Dear editor,

We want to thank the reviewers for their insightful and constructive comments. We have revised the manuscript in the light of their suggestions. The manuscript now read better and we hope that the modified manuscript would be acceptable to the reviewers. We would like to address comments here point by point. The reviewers’ comments and queries are in black and our responses appear in blue. The line numbers in the response letter refer to the blue coloured mark-up version of the revised manuscript.
Comments from Reviewer: 1

This paper presents and evaluates a methodology to derive liquid water content from w-band radar and microwave radiometer liquid water path retrievals. Using optimal estimation, they attempt to evaluate the widely used power law relationship between radar reflectivity and liquid water content. The paper is reasonably well written but could do with editing for grammar as there were a few grammatical errors (not a big distraction). I noted no technical errors with the retrieval development, implementation, and evaluation. I found the use of synthetic data to be a strong aspect of their work. The case study they present with the tethered balloon data was interesting and shows the limitations of the algorithm and comparison methodology. I was hoping that they would use the tethered balloon and CDP in an actual cloud instead of fog but perhaps the practical aspects of that make it too difficult.

Even though it is well presented, I would say that the paper does not present much new. Radiometer and radar synergistic retrievals on liquid clouds implemented via optimal estimation has been in wide use for some time. I don't particularly see that the present paper goes much beyond what we already know. I think it would have been better to have minimized the algorithm aspects and focus on what can be learned about cloud processes.

We appreciate the reviewer's analysis, however we would like to point out that this article not only proposes a synergistic retrieval using radar-radiometer, but also extends the climatology of the retrieval. This retrieval algorithm is further extended for LWC retrieval using radar information and the climatology. Therefore, this article presents two versions of the algorithm which is applied to radar measurements in both synergy configurations (with a microwave radiometer) and in stand-alone radar configurations. The algorithm is based on the idea of combining a traditional approach to link Z and LWC (i.e., $Z = a LWC^b$) with the knowledge of LWP (when available). The LWP information enables the retrieval of the scaling factor $lna$ (logarithm of prefactor ‘$a$’ from the Z-LWC relation) for each cloud profile in addition to LWC. However, microwave radiometer does not always accompany the radar, and therefore, and independent estimate of LWP might not be available. To develop the retrieval for radar stand-alone version, a variety of cloud cases are selected to estimate LWC and $lna$, and the behaviour of retrieved parameters is then analysed. The idea is to learn from the synergistic retrieval and utilize that knowledge to direct the retrieval when synergy is not possible. The scaling factor $lna$ is retrieved because additional information from the microwave radiometer is assimilated in the retrieval, and such climatology of scaling factor $lna$ has not been developed before, which can be helpful in establishing new retrieval methods for low-level clouds.

Therefore, in the radar stand-alone retrieval, when the microwave radiometer is not accompanying the radar, the radar stand-alone retrieval algorithm utilizes the climatology of the scaling factor $lna$ to constrain the LWC. The climatology of $lna$ as a function of radar parameters is chosen as the a priori of the cloud profile in the algorithm that allows the variability in the LWC retrieval. By utilizing the climatology of the scaling factor, this radar stand-alone method can provide continuous retrieval of LWC for warm clouds even in the absence of radiometer and other additional measurements. Although this climatology is developed using measurements from SIRTA observatory for limited cloud scenarios, a more extensive data collection from several measurement locations might be used to generate a more robust climatology of scaling factor.
Additionally, even if radiometer-radar synergy through optimal estimations has already been implemented in the past, to our knowledge, the use of newly developed 95 GHz cloud radar which differ from commonly used 35 GHz cloud radar by their higher sensitivity to small droplets and combining them with microwave radiometers during fog conditions is quite new in the literature.
Comments from Reviewer: 2

The presented manuscript describes a retrieval technique for LWC which is within the scope of AMT. Even if the approach is not brand new, the paper applies the known retrieval techniques to fog, which is still a relatively unexplored type of cloud. It presents interesting results for the scientific community. The conclusions are relevant for focusing on new research directions and are thus worth publishing, after some major revisions.

