. Also, manuscript includes the first to my knowledge, analysis of full covariance matrix including non-diagonal elements. The manuscript describes in detail the inversion procedure and error estimated used by GRASP algorithm. It also presents analysis of performance of the approach by example of two types of ground-based measurements: sun photometer measurements of aerosol optical depth and sky radiances and synergy of sun photometer observation and multi wavelength lidar. This is very detailed analysis done by numerical tests using synthetic measurements and perturbing them by random noise and biases. After numerical tests, approach is applied to real observations combination of sun photometer and lidar at Aeroparque and Villa Martelli stations in Buenos Aires, Argentina. Finally, the manuscript describes GRASP error estimates applied to POLDER/GRASP retrievals. Each case considered to illustrate the approach performance is supplemented by analysis of full covariance matrix which allows fast and efficient analysis of information content of the given set of observations.

I believe that the subject of the manuscript is in scope of AMT. It can be published after the following comments will be addressed.

Major comments.

1. **My major comment is related to using linear approximation in errors estimation because forward model can be highly non-linear in vicinity of solution. This issue is discussed in the manuscript and named as a reason for overestimation of errors by 20 to 30%. I wandering whether any specific research was done to better understand the effect of non-linearity of forward model on error estimate? For example, how variation of increment in calculating derivatives can help diminish the effect of non-linearity? I do not require this research to be added to the current version of the manuscript but recommend doing this analysis in the future research. It would be good adding a couple of sentences outlying doing this analysis in perspective.**

Answering to this comment, we would like to note that the numerical tests in synthetic noise were done with the purpose to check how the developed error estimates agree with "actual" errors obtained in the numerical experiment. The results of these tests showed rather good agreement with ~20-30% of overestimation. There could be many different reasons for this result, but we consider that the use of linear approximation is likely the main reason causing this effect. At the same time, the more complex non-linear formalism for error estimation is practically non-existent. Also, we consider that the overestimation can be considered as high boundary for expected errors that is acceptable for our applications since purely numerical experiments tend to provide more accurate retrieval results than it usually seen in practice. Therefore, we concluded that the developed error estimates are satisfactory for our purposes.

Regarding, sensitivity of our error estimates to different settings of the retrieval, specifically, to value of increment in calculating derivatives, we have done some tests and didn't find any clear tendency. Nonetheless, we agree that some sensitivity exists it is possible that our tests were too limited and we plan to do some analysis of this aspect in future.

2. **On page 11 it is stated "Indeed, this optimization makes the iterations converge from given initial guess to git the data even if the basic linear system is singular. Therefore, ones Levenberg-Marquart optimization is used there is an evident dependence on the initial guess that can bias the solution". This is true if not a priori information is used. Underdetermined linear system has multiple solution and minimized quadratic form has wide maximum. In this case there is dependence on initial guess. However, the goal of adding a priori information is to improve condition of linear system for it not being singular. In this case the dependence on initial guess will decrease or didapper. Is this somehow accounted for in including dependence on initial guess in error estimates?**

We fully, agree that a priori information is used in order to make solution unique and if it is added fully adequately no dependence on initial guess should be observed. At the same time, in practice we can state that retrievals when state vector includes large number of unknowns such dependence is often appears in some extent. Moreover, if the retrieval is not optimally set, such dependence can be rather significant while unnoticed because the retrieval continues to converge to local minima thanks to use of Levenberg-Marquart optimization. Therefore, in order to account for such an effect, we have added the Levenberg-Marquardt contribution into the formalism. According to our evaluation, this term is nearly negligible if there is no dependence on initial guess, while increases if such dependence appears. In order to make this consideration clearer for the reader we have added additional explanations in the revised manuscript on the p. 12.

3. **On page 21 it is stated "In this regard, while the retrieval of multi-component is not a part of the standard AERONET inversion, GRASP algorithm allows the retrieval of several aerosol components from dives remote sensing observations including the case of aerosol retrieval from radiometer measurements only". As far as I know, AERONET retrieval code has an option to retrieve several aerosol components. In particular, at the start of the AERONET project the standard product included aerosol parameters for bi-component mixture. Later it was decided to retrieve only one component. Therefore, I always was under impression that information content of sun photometer observation is not sufficient to separate different aerosol types in external mixture. Mostly because of possible correlation between aerosol parameters of different components. Could you please explain, how this separation is achieved in GRASP algorithm? At what conditions or using some additional constraints?**

We are fully agreeing that retrieval of multi-component aerosol from AERONET observation is very uncertain. At the same, some sensitivity to presence of multi-component aerosols exists, especially in some particular situations when two aerosol components have comparable influence on AERONET measurements (the situation used in our tests). Therefore, the possibility of multi-component retrieval and distinguishing the two components is often asked by aerosol community in spite of the earlier demonstration (e.g. in Dubovik et al., 2000). Therefore, we have chosen such approach for illustrative purposes. It seemed a very good case to demonstrate that if constraints are not sufficient the errors can be unacceptably high and correlated. Also, it was a very good case to show that when some estimates are highly uncertain and strongly correlated they still can be used for accurate estimation of their functions. For example, we showed that such property as total SSA of mixed aerosol can be rather accurately obtained from retrieved SSA of fine and coarse modes. We have also illustrated the improvements in multi-component retrieval and error reduction when is included extra information, such as lidar measurements. That retrieval is often used when co-located AERONET and lidar data are inverted together (Lopatin et al., 2013, 2021)

By the way, in the relation to the discussion above, it can be noticed that the retrieval of multi-component aerosol from AERONET is often non-unique and therefore solution depends on initial guess and the L-M. term added in error estimate is useful.

We believe that this is very nice question that can be asked by many readers, therefore we also added a paragraph discussing this aspect, page 22 in the revised manuscript.

Monir comments.

1. **In Eq (30), what exactly bias proxy set means? Is it set of assumed/modeled biases?**

Yes. This means proxy of possible biases. They can come from the knowledge of the measurements, for example, due to calibration uncertainty in AERONET AOD that known at the level of, ~0.01, the bias in AOD can be + or – 0.01.

2. **In EQ. (35), I wandering would it be more correct averaging standard deviations corresponding to + and – biases of the same type prior to the averaging over contribution from different sources of uncertainties? In this case the multiplier in front of sum would be 2N.**

In the Eq. (35), N is equal 2 because N represents, the systematic component after the addition of both, positive and negative bias.