Response to reviewers for manuscript AMT-2022-74: **Average visibility that has been miscalculated**

We appreciate the editorial team and the reviewers for their time and comments towards improving our manuscript. Considering the relevance of two reviewers' comments, we respond to all of the points together below.

Anonymous Referee #1

The authors proved that the algorithm previously used to estimate average visibility is incorrect, and it causes the problems in previous studies related to reliability and credibility. I don't think it is a 'correct' method to calculate the average of the visibility and other methods in the previous studies were incorrect. In this study, I wonder what the definition of the terms 'correct' and 'incorrect' is. I prefer to utilize the term 'an improved method.' In my opinion, it is one of the scientific methods for investigating the natural world instead of the absolutely correct way to estimate the average visibility.

Anonymous Referee #2

This study claims to provide correct algorithm for calculating average visibility. The authors also argued that the methods used in previous studies were not correct.

First of all, I don't agree with the title of the paper. Saying "Average visibility that has been miscalculated" is completely wrong. There is no evidence that previous studies miscalculated visibility, so the paper's title is misleading. I agree with reviewer 1, making the claim that this study provides a correct method is not appropriate, while it may be one of the methods to calculate visibility.

The message of the current manuscript is misleading, therefore it should not be considered for publication. The authors should rewrite the manuscript by proposing their algorithm as one of the methods of calculating visibility. However, again it is a question of how reliable the proposed method is. If the authors take the issue raised carefully and resubmit the manuscript, I can review it again for any possibility of publication in the AMT.

General response:

For ease of discussion, we have grouped the two reviewers' comments into three questions to reply, in which the responses to questions 1 and 2 are more focused on the comments of Reviewer 1, and the responses to questions 2 and 3 are more focused on the comments of Reviewer 2.

1. Question 1: Is this a change from "incorrect" to "correct" or a "general improvement"?

1. Response 1: This is a very important question, because the answer to this question is directly related to the evaluation of the value of this manuscript.

First of all, we agree that there may be no absolute correctness in the world in an absolute sense, and therefore all research work can only be "improvements" rather than changes from "incorrect" to "correct". However, people do not discuss issues in an absolute sense in specific work, otherwise the word "correct" would have no value. We believe that some improvements can be called changes from "incorrect" to "correct"

according to the content of improvements, while some are just general improvements. In order to clarify the difference between the two, we will start with an example for a detailed explanation.

Example: A car is travelling on a road. The average speed of the car is measured to be v_1, v_2 and v_3 when travelling uphill, on a flat road and downhill respectively. What is the average speed of the car ($\bar{v}$)?

Student A first proposed the first method to calculate the average speed, as shown in Eq. 1.

$$\bar{v} = \frac{v_1 + v_2 + v_3}{3} \tag{1}$$

Student B thought that the measurement error of the speed of the car is related to the slope and should be corrected. Therefore, student B suggested that the average speed should be calculated using Eq. 2, where c_1, c_2 and c_3 are the correction factors.

$$\bar{v} = \frac{c_1 v_1 + c_2 v_2 + c_3 v_3}{3} \tag{2}$$

Student C thought that student A had misunderstood the concept of speed, and that the correct average speed should be calculated by dividing the total distance travelled by the time taken, as shown in Eq. 3, where t_1, t_2 and t_3 correspond to the times the car runs at speeds of v_1, v_2 and v_3 respectively, and t is the total running time of the car.

$$\bar{v} = \frac{t_1 v_1 + t_2 v_2 + t_3 v_3}{t} = \frac{t_1}{t} v_1 + \frac{t_2}{t} v_2 + \frac{t_3}{t} v_3 \tag{3}$$

We think that Eq. 2 and Eq. 3 are both improvements to Eq. 1, but Eq. 2 is only a general improvement, whereas Eq. 3 is an improvement from "incorrect" to "correct". This is because the improvement of Eq. 3 corrects the misunderstanding of the concept of the average speed in Eq. 1 and clearly points out the cause of the error, that is, the "weight" of the values should be considered when calculating the average value. However, the improvement of Eq. 2 does not improve the perception of the concept and is a technical improvement.

The improvement of the proposed algorithm for average visibility to the old algorithm is identical in nature to the improvement of Eq. 3 to Eq. 1. The proposed algorithm is derived considering the "weight" of the values when calculating the average visibility, whereas the old one does not. This improvement is not a technical one, but rather a cognitive one, and we therefore consider our improvement a change from "incorrect" to "correct".