The methodologies applied are clearly stated and outlined. I could not find a clear indication regarding the data and code availability, but the methodology can be followed and understood theoretically. The paper is well structured, and the writing is plain and clear, with a concise abstract and a proper title to describe the manuscript's content.

As a general comment, there is a predominance of passive tenses in the text, which I do not recommend using in favor of active sentences to make understanding easier. In addition, I recommend avoiding sentences with too many subordinates, which often recur. The paper is quite long, some parts can be shortened and/or condensed, and some figures can be put together and/or removed.

Specific comments and technical comments:

- I think that the research gap you want to fill needs to be stated more clearly, in the abstract and the introduction. From my understanding, it is that you apply the LWC retrieval to fog and aim to have a method that also works when MWR is not working. You should state these characteristics (or the ones you think are the main ones of your algorithm) clearly when you introduce the work and why it is crucial.

Thank you for the comments. We have updated the abstract and highlighted the key objectives of the algorithm which is as follows:

Line 8-12: “This paper also aims to propose an algorithm configuration that retrieves the LWC of clouds and fog using radar reflectivity and a climatology of the power law parameters. To do so, variations of the scaling factor \( \ln a \) (the logarithm of pre-factor \( a \) from power-law relation) when MWR observations are available are allowed in each cloud profile to build a climatology of the scaling factor \( \ln a \) that can be used when MWR observations are not available. The algorithm also accounts for attenuation due to cloud droplets.”

Furthermore, in the updated manuscript of the article, we added a new section 4.5 demonstrating the benefit of assimilation LWP from MWR in the algorithm which is also highlighted in the abstract.

- I think that the paper is too long and that you have too many (nice) figures. For example, you can merge figure 1 and 3 into a single figure with two subplots. I hardly looked at figure 3. Think if it is really needed. Maybe figure 4 can go in the supplementary material, as well as figure 6? Can you make a single figure of figure 7 and 8? Regarding the paragraphs, can you maybe shorten section 3? The methodology you explain (in detail and very clearly) is well known in literature, so maybe you can just point out what you do differently from the standard theory? These are just some suggestions.
Thank you for the recommendation. We could not merge the figure 1 and 3 as there is a whole section 2 between these two figures. By merging the figure 3 with figure 1, we felt that the continuity of the manuscript would be interrupted, so we decided to keep it separate. By merging figure 7 and 8, the size of figure will be too long. However, we moved figure 4 and 6 to the supplementary section. We have also shortened section 3.

- I need some clarifications on what you call Doppler velocity. If you are talking about mean Doppler velocity, the second moment of the Doppler spectrum, you would need to consider this measurement as a convolution of hydrometeor properties with air motion, turbulence etc. For this reason, I have some difficulties in agreeing with what you wrote on the interpretation of the radar Doppler velocity values throughout the text. See the detailed comments in the text for more.

Thank you for bringing this to our attention. We meant Doppler velocity as the velocity of targets (cloud droplets here) relative to the radar. However, to avoid this misunderstanding, we removed the discussion about velocity in the article.

- It would greatly help the reader to have a table with the radar mode characteristics, in particular min/max range gate, time resolution, Doppler resolution, min/max Doppler range, at least.

As stated this information was indeed missing in the article. Thank you for the suggestion. A table containing BASTA modes, range resolution and respective target application has now been added in the section 2.1 detailing BASTA radar.

- I had some problems understanding your lna. The variable is not properly introduced and only quite late you describe what it really is…Please, introduce it clearly at the beginning, once and for all, and then refer to that definition.

Thank you for the suggestion. We have updated the definition of \( \text{lna} \) in the abstract at line number 10, so that the reader is informed about this parameter well in advance.

