Supplement of "Evaluation of the quality of a UAV-based eddy covariance system for measurements of wind and turbulent flux"

Yibo Sun[1], Xinwen Lin[4], Bing Geng[5], Bo Liu[1,2,3], Shennan Ji[1,2,3]

[1]State Key Laboratory of Environmental Criteria and Risk Assessment, Chinese Research Academy of Environmental Sciences, Beijing 100012, China.
[2]Institute of Ecology, Chinese Research Academy of Environmental Sciences, Beijing 100012, China.
[3]State Environmental Protection Key Laboratory of Ecological Regional Processes and Functions Assessment, Beijing 100012, China.
[4]Collage of Geography and Environment Science, Zhejiang Normal University, Zhejiang 321004, China
[5]Beijing Academy of Social Sciences, 100101, Beijing, China.

Correspondence to: Yibo Sun (sunyb68@163.com)

The intention of this supplement is to guide the readers through the relevant equations about geo-referenced wind calculation, and to provide the procedures in calibration the mounting misalignment using data from 'box' maneuver. Section 1 provides the formulas necessary to compute the geo-referenced 3D wind vector. Section 2 provides the procedure and results for calibrating the mounting misalignment based on 'box' flight maneuver. References to literature are given at the end of the document.

1 Detailed equations for calculation of the geo-referenced 3D wind vector

Wind measurement by aircraft is challenging. The wind measurement components of the UAV-based EC system consist of sensors that measure air pressure (static and dynamic pressure), air temperature, and aircraft attitude, position, velocity, and angular velocity. From these measurements, two velocity vectors U_a (velocity of the air with respect to the aircraft) and U_p (velocity of the aircraft with respect to the Earth) are derived. The velocity of the wind with respect to the earth U (i.e., geo-referenced wind vector) is the result of adding these two vectors together, as:

$$U = U_a + U_p \tag{S1}$$

For a fixed-wing aircraft, approaches to compute the geo-referenced wind vector based on the combination of a multi-hole probe and navigation system are often similar in principle. This text provides the basic formulas necessary to compute the geo-referenced wind vector measured from an aircraft. Detailed information about airborne wind measurement is found in the literature (e.g., Crawford and Dobosy (1992); Williams and Marcotte (2000); Khelif et al. (1999); Metzger et al. (2011)). Figure S1 illustrates the transformational relation for the geo-referenced wind vector calculation used by the current UAV-based EC system.

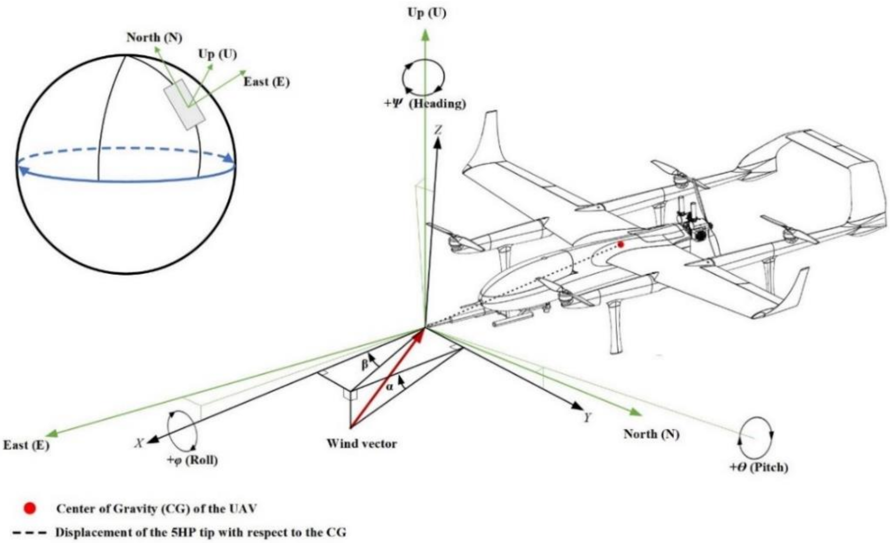

Figure S1. Diagram of the coordination transformational relation for geo-referenced wind vector calculation. The green coordinate represents the geo-referenced coordinate system. The black coordinate represents the aircraft coordinate system.