2. Question 2: Why do you think that the new algorithm is "correct" and the old one is "incorrect"?

2. Respond 2: We have presented a rigorous proof in the manuscript. Here we use an extreme example to illustrate why the new algorithm is "correct" and the old one is "incorrect".

Suppose there are two kinds of boxes of the same volume, box A is filled with gases and aerosols with a horizontal visibility of v, and box B is a perfect vacuum so that the

visibility is infinite. We mix uniformly a certain number of boxes A with boxes B, and then calculate the average visibility after mixing using the new algorithm and the old one, respectively, the results of which are given in Table R1 and Table R2.

First, we mix one box B with a different number of boxes A. The average visibility calculated using the new algorithm and the old algorithm is given in Table R1. It can be seen from Table R1 that as the number of boxes A increases, the average visibility after mixing calculated by the new algorithm gradually converges to the visibility of box A, while the average visibility calculated by the old algorithm is always infinite. Then, we mix one box A with a different number of boxes B. The average visibility calculated by two algorithms is given in Table R2. It can be seen from Table R2 that as the number of boxes B increases, the average visibility calculated by the new algorithm gradually converges to the visibility of box B, while the average visibility calculated by the old algorithm remains infinite. Clearly, the results calculated by the new algorithm are more reasonable than the results of the old algorithm. The difference between the old and new algorithms is essentially a matter of the weight of the values of observed visibility data. The visibility is determined by the extinction coefficient of the medium through which the light propagates, so the weight should match the extinction coefficient of the medium when calculating the average of visibility data. Large weighting factors should be given to the relatively small visibility values corresponding to the large extinction coefficient. But the old algorithm is the opposite, giving large weighting factors to those large visibility data corresponding to relatively small extinction coefficients.

Table R1. The average visibility calculated by the new algorithm and the old one when one box B is mixed with a different number of boxes A.

Number of box A	Average visibility calculated by the new algorithm	Average visibility calculated by the old algorithm
1	v /2	+∞
2	2 v/3	+∞
3	3 v/4	+∞
4	4 v/5	+∞
…	…	…
n	nv / (n+1)	+∞

Table R2. The average visibility calculated by the new algorithm and the old one when one box A is mixed with a different number of boxes B.

Number of box B	Average visibility calculated by the new algorithm	Average visibility calculated by the old algorithm
1	2v	+∞
2	3v	+∞
3	4v	+∞
4	5v	+∞
…	…	…
n	nv	+∞

3. Question 3: Discussion of the relationship between the evidence and the conclusion.
3. Response 3: The argument that "there is no evidence that previous studies miscalculated visibility" does not lead to the conclusion that the algorithm for calculating the average visibility in the past is correct, nor to the conclusion that the title of the manuscript is misleading. This is because in many cases people only look for evidence when they realize that there exists a problem. A well-known example is that before Galileo, it was a common belief that "heavier objects fell faster than lighter ones". No one could give conclusive evidence denying the above conclusion at that time until Galileo's thought experiment.
Returning to the issue of the algorithm for average visibility in this manuscript, we think that we should not decide that the old algorithm is correct and then come to reject the new algorithm from the start, but rather should look at the process of proving the algorithm to determine which is correct. However, the commonly used old algorithm has not been rigorously verified, which probably has been neglected in past research. Instead, we not only present the new algorithm for average visibility, but also prove that the new algorithm is correct and the old one is incorrect. The rigorous proof is presented in the manuscript. In brief, the weight should be considered when calculating the average. The visibility is determined by the extinction coefficient of the medium through which the light propagates. Therefore, the weight should match the extinction coefficient of the medium when calculating the average of visibility data. The answers to Question 1 and Question 2 in this response can help to understand the difference between the old and new algorithms, i.e., the new algorithm considers the weight of the values of observed visibility data, whereas the old one does not. If we cannot find a problem in the process of proving, we should conclude that the new algorithm and the old algorithm cannot be correct at the same time, and the new algorithm is the correct one.

To summarize, neither of the two reviewers denied the significance of discussing the algorithm for average visibility, and did not raise any objections to the proof process of the new algorithm in the referee comments. In other words, the two reviewers did not object to the manuscript at a substantive level, but actually expressed doubts about the conclusions of the manuscript out of caution or habitual thinking. We hope that this response will dispel the doubts of the two reviewers.