- In general, when there is a figure in the text, there is no need to state in the main text of the publication what the figure contains. You should write what readers should see or find in terms of results in the figure, followed by the “(figure n)” in parenthesis. I tried to correct this for every figure, please check.

Thank you for the comment. We have removed such sentences in the main text and detailed the description in the caption of all the figures in the updated manuscript.

- I wonder if you ever considered using the skewness of the Doppler spectra to distinguish drizzle from non-drizzle profiles. I commented on that a couple of times in the text. I am happy to contribute more in this respect, if you think it is an interesting approach.

As stated, drizzle is indeed the challenging part of the LWC retrieval, and we have considered using the skewness of the doppler spectra. But in the context of this article, because the Doppler spectra are not operationally available with the vertically pointing BASTA radar, we decided to concentrate our efforts on developing a method to estimate LWC using radar-microwave radiometer synergy. We foresee utilizing Doppler spectra (and skewness) operationally and making the most of the cloud information from BASTA cloud radar in the future.

The additional comments and technical corrections suggested in the pdf have been incorporated into the updated version of the manuscript.
Comments from Reviewer: 3

General comments

This manuscript combines cloud radar and microwave radiometer observations using a variational framework in order to estimate the liquid water content profile of warm liquid water clouds. Reliable quantification of uncertainties (both instrument and retrieval uncertainties) is a major goal of the profiling community, so the use of optimal estimation in the retrieval methodology is an attractive approach as it enables multiple error sources to be included and provides the uncertainties directly within the retrieval framework.

The manuscript is relatively well written but would benefit from some editing as there are a number of mistakes. The figures are relevant and clear. The explanation of the methodology is clear but could be much more concise in some sections.

A reliable method of retrieving the profile of liquid water content from remote sensing observations is of clear interest to the community. However, the initial assumption that a power law is suitable for deriving liquid water content from radar reflectivity should be examined more closely. The authors try to solve this by varying the exponent a in the power law relationship between radar reflectivity and liquid water content to account for the presence of drizzle, but it seems of more benefit if the authors used this need to modify the exponent as an indication that drizzle is present and that maybe another retrieval method should be used.

Validation of the retrieved profile of liquid water content has historically proved somewhat challenging, with mostly aircraft-based observations being used for validation. Obtaining the vertical profile of liquid water content from aircraft observations usually requires significant averaging in time and space. There is clear potential shown here for validating the retrievals using balloon-borne in situ measurements, and the case study shows that retrievals in non-drizzling clouds match well, whereas those in drizzling situations are not so good. Investigating if these aspects within the retrieval methodology are robust would be novel and of interest. The manuscript requires some major revisions.

Specific comments and questions

• If drizzle is present, then the measured radar reflectivity is essentially responding to the drizzle droplets and not the cloud droplets, hence the wide spread seen in the power law relationships given in the literature. This problem has been discussed previously in numerous articles (which have also been referenced in this manuscript) and nicely summarised in Löhnert et al. (2008, https://doi.org/10.1175/2007JTECHA961.1).

We would like to thank you for highlighting the challenges in retrieving cloud profiles with drizzle. We tried to cover most of the problems associated with drizzling clouds from various references. The suggested reference is also now added in the manuscript as follows:
Line 89-90: “To improve the quality of LWC retrievals in clouds and drizzle, Löhnert et al. (2008) implements a target classification scheme using certain thresholds determined by radar reflectivity and ceilometer extinction profile.”

• One issue with the approach taken in this manuscript is the retrieval of liquid water content profiles for drizzling cloud cases. While there likely is a relationship between the amount of drizzle and the cloud liquid water content or cloud liquid water path, it no longer follows that the shape of the profile of reflectivity in drizzle should necessarily
match the shape of the profile of cloud liquid water content; it will match the shape of the profile of drizzle water content, which can extend far below the liquid cloud base.