Two coordinate systems are involved in the calculation of the geo-referenced wind vector: aircraft coordinate system (black in Figure S1, *X*, positive forward, *Y*, to port, and *Z*, toward the airplane's roof) and geographic coordinate system (green in Fig. S1, *E*, positive eastward, *N*, northward, and *U*, upward). A transformation matrix, which is defined by measurements of the three conventional attitude angles: roll (φ), pitch (θ), and heading (ψ), accomplished rotation from the aircraft to the geographic coordinate. They must be applied in the following order: roll, pitch, and heading to convert from aircraft coordinate to geographic coordinate, and yaw, pitch, roll to convert the other way. The probe need not be at the origin of coordinate system. Based on the basic aircraft kinematics and the wind equation given by Lenschow (1986), considering the influence of tangential velocity of rotation on the probe tip, the full expression to compute the geo-referenced wind by aircraft can be expressed as:

$$U(t) = G(t)\widehat{U}_a(t) + U_p(t) + \Omega_p(t) \times R_p \tag{S2}$$

The unadorned symbols in Eq. (A2) are in geographic coordinate. The aircraft's coordinate is denoted by ('^'). *G* is the transformation matrix. Ω_p is the angular rate of the aircraft. The components of $\widehat{U}_a$ are measured with the 5HP mounted on the nose of the UAV, usually extended on the forward part of the aircraft to reduce the measurable effects of the airflow distortion by the wing. The components of G, U_p and Ω_p are obtained from the integrated navigation system (INS) outputs, which originate from the center of gravity (CG) of the UAV. R_p is the vector distance from the CG of the UAV to the 5HP tip.

The rotation matrix *G* from airplane to earth coordinates is computed in factored form by sequentially removing the dependence of the observed data on roll, pitch, and heading together with a relabeling of axes. The first rotation $T_1(\varphi)$ removes roll: it is a rotation about the *X*-axes with matrix representation:

$$T_1(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \tag{S3}$$

The next rotation $T_2(\theta)$ removes pitch:

$$T_2(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \tag{S4}$$

The last rotation $T_3(\psi)$ removes heading:

$$T_3(\psi) = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{S5}$$

The present coordinates of the hypothetical aircraft frame point north, east, and down, and must be transformed to east, north and up. This is done with the permutation T_4:

$$T_4 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{S6}$$

Then, the total transformation *G* between the frames is the matrix product:

$$G = T_4\, T_3(\psi) T_2(\theta) T_1(\phi) = \begin{bmatrix} sin\psi cos\theta & cos\psi cos\varphi + sin\psi sin\theta sin\varphi & sin\psi sin\theta cos\varphi - cos\psi sin\varphi \\ cos\psi cos\theta & -sin\psi cos\varphi + cos\psi sin\theta sin\varphi & sin\psi sin\varphi + cos\psi sin\theta cos\varphi \\ sin\theta & -cos\theta sin\varphi & cos\theta cos\varphi \end{bmatrix} \tag{S7}$$

In addition, offset corrections ($\varepsilon_\varphi, \varepsilon_\theta, \varepsilon_\psi$) are introduced here to correct for possible misalignment of the CG to the probe's relative wind sensors. The generally small values of these correction constants are determined via dedicated flight maneuvers (in Text 2). Due to the offset in φ had an insignificant effect on the computed wind speed (Van Den Kroonenberg et al., 2008), therefore, ε_φ was not included in the calibration and was set to 0. Then, the three-rotation angle could be expressed as:

$$\begin{cases} \varphi = \varphi_i \\ \theta = \theta_i + \varepsilon_\theta \\ \psi = \psi_i + \varepsilon_\psi \end{cases} \tag{S8}$$

where φ_i, θ_i, and ε_ψ are the INS measured attitude angle.

The cross-product term ($\Omega_p \times R_p$) in Eq. (S2) describe the "lever arm" effect due to the tip of the 5HP not being placed at the CG of the UAV, and all are defined in earth coordinates. In the UAV, the displacement of the 5HP tip with respect to the CG of the UAV along the *X*-axis (*L* = 1.459 m) is larger than the displacement along the *Y*-axis (0 m), and *Z*-axis (0.173 m), so that the lateral and vertical separation distances can be negligible. Then, R_p can be expressed as:

$$R_p = G(t) \cdot \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Lcos\psi cos\theta \\ Lsin\psi cos\theta \\ -Lsin\theta \end{bmatrix} \tag{S9}$$

Similarly, Ω_p can be expressed as:

$$\Omega_p = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} cos\psi cos\theta & -sin\psi & cos\psi sin\theta \\ sin\psi cos\theta & cos\psi & sin\psi sin\theta \\ -sin\theta & 0 & cos\theta \end{bmatrix} \cdot \begin{bmatrix} \dot{\varphi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot{\theta} sin\psi + \dot{\varphi} cos\psi cos\theta \\ \dot{\theta} cos\psi + \dot{\varphi} sin\psi cos\theta \\ \dot{\psi} - \dot{\varphi} sin\theta \end{bmatrix} \tag{S10}$$

Thus,

$$\Omega_p \times R_p = T_4 L \begin{bmatrix} -\dot{\theta} sin\theta cos\psi - \dot{\psi} sin\psi cos\theta \\ \dot{\psi} cos\psi cos\theta - \dot{\theta} sin\psi sin\theta \\ -\dot{\theta} cos\theta \end{bmatrix} \tag{S11}$$

Then, converting to a meteorological and inertial navigation frame of reference:

$$\Omega_p \times R_p = L \begin{bmatrix} \dot{\psi} cos\psi cos\theta - \dot{\theta} sin\psi sin\theta \\ -\dot{\theta} sin\theta cos\psi - \dot{\psi} sin\psi cos\theta \\ \dot{\theta} cos\theta \end{bmatrix} \tag{S12}$$

where $\dot{\psi}$ and $\dot{\theta}$ are the angular velocity of heading (ψ) and pitch (θ) angle. The air velocity component ($\widehat{U}_a$) with respect to the aircraft is measured by 5HP. Generally, wind measurements by aircraft are subject to flow distortion and needed to be corrected. According to Vellinga et al. (2013) and Crawford et al. (1996), considering the influence of lift-induced upwash, the wind components with respect to the aircraft ($\hat{u}_a, \hat{v}_a, \widehat{w}_a$), can be calculated as:

$$\widehat{U}_a = \begin{bmatrix} \hat{u}_a \\ \hat{v}_a \\ \widehat{w}_a \end{bmatrix} = \frac{|U_a|}{D} \begin{bmatrix} -1 \\ tan\beta \\ tan\alpha \end{bmatrix} + w_u \begin{bmatrix} sin\chi \\ 0 \\ -cos\chi \end{bmatrix} \tag{S13}$$

$$D = (1 + tan^2\,\alpha + tan^2\,\beta)^{1/2} \tag{S14}$$

where $|U_a|$ is the magnitude of the true airspeed, α is the angle of attack (the airstream with respect to the aircraft in the aircraft's vertical plane, with positive in the downward direction), β is the angle of sideslip (the angle of the airstream with respect to the aircraft in the aircraft's horizontal plane, with clockwise positive rotation), w_u is the vortex's tangential velocity experienced at the probe tip (i.e., lift-induced upwash), and χ is the vertical separation angle probe to wing (Vellinga et al., 2013). For the UAV applications, the influence of flow distortion effects always be ignored because the probe is long enough to avoid the influence of the flow distortion. For the measurement of true airspeed, the details of converting the pressures (static and dynamic) and total air temperature measured by the 5-hole probe (5HP) to the magnitude of the relative true airspeed were given in Sun et al. (2021). Lastly, integrating the equations above, the final geo-referenced wind vectors (u, v, w) are:

$$u = u_p - |U_a|D^{-1}[sin\psi cos\theta + tan\beta(cos\psi cos\phi + sin\psi sin\theta sin\phi) + tan\alpha(sin\psi sin\theta cos\phi - cos\psi sin\phi)] - L(\dot{\theta} sin\theta sin\psi - \dot{\psi} cos\psi cos\theta) \quad (S15)$$

$$v = v_p - |U_a|D^{-1}[cos\psi cos\theta - tan\beta(sin\psi cos\phi - cos\psi sin\theta sin\phi) + tan\alpha(cos\psi sin\theta cos\phi + sin\psi sin\phi)] - L(\dot{\psi} sin\psi cos\theta + \dot{\theta} cos\psi sin\theta) \quad (S16)$$

$$w = w_p - |U_a|D^{-1}[sin\theta - tan\beta cos\theta sin\phi - tan\alpha cos\theta cos\phi] + L\dot{\theta} cos\theta \quad (S17)$$

The last term on the right-hand of Eqs. S15-S17 is the leverage effect correction term.