The issue is particularly important because a unique scaling factor is assigned for the cloud profile which contains cloud droplets as well as drizzle. We discussed this issue in the conclusion (section 8) of the manuscript as follows:

Line 756-762: “However, drizzle in clouds is a substantial source of error in the retrieval. Because drizzle droplets are significantly larger than cloud droplets, power law may not be applicable in the Mie regime. As a result, the forward model exclusively for drizzle must incorporate Mie scattering or eventually another kind of relationship to link Z and LWC. A prospective work for such cloud columns is planned to separate drizzle and cloud pixels using Doppler velocity information and develop a forward model for drizzle.”

- If the drizzle situations can be identified using the fact that the scaling parameter in the power law relationship has had to be adjusted to match the measured liquid water path, then this does provide additional information. Do the uncertainties in the retrieval increase when the scaling parameter changes?

The variation in $\ln(a)$ gives a clear idea to separate liquid cloud and drizzle profiles. Particularly, when drizzle with cloud droplets co-occur in the profile, the value of scaling factor adapts to constrain the LWP and therefore introduces error in the liquid water content retrievals.

- Lines 61-63: I'm not sure - depends on how you define spectrum shape - its more that any variation in the largest sizes will make much more of a difference to Z than for LWC

We rephrased the sentence as per the suggestion.

Line 69-72: “Thus, a small variation in the larger droplet size strongly influences the Z than LWC, which leads to high uncertainties in estimated LWC profile. Since the cloud droplet size changes significantly within the cloud structure, the retrieval of LWC using only Z information will not be accurate even if the most appropriate empirical relation for the cloud type is used.”

- Lines 118-124: It's not clear which range-resolution mode is used here (highest resolution?), or is it a merged product? If it is the merged product, at what range does the range resolution change? Presumably, although not stated, the range resolution in fog is 12.5 m?

The L2 merged product is used in this study and this is updated in Line 126-127. For the fog the range resolution used is 12.5 m.

- Section 3.1: Much of this section could be condensed considerably and combined with section 3.2

We have condensed and merged the section 3.1 with section 3.2.

- Section 3.3.1: There are a number of recent papers (summarised neatly e.g. in Tridon et al., 2020, https://doi.org/10.5194/amt-2020-159) discussing the uncertainties in models of the attenuation by liquid droplets, particularly with respect to temperature. The radiative model used for generating the gaseous absorption should also be stated and referenced. Is -17 dBZ an appropriate threshold for discriminating between cloud and drizzle droplets? Especially since it would be expected that cloud droplets would dominate the attenuation; drizzle droplets can dominate the reflectivity while contributing negligible amounts of attenuation.
Thank you for suggesting this article reference. We referred this article in the updated version of manuscript at line 264-268 as: “Tridon et al. (2020) compared the attenuation coefficient as a function of temperature using three different models for computing the liquid water refractive index. In this comparison, the attenuation produced by a 1 km thick liquid cloud containing 1 gm$^{-3}$ of liquid water was determined to be around 4 dBkm$^{-1}$ for W(95GHz) band.”

L2 reflectivity is already corrected for gaseous absorption. We use ERA reanalysis profiles of Temperature RH and Pressure extracted above the radar. The model used is from Liebe (1989). However, it is worth noticing that at mid latitudes the gaseous attenuation is not that huge.

Once again, we know that drizzle area is clearly a challenging task for this retrieval. This threshold (-17 dBZ) was used in Vivekanand et al. (2020) for separating cloud and drizzle observed during VOCALS campaign. We also updated this in the manuscript at line 279-281.

- Section 3.5.1: The case study shows an example where the cloud base is not known, particularly during the first four hours of the day (and for the last 30 minutes of the day). The clear presence of drizzle during the first four hours suggests that the cloud base is probably around 800 m at 0000 lowering to 250 m by 0330. Periods from 0430 to 2330 look most appropriate for the retrieval methodology described in this manuscript. Why not include cloud base information from the co-located ceilometer at SIRTA to determine this. Then investigate the different Ina values suggested by the retrieval. Which ones suit drizzle-free periods, and which ones suit drizzling clouds?