2 Calibration results of the ‘box’ flight maneuver

In our calibration flight campaign, the first ‘box’ flight maneuver was used to correct the mounting misalignment in heading (ϵ_ψ) and pitch (ϵ_θ) angles between the 5HP and the CG of the UAV. The offset in roll angle (ε_φ) was not included in the calibration and was set to 0° since its influence on the wind calculation is minimal. The detailed procedure for acquiring the calibration parameter ϵ_ψ and ϵ_θ are given in Vellinga et al. (2013) and Sun et al. (2021). The calibration of the UAV-based EC system should occur in ideal atmospheric conditions, i.e., a constant mean horizontal wind component, near zero mean vertical wind. During the calibration, only the data from the straight sections of the ‘box’ flight maneuver were used. The calibration values ϵ_ψ and ϵ_θ were both determined iteratively until their values reached a steady state.

Before calibration, ϵ_ψ and ϵ_θ were set to their default value (0º). The offset ϵ_θ was first calibrated. The value of ϵ_θ was set to vary within the typical range of $\pm 1°$, and the mean vertical wind component ($\overline{w}$) was iteratively calculated using a step length of 0.2° to find the value of ϵ_θ for which $\overline{w}$ is zero. The individual straight sections of the ‘box’ maneuver are used. The results are shown in Figure S2. The average offset was calculated and served as the final value used in Eq. 8. The final iterative step resulted in an offset of -0.183º for ϵ_θ.

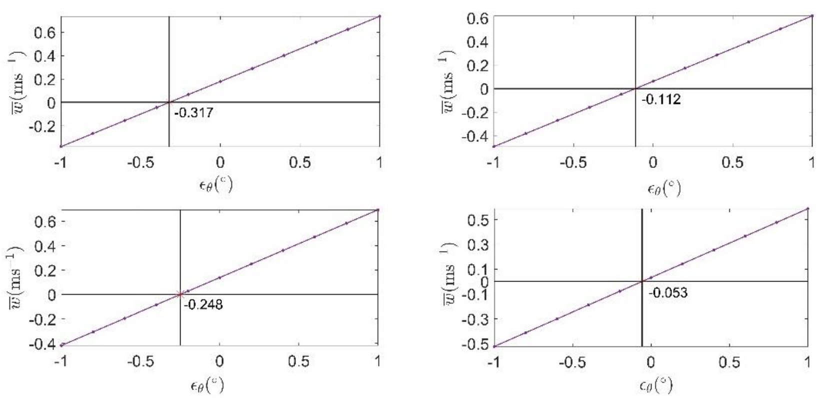

Figure S2. Offset values (ϵ_θ) in the pitch angle corresponding to the zero-averaged value of the vertical wind component ($\overline{w} = 0$). The final offset value (ϵ_θ) in the pitch angle was calculated by averaging the determined offset value from the individual straight sections of the ‘box’ maneuver.

Next, the offset ϵ_ψ was calibrated by setting the possible value to vary within the range between 0º and 4º and by iteratively calculating the horizontal wind speed using a step length of 0.5º. The final offset ϵ_ψ was determined from the straight sections of the ‘box’ maneuver by finding the minimum variances for horizontal wind direction ($\sigma_{U_{dir}}$) and wind speed velocity (σ_U). Figure S3 shows the results for ϵ_ψ from the final iterative session of the calibration. Figure S3b shows that $\sigma_{U_{dir}}$ and σ_U reach their minima at the offset value of 1.822º and 2.178º, respectively. The final offset value ϵ_ψ is determined as their average of 2º.

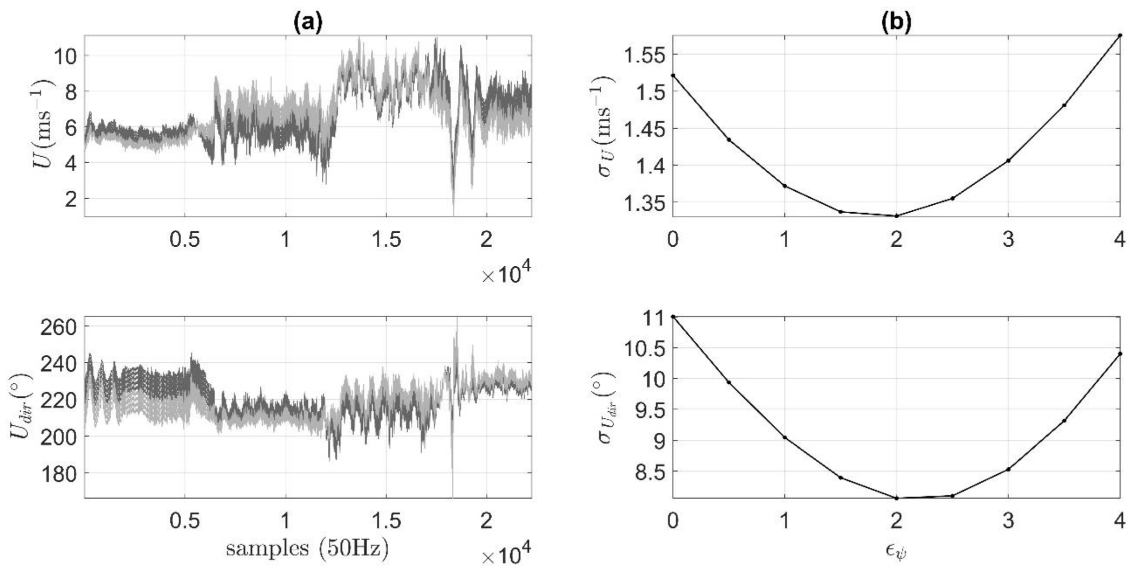

Figure S3. Offset values (ϵ_θ) in the pitch angle corresponding to the zero-averaged value of the vertical wind component ($\bar{w} = 0$). The final offset value (ϵ_θ) in the pitch angle was calculated by averaging the determined offset value from the individual straight sections of the ‘box’ maneuver.

References

Crawford, T. L. and Dobosy, R. J.: A sensitive fast-response probe to measure turbulence and heat flux from any airplane, Boundary-Layer Meteorology, 59, 257-278, 10.1007/BF00119816, 1992.

Khelif, D., Burns, S. P., and Friehe, C. A.: Improved Wind Measurements on Research Aircraft, Journal of Atmospheric and Oceanic Technology, 16, 860-875, 10.1175/1520-0426(1999)016<0860:IWMORA>2.0.CO;2, 1999.

Lenschow, D. H.: Aircraft Measurements in the Boundary Layer, in: Probing the Atmospheric Boundary Layer, edited by: Lenschow, D. H., American Meteorological Society, Boston, MA, Boston, https://doi.org/10.1007/978-1-944970-14-7_5, 1986.

Metzger, S., Junkermann, W., Butterbach-Bahl, K., Schmid, H. P., and Foken, T.: Measuring the 3-D wind vector with a weight-shift microlight aircraft, Atmos. Meas. Tech., 4, 1421-1444, 10.5194/amt-4-1421-2011, 2011.

Sun, Y., Ma, J., Sude, B., Lin, X., Shang, H., Geng, B., Diao, Z., Du, J., and Quan, Z.: A UAV-Based Eddy Covariance System for Measurement of Mass and Energy Exchange of the Ecosystem: Preliminary Results, Sensors, 21, 10.3390/s21020403, 2021.

van den Kroonenberg, A., Martin, T., Buschmann, M., Bange, J., and Vörsmann, P.: Measuring the Wind Vector Using the Autonomous Mini Aerial Vehicle M2AV, Journal of Atmospheric and Oceanic Technology, 25, 1969-1982, 10.1175/2008JTECHA1114.1, 2008.

Vellinga, O. S., Dobosy, R. J., Dumas, E. J., Gioli, B., Elbers, J. A., and Hutjes, R. W. A.: Calibration and Quality Assurance of Flux Observations from a Small Research Aircraft*, Journal of Atmospheric and Oceanic Technology, 30, 161-181, 10.1175/JTECH-D-11-00138.1, 2013.

Williams, A. and Marcotte, D.: Wind Measurements on a Maneuvering Twin-Engine Turboprop Aircraft Accounting for Flow Distortion, Journal of Atmospheric and Oceanic Technology, 17, 795-810, 10.1175/1520-0426(2000)017<0795:WMOAMT>2.0.CO;2, 2000.