Drizzle is present in the first 4 hours and it is LWP that scales the scaling factor. The higher values of scaling factor in the presence of drizzle also supports the range of ‘a’ for drizzle from the Krasnov and Russchenberg (2005). The range of scaling factor can also be used to separate drizzle and drizzle free cloud profiles. As shown in the figure, the thin stratus cloud starting from 4:30 UTC shows the same values of LWP as the drizzling part in the first 4 hours, but the scaling factor is clearly different for these profiles.
Section 5.2: This case study shows the impact of drizzle on the retrievals. Figure 9b shows the heterogeneous nature of the drizzle increasing the reflectivity by significant amounts in some regions. Indeed, the two panels in Figure 10 are very reminiscent of the figures presented in Krasnov and Russchenberg (2002, 2006) and in Löhnert et al. (2008, https://doi.org/10.1175/2007JTECHA961.1) which show two populations with their own relationships, one for drizzle and one for drizzle-free clouds. With very different $Z$-$LWC$ relationships for drizzle and for liquid droplets, it is not surprising that CDP-calculated reflectivities don't always match observed reflectivities. This is quite obvious in Figure 8, where the two measurements for the non-drizzling time period from about 0430 onwards seems to agree very well, but the agreement for the drizzling time period beforehand is not so good.
The problem here is that without the largest droplets the reflectivity of the CDP simulation is higher than the radar. We have clearly shown that the issue is in the fog area and this is not about drizzle.

- Section 6: It may be safer to remove the discussion on vertical velocity relationships. Correcting Doppler velocity for the vertical air motion is a challenge, and without this correction, some of the statements are difficult to corroborate. The typical air motion in low-level liquid clouds can easily exceed +/- 1 m s-1 in turbulent situations, so using Doppler velocity alone to discriminate between drizzle and liquid droplets requires care. Thank for the suggestion, it is also suggested by reviewer 2. We removed the discussion about vertical velocity in the text.

**Technical comments**

- The variable \( \ln a \) needs a clear introduction and description. I assume it is log \( (a) \)? The variable \( \ln a \) (natural log of pre-factor ‘\( a \)’) is defined in abstract in the updated version of manuscript.

- Line 28: Replace 'net radiative forcing in earth's radiation' with 'the net radiative forcing in the Earth's radiation'. We made the suggested change in the updated version. Thank you.

- Line 34: Not sure that fog and haze are always 'disastrous'. We updated these line as, “On the other hand, low visibility phenomena like fog and haze have economic implications in transportation, especially in the aviation sector. Short-range fog forecasts are still inaccurate due to the complexity of fine-scale processes involved in the fog life cycle (Martinet et al., 2020).”

- Line 43: Replace 'longer' with 'larger'. Suggest adding that droplets larger than this size have appreciable terminal velocity and fall out of the cloud, and are termed drizzle droplets. We updated the manuscript as suggested.

- Line 45: Replace 'spectrum and whereas, LWC' with 'spectrum, whereas LWC'. Thank you, replaced as suggested.

- Line 68: Would be clearer if LWC and LWP are defined together. Then line 71 should not start as a new paragraph but follow the sentence introducing the Frisch algorithm. LWC is defined when power law is defined, this is why LWP is defined separately. The paragraph is merged with the paragraph introducing Frisch algorithm.

- Line 128: How many channels in the water vapor absorption band? What is the frequency range? Thank you for bringing our attention to this. There are 7 channels in each band (WV band and oxygen band) and updated in the manuscript on line 142-143.

- Line 76: Should state why the presence of drizzle causes problems for the retrievals (a few drizzle droplets dominate the reflectivity without contributing much to LWC). Thank you for the suggestion. We added this sentence at line number 84-86.
• Line 80: There are some LWC profile retrievals in the literature that are applicable to both precipitating and non-precipitating clouds, although they may have their own drawbacks. It's worth stating here that the issues for fog retrievals have historically been due to the cloud radar blind zone, which can now be mitigated for FMCW radars.
We updated the manuscript with the suggested information at line 90-93.

• Lines 112-113: Suggest using 'range-resolution modes' rather than 'resolution modes' both here and elsewhere in this paragraph. Include the minimum range for the 12.5 m range-resolution mode.
We replaced the 'resolution modes’ with range-resolution modes throughout the updated manuscript.

• Line 128: How many channels in the water vapor absorption band? What is the frequency range?
The update has been made as follows.
Line 141-143. “HATPRO MWR is a passive instrument, converting the naturally emitted downwelling radiative energy emitted from the atmosphere within two spectral bands with seven channel each: the first one focuses on the water vapor absorption band (22.24–31 GHz) while the second one is centered on the 60 GHz oxygen complex band (51–59 GHz).”

• Line 137: The statement starting on this line is not strictly true, suggest revising.
We have changed the sentence as follows:
Line 146: “MWRs are sensitive to the total liquid water content in the cloud column.”

• Line 141: For a column containing a single liquid layer, MWR provides the LWP for the cloud layer.
Thank you for the suggestion, we replaced the previous sentence with the suggested one.

Line 145: This uncertainty is also due to uncertainty in the microwave radiative transfer model.
We added this sentence at line 161.

• Table 2: How many size bins does the CDP have?
We added the information as follows:
Line 166: “The sampling rate of CDP was 10 sec and had 50 size bins each with 1 µm resolution during SOFOG-3D campaign.”

• Lines 172-174. Sentence needs revising
We rephrased the sentence as:
Line 195: “Figure 1 illustrates how the input parameters (Z and LWP) are used to retrieve the output variables (LWC and Ina).”

• Line 264: To be consistent, can use the same size limit as stated in line 44.
Thank you so much, we have updated the size limit.

• Lines 288-289: This statement is not strictly true. Some of the liquid water attenuation estimates were calculated for a wide range of liquid cloud microphysical properties.
We removed the sentence to avoid the ambiguity.
• Lines 327-329: Why choose a hard limit of 2.5 km? Why not use a temperature profile (e.g. from MWR, NWP model, or nearby radiosonde) to select an appropriate freezing level for each day, since you state later on in the paragraph that it changes from day to day.

We rephrased the sentence as follows:
Line 326-328: “In this data set, majority of the liquid clouds are observed below 2500 m. We selected the cloud cases where cloud height remained below 2500 m as the clouds above are anticipated to be mixed phase or ice clouds.”

• Lines 519-521: This statement is not true, calculations for all types of liquid clouds have been examined in the literature.

Thank you for highlighting this, we removed the sentence.

• Lines 574-575: Is this true if the CDP is limited to sizes less than 50 microns? A few drizzle droplets (e.g. 100-500 microns in diameter) will immediately increase the radar reflectivity far above that calculated from the CDP.

Yes, this is true, we rephrased the sentence for clarity.
Line 580-583: “If a well-calibrated radar is sampling the same cloud column and has a similar sensitivity to DSDs, the in-situ reflectivity estimate should match the radar reflectivity. However, the sensitivity of the CDP sensor is limited to sampling the droplet diameters from 2 to 50 µm, while radar can sample a wider range of DSDs and is more sensitive to the largest droplets.”

• Lines 583-584: The standard term for PPI is Plan Position Indicator.

Thank you for the correction. We corrected the term in the updated manuscript.

• Lines 772-779: Most liquid clouds, by their very nature, are unlikely to be homogeneous in the sense suggested as suitable here. Maybe a more statistical approach is necessary for some aspects of the retrieval comparisons.

Thank you for the sentence suggestion, the update has been made at line 775.

• References: Some references are not complete.

Updated and completed the references.